PLANNING FOR THE UNEXPECTED: THE VALUE OF RESERVE CAPACITY FOR PUBLIC TRANSPORT NETWORK ROBUSTNESS

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ABSTRACT

Public transport networks (PTN) are subject to recurring service disruptions. Most studies of the robustness of PTN have focused on network topology and considered vulnerability in terms of connectivity reliability. While these studies provide insights on general design principles, there is lack of knowledge concerning the effectiveness of different strategies to reduce the impacts of disruptions. This paper proposes and demonstrates a methodology for evaluating the effectiveness of a strategic increase in capacity on alternative PTN links to mitigate the impact of unexpected network disruptions. The evaluation approach consists of two stages: identifying a set of important links and then for each identified important link, a set of capacity enhancement schemes is evaluated. The proposed method integrates stochastic supply and demand models, dynamic route choice and limited operational capacity. This dynamic agent-based modelling of network performance enables to capture cascading network effects as well as the adaptive redistribution of passenger flows. An application for the rapid PTN of Stockholm, Sweden, demonstrates how the proposed method could be applied to sequentially designed scenarios based on their performance indicators. The method presented in this paper could support policy makers and operators in prioritizing measures to increase network robustness by improving system capacity to absorb unexpected disruptions.

Keywords: Robustness, Public Transport, Disruption, Dynamic Assignment, Capacity, Mitigation

1. INTRODUCTION

A shift of travel from personal cars to public transport is generally seen as one of the most important means of meeting the challenges of congestion, noise and emissions facing many urban areas (e.g. European Council, 2006). One significant barrier towards this goal is the perceived unreliability of public transport services (Friman et al., 2001; Beirão and Sarsfield Cabral, 2007). From a traveller's perspective, reliability implies that travel times (and other travel conditions such as crowdedness) should be predictable and consistent from day to day. Furthermore, *robustness* implies that the performance of the system (in terms of travel times etc.) should be able to withstand or quickly recover from disturbances such as infrastructural and vehicular malfunctions and planned maintenance closures. Although variability in travel conditions arises also from normal fluctuations, it is expected that a more robust public transport system leads to fewer negative experiences and greater service satisfaction for travellers, and lower costs for society.

Public transport networks (PTN) are characterized by services superimposed on physical networks of roads and railways. The importance of transfers, multimodality, transport hubs and intermediate walking links means that PTN are relatively complicated systems; however, the connectivity is in general low compared to the personal car alternative, and the level of service varies non-continuously in time. Furthermore, successful operations are dependent on the availability of staff (e.g., drivers), electric power, telecommunications, vehicles and so on. All together, these factors imply that PTN are less flexible and hence possibly less robust than travelling in personal cars. At the same time, the multimodality of PTN can potentially allow alternative modes to provide reserve capacity in case of failures. There are also greater possibilities for coordinated restoration and mitigation efforts in case a disruption occurs.

Disruptions of PTN may be caused by degradations of the technical or physical infrastructure, for example electrical failures or malfunctioning vehicles, but can also arise from degradations of the services, for example crew strikes or accidents. Compared with road networks, service disruptions in the PTN are prone to have wider direct implications. In addition to the immediate effect on the network links directly concerned, the dynamic nature of public transport supply results in impacts on service availability and capacity further downstream. Depending on its duration, a disruption may even have consequences on service availability upstream and spread to other lines due to its impact on vehicle scheduling. The impacts of service disruptions depend on local crowding levels as well as on how travellers react to changes in supply. PTN connectivity determines the availability of alternative routes in case of disruptions.

Regarding the robustness of PTN, most studies have focused on network topology and how degradations of physical infrastructure links in a particular modal network – typically underground networks – affect connectivity and distances between stations (Angeloudis and Fisk, 2006; Criado et al., 2007). von Ferber et al. (2009, 2012) examined the impact of random and targeted attacks on network performance where the latter were defined based on network centrality measures. Graph theory principles were used in order to compute the largest connected sub-network when a growing share of links fails. They concluded that network connectivity in terms of the size of the largest connected subset is more sensitive to disruptions based on betweenness centrality than on node degree. The same conclusion was reached when vulnerability was defined in terms of the speed in which the system becomes fragmented (Colak et al., 2010).

PTN worldwide differ with respect to the extent they can absorb various attack scenarios (Derrible and Kennedy, 2010; von Ferber et al., 2012). Derrible and Kennedy (2010) suggested that robustness of metro systems corresponds to the number of cyclic paths available in the network. Similarly, Rodriguez-Nunez and Garcia-Palomares (2014) emphasized the importance of circular lines in providing travel alternatives in case of disruptions. Ash and Newth (2007) developed an evolutionary algorithm for adding links that improve network robustness. Based on the algorithm results they concluded that the most robust networks were characterized by high clustering, modularity and long path length compared with random network evolution.

Network vulnerability is investigated either through a full scan approach or by identifying a limited set of the most critical links. Previous studies of road networks concluded that different links were found to be the most critical depending on the criteria used in the evaluation (Knoop et al., 2012; El-Rashidy and Grant-Muller, 2014). A network-wide screen was applied by Rodriguez-Nunez and Garcia-Palomares (2014) by testing the impact of each link closure while carrying out a deterministic and static assignment of passengers to the shortest paths. While their study accounts for passenger demand and travel times, link failure was considered equivalent to the removal of the disrupted link from the network with perfect information. De-Los-Santos et al. (2012) performed a similar network scan evaluation while considering also the case of a replacement service for the closed link.

None of the evaluations carried out in the studies aforementioned have taken into consideration the underlying system dynamics and their implications on cascading effects and adaptive rerouting travel decisions. In contrast, a dynamic, stochastic and multimodal notion of PTN vulnerability, accounting for interactions between supply and demand and the accumulated effect of disruption on system performance, was introduced by Cats and Jenelius (2014). Candidate critical links were then identified by extending the measures of betweenness centrality and link importance to a dynamic-stochastic setting from the perspectives of both operators and passengers. The criticality of a link was evaluated as the reduction in welfare (considering travel time, number of transfers, etc.) due to a capacity reduction of the link. Their findings suggest that the dynamic betweenness centrality measure is a better indicator for disruption impact on passenger welfare than the static betweenness centrality which is based solely on network topology.

There is lack of knowledge concerning the effectiveness (taking both benefits and costs into account) of different strategies to reduce the probability of disruptions, or to reduce the impacts when disruptions do occur. Arguably, this is partly because of limitations in previous PTN models: the insights that can be gained from studies of network topology, which have dominated the literature, are largely limited to general design principles. The dynamic modelling approach, however, allows for the evaluation of a much wider range of design, management and real-time control strategies. Cats and Jenelius (2014) studied the mitigating impact of real-time information (RTI) provision, and found that RTI may have significant positive influence, although counter-examples also exist due to secondary spill-over effects.

