Observer-based Tracking Problem of Leader-follower Multi-agent Systems

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Abstract: In this paper, we consider the tracking problem for leader-follower multi-agent systems moving in the plane. In our model, the system matrix of the leader can be of any form and the control input of the leader is not known to the followers even if they are connected to the leader. To track such a leader, an observer-based decentralized feedback controller is designed for each follower. We show that if the adjacent graph of the system is connected, with the proposed control law, all the followers can track the leader eventually. At last, we generalize the result to the higher dimension case for some special system matrix of the leader.

Key Words: Multi-agent systems, leader-follower, decentralized control, observer

1 INTRODUCTION

In the past few years the study of multi-agent systems has attracted a considerable attention from various research communities. Multi-agent systems appear in various areas, such as cooperative control of unmanned aerial vehicles [1], consensus problem of communication networks [2, 3], formation control of mobile robots [4, 5], etc.

There is a large amount of literature concerning the coordination problem of multiple mobile agents, in which local rules are usually applied to each agent mainly based on the relative information between its own and its neighbors (or its possible leaders), e.g., [2, 3, 6, 7, 8], to name a few. This is inspired originally by the flocking behaviors of animals in nature. In the engineering application, the coordination motion of multiple robots also needs distributed control due to the limitation of sensing and measurement.

In collective behaviors of multiple agents, the leader-follower model has attracted a great deal of attention lately. Some literature can refer to [2, 9, 10, 11, 12, 13, 14], etc. The leaders have the desired trajectory or target, which can help guide the whole group. The other agents follow the leaders by the relative information obtained from their neighbors and the leaders. Jadbabaie et al.[2] considered the modified Vicsek model and proved that all agents will move with the same direction as the leader if the adjacent graphs are jointly connected. In [9, 10], to reach consensus, the connectivity keeping problem was considered by computing the ratio of the numbers of leaders and followers. In practice, some state variables of agents (or even their leaders) may not be measured. In this case, decentralized observer design is needed for each agent. Hong et al. [11] proposed a neighbor-based observer to estimate the unmeasurable variables (velocity) of an active leader. Then they extended the result to second-order case [12]. [14] considered the observer-based synchronized output regulation problem of linear networked systems, in which only part of the agents (called leaders) can measure the state of the exosystem and the others must estimate the exosystem state. However, the proposed control law is not based on the relative state error but the absolute state.

In this paper we consider the tracking problem of leader-follower multi-agent systems. The motivation is the platooning problem of multiple vehicles following a priori given paths. Using the leader-follower approach, one could only provide the leader with the specifics of the path while the followers can follow the path by tracking each other and the leader using onboard sensors. In our model, there are N followers and one leader moving in the plane. Different from [11, 12], in this paper, the control input $u_l = A_l x_l + b_l$ of the leader is unknown to each follower even if it is connected to the leader and A_l can be of any form, which makes the states of the leader coupling. With different A_l, the leader moves along different trajectories. To track such a leader, we first estimate the control input u_l, based on which a decentralized feedback controller is designed for each follower. Both the observer and the controller are based on local relative information. We prove that if the adjacent graph of the system is connected, all the followers will track the leader eventually. Moreover, this result can be generalized to higher dimension case for some special system matrix of the leader.

The rest of the paper is organized as follows. Section 2 provides some necessary preliminaries. Section 3 is our problem formulation. Section 4 presents the main results. We deals with the tracking problem by constructing neighbor-based decentralized controller and observer. In Section 5, an illustrative example is given. Section 6 is the conclusion.

2 PRELIMINARIES

Graph theory is helpful in coordination problems of multiple agents. In this section, we recall some basic concepts and results on graph theory and other knowledge related to this paper. More details can be found in for example [4, 15].

An undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ of order N consists of a vertex set $\mathcal{V} = \{1, 2, \cdots, N\}$, an edge set $\mathcal{E} = \{(i, j) : i, j \in \mathcal{V}\} \subset \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N\times N}$, where $a_{ii} = 0$ and $a_{ij} = a_{ji} \geq 0$. $a_{ij} > 0$

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if and only if there is an edge between vertex i and vertex j and the two vertices are called adjacent (or they are mutual neighbors). The set of neighbors of vertex i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (i,j) \in \mathcal{E},\ j \neq i\}$.

A path of length l from vertex i to j is a sequence of $l+1$ distinct vertices starting from i and ending at j such that the consecutive vertices are adjacent. For an undirected graph $\mathcal{G}$, if there is a path between any two vertices, then $\mathcal{G}$ is called connected, otherwise, disconnected. A subgraph $\mathcal{G}_1$ of graph $\mathcal{G}$ is an induced subgraph if two vertices in $\mathcal{G}_1$ are adjacent if and only if they are adjacent in $\mathcal{G}$. An maximal connected induced subgraph of $\mathcal{G}$ is called a component of $\mathcal{G}$.