One important set of strategies for enhancing PTN robustness is the increase of capacity on alternative lines. This implies operational strategies such as increasing the frequency on existing lines, or running replacement lines for the disrupted line. There are several benefits of such strategies: First, increasing capacity through increased line frequency means that the transfer and waiting times will decrease for travellers who are forced to choose alternative travel routes. Second, increasing capacity reduces the risk that the total demand from rerouting and existing passengers exceeds capacity, which could otherwise lead to denied boarding for passengers and delays for vehicles. With demand close to capacity, these spill-over effects could cascade through the network and lead to significant welfare losses even for travellers not directly affected by the disruption.

Capacity enhancement can be implemented at different stages of planning, management and operations. *Strategic* increases of capacity affect long-term levels and patterns of travel demand. A permanent increase in service capacity is normally motivated by demand levels under normal operations. However, it is sometimes argued that additional capacity will also make the system more robust in case of service disruptions. For *tactical* increases in capacity for a planned temporary degradation of the network (e.g., due to maintenance and construction), capacity may be increased on lines that serve as alternative travel options for the affected passengers. Since the degradation is planned in advance, informed decisions can be made to reallocate vehicles, drivers, etc. between lines. Finally, *operational* increases in capacity could be made in response to unexpected degradations. The allocation of reserve vehicles and crew to services that become overloaded due to service disruptions constitutes a real-time mitigation strategy.

This paper proposes and demonstrates a methodology for evaluating the effectiveness of a strategic increase in capacity on alternative PTN links to mitigate the impact of unexpected network disruptions. For a given disruption scenario, a set of links is first identified as candidates for capacity enhancement based on (i) their initial saturation levels in terms of volume to capacity ratios, and (ii) the overloading in terms of increased saturation that occurs due to the disruption. Second, the effect of capacity increase is evaluated for each candidate link by comparing the disruption impacts with and without increased capacity. Based on the evaluation the most effective of the mitigation actions can be identified.

The proposed methodology is integrated in a framework for identifying the most effective capacity enhancement schemes under a larger set of feasible disruption scenarios. The approach builds an additional layer on the methodology for identifying critical PTN links introduced in Cats and Jenelius (2014). The methodology is applied in a case study for the high frequency PTN of Stockholm, Sweden. To evaluate the public transport system performance under varying conditions, a dynamic public transport operations and assignment model, BusMezzo, is used (Cats, 2013).

The paper is organized as follows. Section 2 describes how the impact of service disruption is measured and how the benefit of capacity enhancement is evaluated. By analysing the impact of service disruptions on flow redistribution we propose measures to identify oversaturated links which are candidates for capacity enhancement. A framework is then proposed for identifying enhancements of reserve capacity in PTN with the potential to increase robustness. Section 3 details how the framework is operationalized using a dynamic public transport operations and assignment model. The model is then applied to the rapid public transport system of Stockholm, Sweden. Section 4 provides the details and the results of the case study including the dynamic scenario design process and the impact analysis. We conclude with a discussion on the implications of the proposed framework on appraisal schemes and PTN design, and outline related directions for future studies.

2. METHODOLOGY

This section presents an approach to identifying links where capacity enhancements would be the most effective for increasing network robustness. First the components of the PTN model are defined. Second, the approach to evaluating the impacts of link disruption is presented. Third, a framework for identifying the most effective capacity enhancement schemes across a range of severe disruption scenarios is proposed. Finally, the potential consideration of capacity enhancement in a cost-benefit appraisal scheme is outlined.

2.1 Public transport network definition

The physical PTN is defined by a directed graph $G(S,E)$, where the node set S represents stops and rail stations (all called stops here for simplicity), and the link set $E \subseteq S \times S$ represents direct connections between stops.

Each link $e \in E$ may be operated by one or several public transport lines. A line l is defined by a sequence of stops $l = (s_{l,1}, s_{l,2}, \ldots, s_{l,|l|})$, where $s_{l,1}$ is the origin terminal and $s_{l,|l|}$ is the destination terminal. The set of all lines is denoted L. We let $e \in l$ mean that link e is in line l, i.e., that $e = (s_{l,i}, s_{l,i+1})$ for some i. Thus, each link e is associated with a set of lines $L_e = \{l \in L \mid e \in l\}$ traversing the link.

Each link e is associated with a *riding time*, which is the time from the departure from the upstream stop to the arrival at the subsequent downstream stop. The riding time may vary between trips and between days depending on the current traffic conditions. Similarly, each stop is associated with a *dwell time*, which is the time required for a vehicle to stop for boarding and alighting. Like the riding times, the dwell times may vary between trips and days, depending on the current number of passengers, vehicle type, etc., and may be considered stochastic.

Each line l is operated with a set of vehicle trips according to a schedule. The departure time of a trip on line l from the origin terminal is in general a function of a scheduled departure time and the arrival time of the previous trip, which may be stochastic due to the stochastic riding and dwell times.

Travel demand is connected to the network through a subset of origin-destination (OD) nodes, $S_{od} \subseteq S$. The set of travellers from origin $o \in S_{od}$ to downstream destination $d \in S_{od}$ during time interval $(t, t+\tau)$ is denoted $N_{od}(t,\tau)$. The demand for public transport is assumed here to be inelastic, that is, not affected by changes in travel times etc. However, the number of travellers during the time interval may be stochastic to represent day-to-day variations.

Similarly to the vehicle lines, the physical path of a traveller is defined by a sequence of stops from the origin to the destination, that is, $j = (s_{j,1}, s_{j,2}, \ldots, s_{j,|j|})$, where $s_{j,1}$ is the origin stop and $s_{j,|j|}$ is the destination stop. In general, which physical path a given traveller chooses a given day and time-of-day will depend on the properties of the different public transport lines and on the conditions that day, according to the preferences of the individual. Considering the dynamics and the stochasticity of the system, the probability that traveller n uses physical path j is denoted $p_n(j)$. In Section 3 we present a dynamic public transport route choice model that is used in this paper to calculate $p_n(j)$.

2.2 Disruption scenarios and impacts

From the perspective of the travellers, evaluating the impacts of network disruptions involves comparing and summing the various aspects of the impacts for different travellers. The impacts must therefore be expressed in units such that interpersonal comparisons and summations are meaningful. For many reasons, not least in cost-benefit analyses of capacity enhancements as considered in this paper, it is desirable to express the disruption impacts in economic terms. This allows prevention, repair and restoration costs to be added and compared to other impacts such as late arrivals. With these aims, it is reasonable to express the impacts for passengers in terms of changes in *welfare*, essentially the total utility of the passengers expressed in monetary terms (Jenelius and Mattsson, 2014).

In each scenario a network segment is disrupted at time t_0 and is restored to nominal capacity at time $t_0 + \tau_D$. Meanwhile, the system recovers to normal operations only after a certain *recovery time* $\tau_R > \tau_D$ from the beginning of the disruption. The recovery time will vary between scenarios and is governed by the dynamic interactions of supply and demand. Before the disruption and after the recovery, the expected welfare is by definition the same in the disruption scenario and in the baseline scenario without disruption. To evaluate the impacts of a disruption scenario σ, it is therefore sufficient to compare the welfare with that in the baseline scenario σ_0 during the recovery time. For simplicity, the total welfare for all passengers during the recovery time is written as $W(\sigma)$. The impact of disruption scenario σ from the passengers' perspective is thus the total change in welfare from the baseline scenario, i.e.,

$$\Delta W(\sigma) = W(\sigma) - W(\sigma_0) \tag{1}$$

Since travel demand levels and line schedules may vary with time, so do the impacts of network disruption. Day-to-day variations in demand and supply further imply that the total number of passengers travelling between o and d, $|N_{od}(t_0, \tau_R)|$ is stochastic. The stochasticity arises in part from the fact that variations in traffic conditions (in-vehicle time, waiting times, etc.) influence the path choice probabilities $p_n(j)$ for a given individual.

In the case study of this paper, the welfare of each individual traveller is evaluated as a generalized cost function. With $W_n(\sigma)$ denoting the welfare of passenger n in scenario σ, the expected total welfare in scenario σ is

$$W(\sigma) = E[\sum_{o\in S_{od}} \sum_{d\in S_{od}} \sum_{n\in N_{od}(t_0,\tau_R)} W_n(\sigma)] \tag{2}$$

The focus of this paper is on two dimensions of the disruption scenarios: the network element that is disrupted, i.e., the set of disrupted links, and the type of capacity enhancement provided to mitigate the impacts. Section 3 details how public transport supply and service disruptions are represented in this study. Other factors such as the start time t_0 and duration τ_D of the disruption are held fixed in all scenarios. A disruption scenario σ involving network element δ and capacity enhancement scheme h can then be summarized as the pair $\sigma = (\delta, h)$. Let $\delta = 0$ denote a scenario with no disruption and $h = 0$ a scenario with no capacity enhancement.

Following Jenelius et al. (2006), the magnitude of impact of a link disruption is referred to as the *importance* of the link. In the present framework, the importance of an element is evaluated conditional on that a certain capacity enhancement scheme is implemented. Given scheme h, the importance from the passengers' perspective (*PI*) of network element δ is then

$$PI(\delta \mid h) = W(\delta, h) - W(0, h) \quad \forall h,\ \forall \delta \neq 0. \tag{3}$$

Under a given disruption situation δ, the *value of capacity enhancement* (*VCE*) for scheme h is the difference in welfare with and without the scheme implemented,

$$VCE(h \mid \delta) = W(\delta, h) - W(\delta, 0) \quad \forall h \neq 0,\ \forall \delta. \tag{4}$$

The *value of strategic disruption mitigation* (*VSDM*) of capacity enhancement scheme h for a disruption of element δ is calculated as the difference in disruption impacts with and without the scheme implemented,

$$VSDM(h \mid \delta) = W(\delta, h) - W(0, h) - (W(\delta, 0) - W(0,0)) = PI(\delta \mid h) - PI(\delta \mid 0) \quad \forall h \neq 0,\ \forall \delta \neq 0. \tag{5}$$

Table 1 summarizes the indicators used for analysing the impacts of reserve capacity on network robustness. All the indicators are based on differences in total passenger welfare under alternative network scenarios.

Table 1: Impact indicators used in the evaluation framework

	$h = 0$	$h \neq 0$	VCE
$\delta = 0$	$W(0,0)$	$W(0,h)$	$VCE(h\|0) = W(0,h) - W(0,0)$
$\delta \neq 0$	$W(\delta,0)$	$W(\delta,h)$	$VCE(h\|\delta) = W(\delta,h) - W(\delta,0)$
PI	$PI(\delta\|0) = W(\delta,0) - W(0,0)$	$PI(\delta\|h) = W(\delta,h) - W(0,h)$	
$VSDM$	$VSDM(h\|\delta) = PI(\delta\|h) - PI(\delta\|0)$		

2.3 Evaluation framework for unexpected disruptions

Since it is not known in advance where unexpected disruptions will occur, it is of interest to identify capacity enhancement schemes that are effective under a range of severe disruption scenarios. In this section a two-stage evaluation approach is proposed: In the first stage, a set of important links is identified using the approach of Cats and Jenelius (2014). In the second stage, for each identified important link, a set of capacity enhancement schemes are identified based on criteria defined in Section 2.5. The framework illustrated in Figure 1 consists of the following steps:

1. The baseline scenario assesses the performance under normal conditions and identifies a set of the most central links which constitute candidates for important links.

2. A disruption scenario is simulated for each of the candidate central links. The importance of each link is evaluated based on the welfare impact of the disruption.

3. For each important link, a set of the most overloaded links are determined based on specific volume-over-capacity criteria as candidates for capacity enhancement.

4. A scenario is simulated for each combination of disrupted link and capacity enhancement scheme. The most effective capacity enhancement schemes are identified based on the impacts they have on welfare changes.

Further details on the indicators employed in the dynamic scenario design are given in the following sections.