Define the Laplacian L with respect to graph $\mathcal{G}$ as

$$L = D - A \tag{1}$$

where the diagonal matrix $D = \mathrm{diag}\{d_1, \cdots, d_N\} \in \mathbb{R}^{N\times N}$ is called degree matrix with the diagonal elements $d_i = \sum_{j=1}^{N} a_{ij}$ for $i = 1, \cdots, N$.

By the definition, for undirected graph, L is symmetric and every row sum of it is zero. 0 is an eigenvalue of L and $\mathbf{1}_N$ is the associated eigenvector, that is, $L\mathbf{1}_N = 0$, where $\mathbf{1}_N = [1, 1, \cdots, 1]^T \in \mathbb{R}^N$. If $\mathcal{G}$ is connected, then 0 is the algebraically simple eigenvalue of L and all the other eigenvalues are positive.

To determine the stability of a polynomial (or a matrix), Hurwitz stability test is helpful.

For a given polynomial $p_n(s) = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1}s + a_n$, define its Hurwitz matrix $H_n \in \mathbb{R}^{n\times n}$ as

$$H_n = \begin{bmatrix} a_1 & 1 & 0 & 0 & \cdots & 0 \\ a_3 & a_2 & a_1 & 1 & \cdots & 0 \\ a_5 & a_4 & a_3 & a_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & a_n \end{bmatrix}.$$

As a classical result recall that

Lemma 2.1 *[17](Routh-Hurwitz Theorem) The polynomial $p_n(s)$ is stable if and only if all the leading principal minors of H_n are positive.*

At last, we list some properties of Kronecker product of matrices with appropriate dimension [16].

- $\alpha(X \otimes Y) = (\alpha X) \otimes Y = X \otimes (\alpha Y), \quad \alpha \in \mathbb{R}$;
- $(X + Y) \otimes Z = (X \otimes Z) + (Y \otimes Z)$,
  $Z \otimes (X + Y) = (Z \otimes X) + (Z \otimes Y)$;
- $(X \otimes Y) \otimes Z = X \otimes (Y \otimes Z)$;
- $(X \otimes Y)(Z \otimes W) = XZ \otimes YW$;
- $\sigma(X \otimes Y) = \{\lambda_X \lambda_Y \mid \lambda_X \in \sigma(X),\ \lambda_Y \in \sigma(Y)\}$, where $\sigma(X)$ is the spectrum of X.

3 PROBLEM FORMULATION

In this paper, we consider a multi-agent system with N followers and one leader moving in the plane. The topology relationship of the N followers and the leader can be described by a graph $\bar{\mathcal{G}}$. We assume the information transfer between followers is undirected (denoted by an undirected graph $\mathcal{G}$), while the agents in graph $\mathcal{G}$ is connected to leader by directed edges, that is, the information can only be transferred from leader to followers. Graph $\bar{\mathcal{G}}$ is said to be connected if at least one agent in each component of graph $\mathcal{G}$ is connected to the leader by a directed edge [11]. Note that graph $\bar{\mathcal{G}}$ is connected does not mean graph $\mathcal{G}$ is also connected. Define $\Delta = \mathrm{diag}\{\alpha_1, \cdots, \alpha_N\}$ as the leader adjacency matrix associated with graph $\bar{\mathcal{G}}$, where $\alpha_i > 0$ if the leader is a neighbor of agent i (that is, there is a directed edge from agent i to the leader) and otherwise $\alpha_i = 0$.

The dynamics of each follower is

$$\dot{x}^i = u^i, \quad i = 1, \cdots, N \tag{2}$$

where $x^i \in \mathbb{R}^2$ is the state and $u^i \in \mathbb{R}^2$ is the control input of the i-th follower, $i = 1, \cdots, N$.

The dynamics of the leader is

$$\dot{x}_l = u_l \tag{3}$$

where $x_l = [x_{1l}, x_{2l}]^T, u_l \in \mathbb{R}^2$ are the state and control input, respectively. For simplicity, choose

$$u_l = A_l x_l + b_l \tag{4}$$

where $A_l \in \mathbb{R}^{2\times 2}, b_l = [b_{1l}, b_{2l}]^T \in \mathbb{R}^{2\times 1}$ can be any matrix such that the leader can move along different trajectories. In our problem, neither u_l nor $x_l(0)$ is known to any follower even if it is connected to the leader.