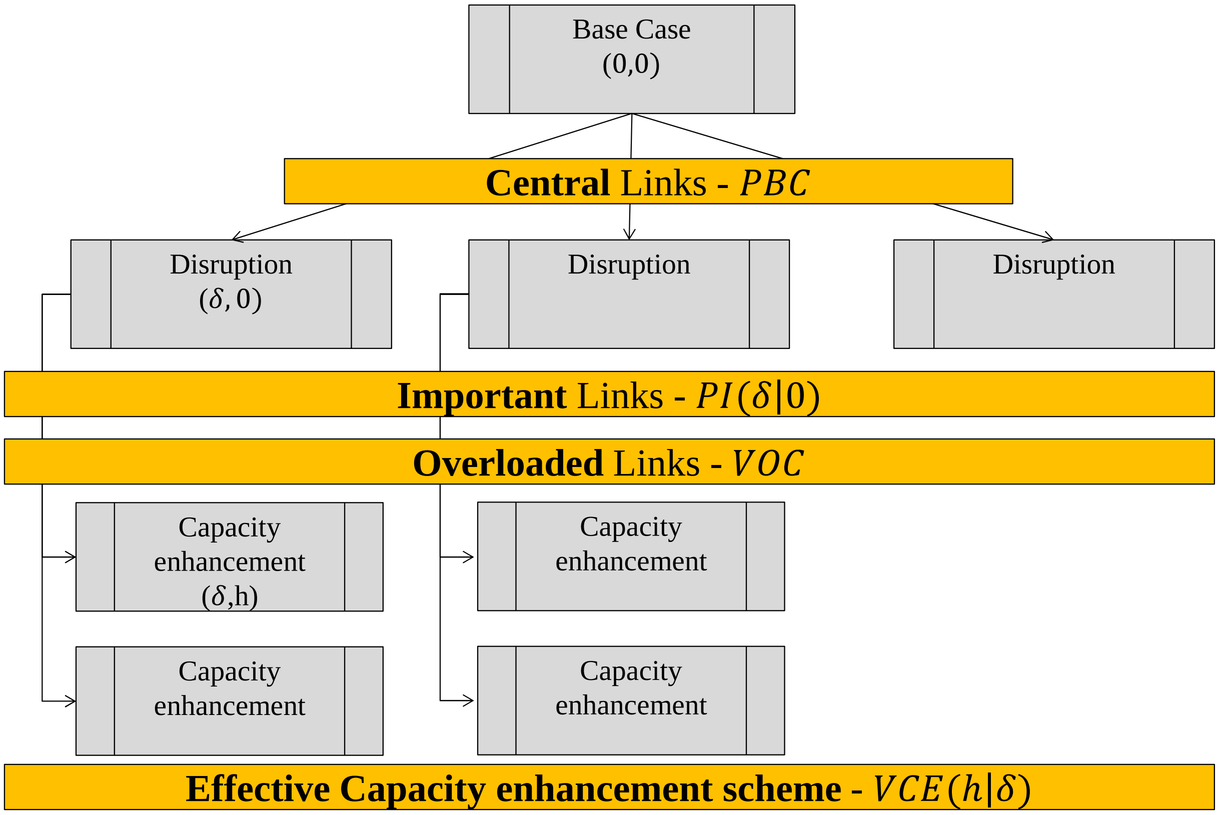

Figure 1: Evaluation framework

2.4 Identifying important links

To identify the *n* most important links for some chosen number *n*, one must in principle evaluate all possible link failures or use some form of optimization algorithm. The exhaustive-scanning approach is computationally very demanding with a dynamic and stochastic simulation model capturing the interactions between demand and supply, and the optimization problem is difficult. Instead, Cats and Jenelius (2014) propose a heuristic approach based on a generalization of the *betweenness centrality* measure. In network topology, the betweenness centrality of a link is the fraction of shortest paths between all pairs of network nodes that include the link. The *passenger betweenness centrality* (PBC) is defined by Cats and Jenelius (2014) as the expected fraction of all passengers who travel during a given time interval that traverse the link. In other words, the PBC reflects the expected passenger volume on the link during the time interval. The PBC is used in the baseline scenario as proxy for importance and identify the *n* most central links, for which the importance is then evaluated.

In the identification of central links, the time interval from the start to the end of the disruption is considered, i.e., the interval from t_0 to $t_0 + \tau_D$. Formally, let $|N_{ode}(\sigma_0, t_0, \tau_D)|$ be the number of passengers from o to d entering link e during the time interval in the baseline scenario σ_0. The expected passenger volume on link e is then

$$V(e|\sigma_0) = \sum_{o \in S_{OD}} \sum_{d \in S_{OD}} E\left[\left|N_{ode}(\sigma_0, t_0, \tau_D)\right|\right] \tag{6}$$

The passenger betweenness centrality is thus the passenger volume normalized by the total number of passengers during the time interval,

$$PBC(e) = \frac{V(e|\sigma_0)}{\sum_{o \in S_{OD}} \sum_{d \in S_{OD}} E[|N_{od}(t_0, \tau_D)|]} \tag{7}$$

2.5 Identifying overloaded links

Given a disruption of a network link, it is of interest to identify the most effective capacity enhancement schemes for mitigating the impacts. The type of capacity enhancement considered in this paper is an increase of the capacity on one or several public transport lines. Here we propose a method for identifying candidates lines by considering PTN links that become overloaded due to the link disruption. Once the most overloaded links are identified, capacity may be enhanced on one or several lines operating on each link. The approach is based on two conditions:

1) The volume-over-capacity ratio (VOC) on the link in the disruption scenario should exceed a threshold value α. This identifies links that are highly saturated under the disruption, but the saturation may not necessarily be due to the disruption.

2) The relative increase in the VOC from the baseline scenario should exceed a threshold value β. This identifies links that experience a large additional load due to the disruption, but the saturation level during the disruption is not necessarily high.

Together, conditions 1) and 2) identify links that are overloaded due to the disruption. For each identified important link, a set of overloaded links are identified using the conditions in Section 2.4. In total, this results in a set of *m* links, where *m* depends on the values chosen for the parameters α and β. Note that the same link may be among the most overloaded for disruptions of several of the important links. For each identified overloaded link, an appropriate capacity enhancement scheme is designed for the line(s) operating the link.

In the identification of overloaded links, the time interval from t_0 to $t_0 + \tau_D$ is considered. The flow and capacity of link e during this time interval under scenario σ are denoted $V(e|\sigma)$ and $C(e|\sigma)$, respectively. The VOC, finally, is obtained as $VOC(e|\sigma) = V(e|\sigma)/C(e|\sigma)$. The conditions for identifying overloaded links can thus be formalized as

1. $VOC(e|\sigma) > \alpha$
2. $\dfrac{VOC(e|\sigma)}{VOC(e|\sigma_0)} > \beta$

The passenger volume $V(e|\sigma)$ is calculated just like in Eq. (6). Meanwhile, the capacity of the link is computed based on the scheduled number of vehicles that traverse the link and their corresponding vehicle capacities (number of seats plus maximum number of standees). The capacity is thus a deterministic quantity. Let $C_{le}(\sigma, t_0, \tau_D)$ denote the total scheduled capacity of the trips of line l that enter link e during the disruption time given scenario σ. The total capacity on link e is obtained by summation over all lines,

$$C(e|\sigma) = \sum_{l \in L} C_{le}(\sigma, t_0, \tau_D) \tag{8}$$

2.6 Embedding robustness effects into the appraisal scheme

The same capacity enhancement scheme may have beneficial effects under several different link disruptions. It is thus of interest to sum up the total disruption mitigating value for each scheme. It is then reasonable to consider the frequency with which the different disruption scenarios are expected to occur, if such estimates are available. Let $F(\delta)$ denote the frequency (e.g., number of occurrences per year) of disruptions of element δ. It is assumed here that the capacity enhancement itself does not affect the disruption frequency. The total value of disruption mitigation of capacity enhancement scheme *h* (for example, per year) is then

$$VSDM(h) = \sum_{\delta \neq 0} F(\delta) \cdot VSDM(h \mid \delta). \tag{9}$$

In the lack of information concerning the probability of disruption across network links, approximations could be based on the share of critical units (e.g., kilometres of track, signals, stations and vehicles).

Eq. 9 indicates a way to incorporate robustness effects of capacity enhancement schemes in economic appraisal schemes such as cost-benefit analysis. Different feasible schemes can thus be evaluated based on their overall benefits in relation to their costs.

3. IMPLEMENTATION

The operationalization of the evaluation framework described in Section 2 requires a dynamic public transport operations and assignment model. BusMezzo, an agent-based public transport simulation model, is used in this study for emulating the dynamic loading of travellers and their interactions with the underlying public transport system (Cats et al., 2011). The following sections describe briefly how the supply and demand are modelled dynamically and how service disruptions are simulated in particular.

3.1 Dynamic public transport model

Public transport performance is an outcome of numerous agent decisions and interactions. The dynamic representation of the public transport system involves primarily vehicles and travelers. Each individual vehicle is assigned to an ordered set of runs and the corresponding timetable which is used for monitoring (e.g., dispatching, holding at stops along the route) and evaluation (e.g., punctuality) purposes. Trip departure times from the origin terminal depend on the scheduled departure time and vehicle availability upon completion of the previous trip and a stochastic recovery time at the terminal. Modeling the chain of trips assigned to each individual vehicle enables the impact of delay propagation from trip to trip to be captured.

The progress of a public transport vehicle along the route consists of a sequence of travel times between stops and dwell times at stops. Travel times are determined by traffic dynamics which are represented at a mesoscopic level. Travel times between stops are composed of running times on links and delays at intersections which are computed based on speed-density functions and stochastic queuing models, respectively. Different public transport modes have different vehicle types, capacities, operating speeds and control strategies. Furthermore, they exercise a varying level of interaction with other vehicles (e.g., busses in mixed traffic, bus lanes, underground), which results in different characteristics in terms of traffic regimes and travel time variability. The effects of background traffic are modeled implicitly in this paper by representing travel times and delays at intersections as random variables sampled from distributions that were derived from empirical travel time data.

Dwell times are determined by passenger activity at each stop – the number of passengers boarding and alighting, congestion on-board, as well as the vehicle and stop characteristics. The stochastic demand-dependent dwell times generate a positive feedback loop that contributes to the deterioration of service reliability along the route. The occupancy on-board each vehicle is constantly updated and enables the enforcement of vehicle capacity constraints. This implies that travelers might be subject to denied boarding. The framework and further details concerning the supply representation in BusMezzo are described in Toledo et al. (2010).

Public transport travelers are generated based on a time-dependent OD matrix where origins and destination may correspond to any geographical location that is accessible from a public transport stop. Passenger generation follows a random Poisson process. A non-compensatory rule-based choice-set generation model produces a set of alternative paths for each OD pair. A path alternative $a \in A^{od}$ is a member of the path set for origin o to a destination d and is defined by an ordered set of stops, lines and connection links. Connection links are access, egress and transfer links that can be traversed by various non-public transport travel modes (e.g., walking or cycling). Each element in the path alternative is a set, or hyper-path, created by grouping those public transport lines that provide an equivalent connection between a given pair of stops or several public transport stops which are connected by the same public transport lines.

The dynamic public transport path choice is modeled as a sequence of decisions – connection, boarding and alighting – which are triggered by the evolving public transport system conditions. Alternative actions that could be undertaken are evaluated based on traveler's preferences and expectations concerning downstream attributes. All travel decisions are modeled within the framework of discrete random utility models. The utility that traveler n attaches to action g is computed as the logsum over the path set $A^g \subseteq A^{od}$ associated with the action:

$$v_{g,n} = ln \sum_{a \in A^g} e^{v_{a,n}} \quad \forall g \in G \tag{10}$$

where G is the set of alternative actions that are available in the particular decision context. The logsum term expresses the utility of an action as the joint utility for a bundle of path alternatives. The deterministic part of the utility function for a single path alternative takes the form

$$v_{a,n} = \beta_{a,n} X_{a,n} \, , \tag{11}$$

where $\beta_{a,n}$ is a vector of coefficients and $X_{a,n}$ is the corresponding vector of expected values of path alternative attributes. A multinomial logit (MNL) model is used for computing the probability of choosing a certain action. The independence of irrelevant alternatives property of the MNL model is counteracted by the nested choice tree structure as well as the hyper-path definition.

Travelers' expectations depend on their prior knowledge, experience and the availability of real-time information provision. In the context of high-frequency urban public transport systems, travelers are assumed to have prior knowledge concerning network topology, planned headways and timetable travel times. The availability of real-time information depends on the dissemination means, the locations (e.g., public stop displays) and individual characteristics (e.g., access to a personal mobile device). The information is generated based on a commonly applied real-time information generation scheme (Cats et al., 2011).

The supply and demand of the public transport system interact dynamically. Modeling individual travelers that undertake successive decisions enables the analysis of the interaction of passenger decisions and public transport performance. Travelers' decisions are triggered and influenced by how the public transport service evolves and their expectations are formed by past experience and real-time information that is generated based on instantaneous supply conditions. Travelers' ability to carry out their decisions depends on service availability and vehicle occupancy.

3.2 Modelling service disruptions

Network robustness is evaluated by assessing system performance under service disruptions. Disruptions are simulated in BusMezzo by specifying the incident start time (t_0), duration (τ_D) and the affected links (δ). Vehicles cannot traverse the disrupted links as long as the disruption is in effect. Most previous studies of PTN robustness assumed that only the disrupted links are affected by the disruption while all other links continue to function regularly (e.g., von Ferber et al., 2012). In contrast, the following effects of an unexpected link failure are explicitly captured in the dynamic public transport model:

- *Downstream* – passengers waiting at downstream stops (including along the disrupted segment) can reconsider and revise their travel decisions (e.g., take another line, walk to a nearby stop)
- *Upstream* – public transport vehicles progress until they queue upstream of the link closure
- *Restrained passengers* – on-board passengers are unable to alight and have to wait until the service is restored
- *Spill-over* – secondary effects are caused by either supply processes (e.g., vehicle scheduling) or passenger rerouting (e.g., delays, denied boarding due to vehicle capacity constraints)

The model thus facilitates the analysis of upstream, downstream and horizontal cascading effects. When the link returns to normal operations at time $t_0 + \tau_D$, traffic on the previously disrupted link resumes and the system recovers gradually.

The breakdown on the disrupted link causes a redistribution of passenger flows in the network. Studies in network science typically apply flow distribution models that imply a deterministic shortest path algorithm with perfect information (e.g., Ash and Newth 2007, von Ferber et al., 2012). The path choice model in BusMezzo is dynamic and probabilistic and takes into account the limited information that is available to individual travelers along their journey. Travelers can adapt their travel decisions in real-time based on their expectations concerning downstream travel conditions and the information available to them. This network loading model can mimic the dynamic rerouting and evolution of flows in response to unanticipated changes in system performance.

4. APPLICATION

4.1 Case study description

The proposed evaluation framework was applied to the rapid public transport system of Stockholm, Sweden. All existing services that operate with an average scheduled headway shorter than 5 minutes during the morning peak period (6:00-9:00) were implemented in the case study network. The system consists of the seven metro lines (blue line 10-11, red line 13-14, green line 17-19), four trunk bus lines (1-4) and one light rail train (22). Figure 2 presents two representations of the network graph where nodes correspond to either stops or transfer hubs and links to line segments. The Stockholm metro network is characterized by a radial structure that supports high regional accessibility, but low degree of connectivity and medium degree of directness when compared with other metro systems in the world (Derrible and Kennedy, 2010). Trunk lines provide high coverage in the inner city and the light rail train line functions as an orbital line that connects major interchange stations that are strategically located along the southern and western edges of the inner city.

Each public transport mode is simulated with distinguished vehicle types, vehicle capacities, operating speeds, traffic regimes, dwell time functions and control strategies. These sets of operational attributes yield different levels of reliability and capacity depending on service design and right-of-way. Given the service frequency, travelers are assumed to depart randomly from the origins without consulting timetables. The case study system serves 437 stops with approximately 700 vehicle runs performed by 200 vehicles during the morning peak period.

An OD matrix was constructed by applying an iterative proportional fitting method based on a base OD matrix and passenger counts data that were available from the metropolitan public transport agency. In order to allow for a warm-up period for the public transport supply, passenger demand data was simulated only for the peak hour (7:00-8:00) resulting in approximately 125,000 passenger trips.

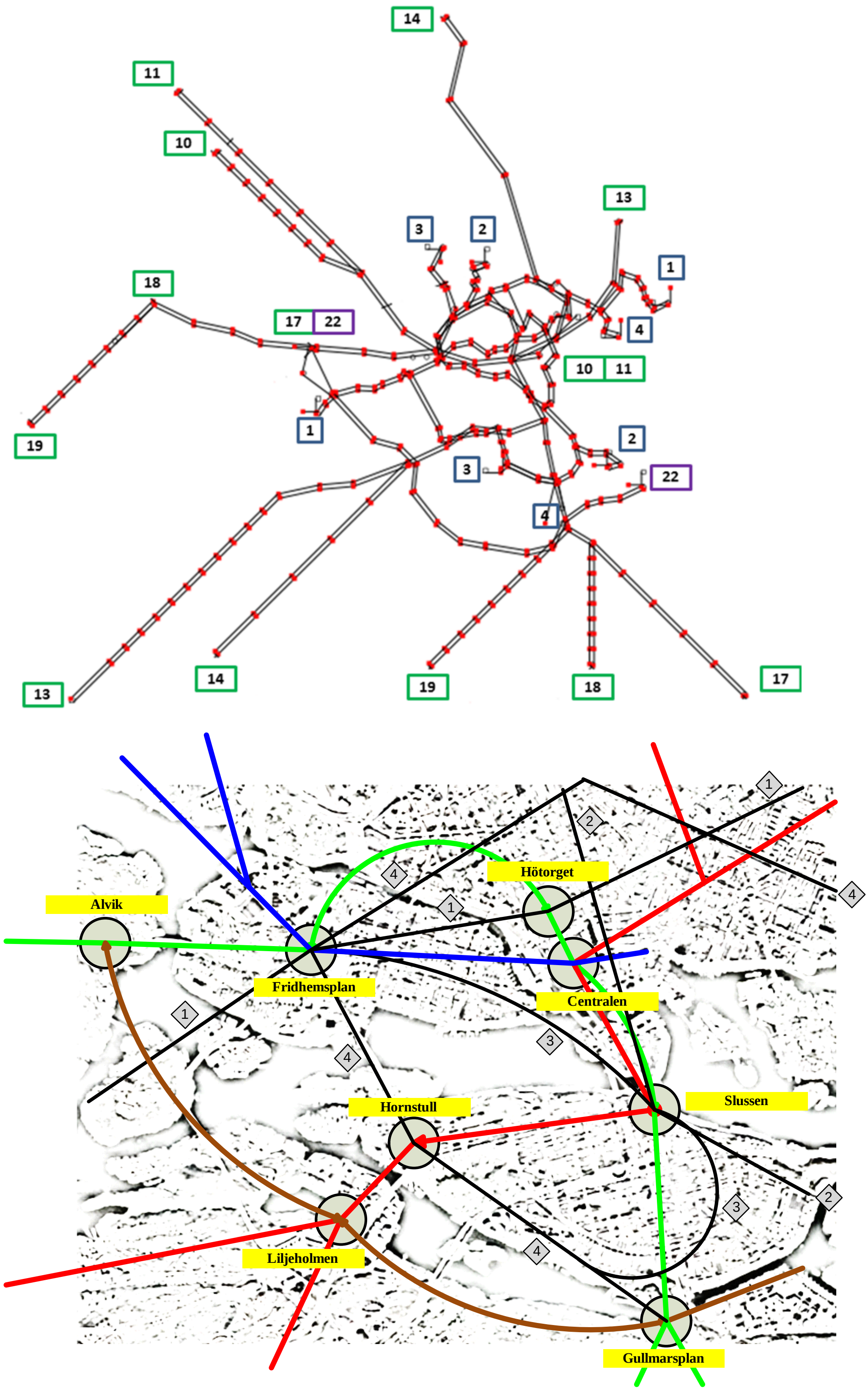

Figure 2: The Stockholm rapid public transport network. Top: as shown in BusMezzo graphical editor. Bottom: A schematic representation of the network core and transfer hubs. The three metro corridors (green (17-19), blue (13-14) and red (10-11)) are drawn in their respective colors, the orbital LRT (22) is drawn in brown, and the 4 trunk bus lines are drawn in black with their line numbers marked.

The choice set generation algorithm was applied as an initialization stage prior to the simulation runs. It resulted in 99,270 alternative hyper-paths for the case study network. This master set is then used in constructing path sets for specific actions and choice contexts. The parameters of the choice set generation model and the dynamic path choice model were estimated based on a stated-preferences survey on public transport route choice decisions (Cats, 2011). The utility associated with path $a \in A^g$ is defined as

$$v_{a,n} = \beta_a^{wait} t_{a,n}^{wait}(t) + \beta_a^{ivt} t_{a,n}^{ivt}(t) + \beta_a^{walk} t_{a,n}^{walk} + \beta_a^{trans} trans_a \ , \tag{12}$$

where $t_{a,n}^{wait}(t)$ and $t_{a,n}^{ivt}(t)$ are the time-dependent anticipated waiting time and in-vehicle time, respectively. $t_{a,n}^{walk}$ is the expected walking time and $trans_a$ is the number of transfers involved with the path alternative. $\beta_a^{ivt} = -0.04$, $\beta_a^{wait} = -0.07$, $\beta_a^{walk} = -0.07$ and $\beta_a^{trans} = -0.334$ are the corresponding coefficients. Each traveler is assigned coefficients sampled from a normal distribution to account for the heterogeneity of preferences in the population. Trip fare is fixed in the Stockholm network for a given OD pair and therefore does not affect passenger path decisions. Real-time information is available at all stops concerning the expected remaining time to the next vehicle arrival from each service line.

Each scenario was analyzed based on the results of 10 simulation runs. This number of replications yielded a maximum allowable error of less than 1% for the average passenger travel time. The execution time for a single run was less than 1 minute on a standard PC.

4.2 Applying the evaluation framework to the Stockholm network

The network robustness evaluation framework implies a sequential path-dependent process of scenario design. First, a set of candidate important links is identified based on the reference scenario followed by the analysis of disruption scenarios on the respective links. Second, a set of overloaded links is identified for each disruption scenario followed by the analysis of a capacity enhancement scenario for the respective link.

4.2.1 Disruption scenario design: identifying important links

The reference scenario of normal operations was first simulated and analyzed to allow the calculation of the passenger betweenness centrality measure, PBC, across the network (see Section 2.4). This measure is used to identify the most central links which are the candidate important links. The five most central network *segments* ($n = 5$) were selected for further analysis. A segment is here defined as a sequence of consecutive links which have similar PBC values and where there is no other link along the line that has a higher PBC value.

A disruption scenario was simulated for each of the central segments. The case study considers a short-term and unexpected disruption. Each disruption was simulated between 7:15 and 7:45 (t_0=7:15, $\tau_D = 30$ min). It is thus assumed that the service operators cannot deploy any special measures to mitigate the impacts of the disruption. Note that in case of a disruption on a metro link, link closure does not imply disruptions on other lines since each metro line has a separate infrastructure – tracks and platforms – as well as a distinguished fleet.

Table 2 presents the five disrupted segments along with the respective disruption scenario results. The average passenger load during the rush hour is equivalent to the numerator of the PBC measure. The average travel time which consists of walking, waiting and in-vehicle time is 24 minutes under normal operations in the case study network. The disruption scenarios induce an increase of 2-11% in total travel time which is caused by a combination of primary (upstream and downstream) and cascading (spill-over) effects. In addition, passenger rerouting results in more complicated trips which involve a higher number of transfers compared with normal operations. Longer travel times and more numerous transfers lead to a decrease in passenger welfare under all disruption scenarios. An average reduction of one welfare unit per passenger is equivalent to a loss of approximately 100,000 Swedish Crowns (15,000 USD) for all passengers during a single rush hour of operations.

Table 2: Passenger travel time and relative welfare changes for each disruption scenario

Service disruption scenario	Disrupted segment δ	Average passenger load during the rush hour on the disrupted link during normal operations	Total travel time $t^{walk} + t^{wait} + t^{ivt}$ [sec]	Transfers $trans$	Welfare for service disruption scenario $W(\delta, 0)$	Relative welfare change due to disruption $PI(\delta\|0)$
Base case	δ =0		1424	1.37	-71.65	
17-19n	**Green (17-19) northbound,** Gullmarsplan-Hötorget	27,186	1584	1.60	-79.72	-11.23%
13-14n	**Red (13-14) northbound,** Liljeholmen-Centralen	18,363	1503	1.42	-75.63	-2.48%
17-19s	**Green (17-19) southbound,** Alvik-Centralen	14,785	1498	1.44	-75.36	-5.18%
10-11s	**Blue (10-11) southbound,** Fridhemsplan-Centralen	11,510	1525	1.40	-76.71	-7.10%
13-14s	**Red (13-14) southbound,** Centralen-Hornstull	10,508	1453	1.43	-73.12	-2.03%

The results indicate that link centrality does not necessarily imply link importance as measured by the impact of its closure on total welfare (Section 2.2). For example, the northbound direction of the red line (13-14n) is ranked second in terms of PBC but results in less substantial impacts than segments that were ranked lower (17-19s and 10-11s).

A subset of the important links was selected for reserve capacity analysis on overloaded links. A robust identification of potential effective capacity enhancements requires the representation and availability of high capacity path alternatives. Disruption scenarios on segments 10-11s and 13-14s were therefore selected for further analysis as both ends of the disrupted segments are within the core network boundaries.

4.2.2 Capacity enhancement scenario design: identifying overloaded links

For each of the disruption scenarios concerning segments 10-11s and 13-14s, a set of overloaded links was identified by analysing the impact of the disruption on cascading failures through the redistribution of saturation levels across the network. The selection criteria described in Section 2.5 were applied as follows:

(1) saturation levels measured by the volume-over-capacity ratio exceed 75% of the link capacity ($\alpha = 0.75$); this value was selected as it often indicates saturation when describing system level-of-service.
(2) saturation levels increase by more than 5% due to the disruption ($\beta = 0.05$); this value was used in order to consider significant changes in flow distribution.

The combination of these two criteria seeks to identify links that are heavily loaded and are significantly affected by the redistribution of flows in the network following the critical segment failure.

Figure 3 presents the impact of each disruption scenario on link saturation levels as measured by the volume-over-capacity ratio for the network core. It is evident that distinctive patterns emerge for the two disruption scenarios in terms of their cascading effects across the network. In both cases, congestion increases significantly on a large number of links. Upstream links are negatively affected as well as links that act as direct substitutes for passengers boarding at the upstream node of the disrupted segment. Note that these affects are not limited to links that are in geographical proximity to the failed segment as both upstream and downstream links of alternative lines are affected due to rerouting. Furthermore, network effects lead to a spillover to other lines that connect transfer hubs that overtake the failed segment. Interestingly, the saturation on some links is relieved following the closure of an important link, because the outgoing flow of the latter constitutes a significant share of the ingoing flow of these links. As shown to the right in Figure 3, this is for example the case of the light rail line 22 in scenario D13-14s west of a major transfer hub (Liljeholmen) between the light rail line 22 and the disrupted red line (13-14).

Although a large number of lines were affected significantly in terms of the relative increase in saturation levels because of cascading effects ($\beta > 0.05$) in both scenarios, only a subset of these line-segments fulfilled both selection criteria yielding the sets {4,19} and {1,4,19} for the 10-11s and 13-14s scenarios, ($m = 2$ and $m = 3$), respectively. Metro line 19 is the most heavily loaded line in the entire Stockholm network. In particular, it provides an alternative connection between two major transfer hubs – Fridhemsplan and T-Centralen – which are the upstream and downstream stations of the segment closed in scenario 10-11s. Bus trunk line 4 is also particularly vulnerable to cascading effects because it provides the only alternative inner city north-south connection that bypasses the city center and the old town. Passenger flows on some segments of these lines are close to capacity even under normal operations. In the case of scenario 13-14s, bus trunk line 1 provides an alternative connection between the upstream stations of the disrupted line in the north-east towards the west and connections to south-west.

Figure 3: Volume over capacity change due to disruption scenario 10-11s (left) and 13-14s (right); Black: disrupted links. Red: a relative increase in VOC of more than 5%. Green: a relative decrease on VOC of

more than 5%. Yellow: an insignificant relative change in VOC. The numbers shown refer to line numbers.

Capacity enhancement scenarios were evaluated for each overloaded link. These scenarios are used to analyze whether the cascading effect may be mitigated by allocating reserve capacity to overloaded links. In order to assess the distinctive contribution of reserve capacity to network robustness, the impact of increased capacity on system performance under normal operations is also evaluated.

In general, service capacity could be enhanced by increasing either vehicle capacity or service frequency. The former option may be restricted by vehicle design and infrastructure limitations. In this paper, increased capacity was therefore implemented through increased service frequency on the respective line that traverses the overloaded link. Furthermore, the capacity enhancement considered in this analysis took the form of a 50% increase in service frequency along the entire route of the considered overloaded link. This share was chosen in order to illustrate the impact of a substantial strategic capacity increase. Note that increased service frequency implies not only that links can accommodate higher flows bus also shorter waiting times for passengers. This will also be reflected in passengers' expectations and path choice decisions.

4.3 Impact analysis

The results of each disruption and capacity enhancement combination are summarized in Table 3. As expected, the introduction of capacity enhancement schemes yields shorter passenger travel times. In all network operations scenarios the selected enhancement schemes lead to a reduction of approximately 360 seconds or 25% of the average travel time. Furthermore, travelers carry out fewer transfers, resulting in a reduction of 10-15% in the average number of transfers. This leads to an increase in total welfare of in the range of 17-21 welfare units due to capacity enhancement (*VCE*) under both normal operations and link closures, which is equivalent to 1.7-2.1 million Swedish Crowns (circa 255,000-315,000 USD) for all passengers during a single rush hour of operations.

The impact of link failure on the total welfare for all passengers diminishes in scenarios with enhanced capacity on overloaded links. For disruption scenario 10-11s, capacity enhancements relieve the welfare reduction associated with segment breakdown from more than 5 welfare units to less than 1.5 welfare units if service frequency on line 19 is increased by 50% when compared with the corresponding normal operations cases. This implies a significant improvement in network robustness with 70% of the disruption effect being mitigated thanks to reserve capacity on the overloaded link compared to the corresponding normal operations scenario.

The same capacity enhancement on line 19 yields a more modest reduction of 15% in the case of scenario 13-14s. Moreover, while a 50% increase in line 1 frequency leads to significant welfare benefits; it also makes the network more vulnerable. The reduction in total welfare in scenario 13-14s is higher than in the lack of capacity enhancement on line 1, although still resulting with much better performance. This seemingly paradoxical case where greater capacity leads to greater vulnerability albeit with better welfare results arises from changes in travel patterns which may induce changes in the relationship between important and overloaded links. The assignment results indicate that this is caused by the fact that line 1 becomes even more saturated as its increased frequency attracts a large number of new travelers. The short waiting times attract more passengers than the increase in supply. Under normal operations, many passengers benefit from this capacity enhancement scheme. In the case of disruption of 13-14s, however, the system becomes slightly more vulnerable than in the base case as line 1 is even less capable to accommodate travelers that reroute due to the disruption.

The last column in Table 3, *VSDM*, shows the change in total welfare due to planning for the unplanned disruption. In other words, this is the contribution of the capacity enhancement scheme to network robustness. It is evident that the value of planned disruption mitigation is substantially higher for the disruption scenario 10-11s compared to 13-14s. In the latter case, capacity enhancement schemes yield very marginal improvements and for line 1 even a marginal reduction in total welfare.

Table 3: Passenger travel time and relative welfare changes for each disruption-capacity scenario

Service disruption scenario δ	Capacity enhancement scheme h	Total travel time $t^{walk} + t^{wait} + t^{ivt}$ [sec]	Transfers $trans$	Welfare for scenario combination $W(\delta, h)$	Welfare change due to disruption $PI(\delta\|h)$	Value of capacity enhancement $VCE(h\|\delta)$	Planned disruption mitigating value $VSDM(h\|\delta)$
0	0	1424	1.37	-71.65			
	1	1085	1.24	-54.65		+17.00	
	4	1084	1.24	-54.61		+17.04	
	19	1072	1.19	-53.98		+17.67	
10-11s	0	1525	1.40	-76.71	-5.06		
	4	1126	1.22	-56.72	-2.11	+19.99	+2.95
	19	1102	1.17	-55.47	-1.49	+21.24	+3.57
13-14s	0	1453	1.43	-73.12	-1.47		
	1	1117	1.28	-56.29	-1.64	+16.83	-0.17
	4	1109	1.26	-55.87	-1.26	+17.25	+0.21
	19	1096	1.23	-55.23	-1.25	+17.89	+0.22

The appraisal of the investment associated with increased capacity should take into consideration its contribution to mitigating the impact of critical link failure. For scenario 10-11s, the welfare benefits from a 50% increase in line 4 frequency increase from 1.7 to 2.0 million Swedish Crowns (255,000 to 300,000 USD) for all passengers during a single rush hour of operations. As formulated in Section 2.6, the robustness benefits from reserve capacity could be incorporated into cost benefit appraisal schemes by accounting for the corresponding probability of failure.

5. CONCLUSIONS

This paper presents a method for evaluating how reserve capacity should be introduced in a PTN in order to improve its robustness. The method requires the identification of important links, meaning that their failure would result in the greatest impact in terms of welfare loss. Links that are highly saturated and become even more so under service disruption are shortlisted for potential capacity enhancement schemes. The effectiveness of these schemes is evaluated through the value of strategic disruption mitigation (*VSDM*) while their efficiency could be derived through their incorporation into the appraisal scheme.

The application of the proposed method requires a tool to evaluate system performance under service disruptions. In contrast to topological network analysis with deterministic shortest-path assignment, the impacts of an unplanned link failure tend to escalate through downstream, upstream and spillover cascading effects. Furthermore, in order to analyze changes in reserve capacity it is necessary to model the adaptive route choice that leads to the redistribution of passengers in response to service disruptions. The case study presented in this paper is facilitated by a dynamic agent-based public transport simulation model, BusMezzo. The analysis considers the impact of an unplanned link failure with real-time information at the stop-level and planned capacity enhancement.

The case study demonstrates how the proposed method could be applied to sequentially designed scenarios based on their performance indicators. This process was applied for the rapid PTN of Stockholm. The results illustrate the potential benefits for PTN robustness from increasing capacity on selected services. While the availability of reserve capacity leads to substantial welfare benefits as measured by the value of capacity enhancement (*VCE*), the *VSDM* might be marginal or even negative. The latter constitutes a seemingly paradoxical case which is caused by changes in network vulnerability due to the redistribution of passenger flows caused by the introduction of reserve capacity. This suggests that an iterative procedure for identifying important and oversaturated links is required in order to account for the interaction between service capacity, disruptions and passenger flows. In other words, since the definition of what constitutes a disruption depends on network capability to absorb a failure then changes in network robustness will influence what will be regarded as a disruption. Changes in link capacities throughout the network lifetime will therefore lead to changes in the identification of important and oversaturated links similarly to the iterative consideration of betweenness centrality in simulations of targeted attacks (e.g., von Ferber et al., 2012).

The method presented in this paper could support policy makers and operators in prioritizing measures to increase network robustness by improving system capacity to absorb unplanned disruptions. The service disruption need not be a full link closure but could also be a partial capacity reduction. The evaluation framework is used to quantify the contribution of strategic and operational capacity increase to network robustness. The effectiveness of operational measures depends on fleet management strategies and their impact on the response time required for deploying the additional capacity.

Tactical capacity increase in response to a planned disruption could also be accommodated in our method. Public transport supply would correspond to the revised service under normal operations. Note that this may differ from simply removing the closed links (e.g. replacement service, line closure vs. operating services on the remaining segments). Moreover, passengers’ route choice will be based on their expectations given the planned disruption which may not be equivalent to perfect network knowledge due to the unusual conditions. Similarly to unplanned disruption, PTN connectivity will determine the availability of alternative routes and the availability of reserve capacity to accommodate rerouting. By applying this method for various PTN, future studies could examine the relationship between network connectivity and resilience and draw conclusions on network design principles.

The impact of service disruptions and capacity enhancements was analyzed in this paper in terms of total passenger welfare. In addition to the overall impact, future studies may investigate the implications on recovery time – the time required to rebound to the normal system functionality. Furthermore, in order to assess the overall impact of capacity increase on network robustness, the probabilities for link failures have to be integrated. The probability of failure could be derived from historical empirical data, or estimated based on network indicators in order to incorporate the robustness effect into an economic appraisal scheme. This will allow performing a cost benefit analysis for investing in new links which account for their implications on network robustness. Note that this implies a full vulnerability scan of the network links.

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