The purpose of this paper is to design decentralized control laws such that all the followers can track the leader. The control input u^i is designed only by the relative information between itself and its neighbors. Since u_l is not available directly by the followers even when they are connected to the leader, it can not be used in the control design. So we have to estimate u_l in the evolution. The estimated value of u_l by agent i is denoted by $\hat{u}_l^i = A_l \hat{x}_l^i + b_l,\ \ i = 1, \cdots, N$.

Remark 3.1 *In some sense, A_l denotes the shape of the leader's trajectory and b_l denotes the translation of the trajectory. In this paper, we assume A_l and b_l are known by each agent. That means each follower may know the shape of the leader's trajectory, but not the initial value. In fact, it is necessary to know the shape of the target's trajectory in the tracking problem. Next, we can only estimate x_l for each follower.*

For each follower, we propose the following neighbor-based control law

$$u^i = -K[\sum_{j\in\mathcal{N}_i} a_{ij}(x^i - x^j) + \alpha_i(x^i - x_l)] + A_l \hat{x}_l^i + b_l \tag{5}$$

and the decentralized observer

$$\dot{\hat{x}}_l^i = -K_l[\sum_{j\in\mathcal{N}_i} a_{ij}(x^i - x^j) + \alpha_i(x^i - x_l)] + A_l \hat{x}_l^i + b_l \tag{6}$$

where $K,\ K_l \in \mathbb{R}^{2\times 2}$ need to be determined, $\hat{x}_l^i$ is the estimate of x_l by agent $i, i = 1, \cdots, N$.

Let $e^i = x^i - x_l,\ e_l^i = \hat{x}_l^i - x_l$, then associate with (2) and (3), we have the error closed-loop system

$$\begin{cases} \dot{e}^i = -K[\sum_{j\in\mathcal{N}_i} a_{ij}(e^i - e^j) + \alpha_i e^i] + A_l e_l^i \\ \dot{e}_l^i = -K_l[\sum_{j\in\mathcal{N}_i} a_{ij}(e^i - e^j) + \alpha_i e^i] + A_l e_l^i,\ i = 1,\cdots,N. \end{cases} \tag{7}$$

The aim of this paper is to find K, K_l such that

$$e^i \to 0,\ A_l e_l^i = \hat{u}_l^i - u_l \to 0,\ t\to\infty,\ i = 1,\cdots,N. \tag{8}$$

Denote by e and e_l the concatenation of vectors $\{e^1,\cdots,e^N\}$ and $\{e_l^1,\cdots,e_l^N\}$, respectively. From (7), we have

$$\begin{cases} \dot{e} = -(H\otimes K)e + (I_N\otimes A_l)e_l \\ \dot{e}_l = -(H\otimes K_l)e + (I_N\otimes A_l)e_l \end{cases} \tag{9}$$

where $\otimes$ denotes the Kronecker product, $H := L + \Delta$, L is the Laplacian of graph $\mathcal{G}$ defined in (1) and $\Delta = \mathrm{diag}\{\alpha_1,\cdots,\alpha_N\}$ is the leader adjacency matrix of graph $\bar{\mathcal{G}}$ defined in the beginning of this section.
The following lemma derived from [11] gives a property of matrix H.

Lemma 3.2 *[11] If graph $\bar{\mathcal{G}}$ is connected, then the symmetric matrix $H = L + \Delta$ associated with $\bar{\mathcal{G}}$ is positive definite.*

Since H is symmetric, there is an orthogonal matrix T such that

$$THT^T := \Lambda = \mathrm{diag}\{\lambda_1,\lambda_2,\cdots,\lambda_N\} \tag{10}$$

is diagonal, where $\lambda_i,\ i = 1,\cdots,N$ are the eigenvalues of H. By Lemma 3.2, if graph $\bar{\mathcal{G}}$ is connected, then all $\lambda_i > 0$. Now by the orthogonal transformation

$$\tilde{e} = (T\otimes I_2)e,\ \ \tilde{e}_l = (T\otimes I_2)e_l \tag{11}$$

system (9) becomes

$$\begin{bmatrix} \dot{\tilde{e}} \\ \dot{\tilde{e}}_l \end{bmatrix} = \begin{bmatrix} -\Lambda\otimes K & I_N\otimes A_l \\ -\Lambda\otimes K_l & I_N\otimes A_l \end{bmatrix} \begin{bmatrix} \tilde{e} \\ \tilde{e}_l \end{bmatrix}. \tag{12}$$

Then the decoupled subsystems are

$$\begin{bmatrix} \dot{\tilde{e}}^i \\ \dot{\tilde{e}}_l^i \end{bmatrix} = \begin{bmatrix} -\lambda_i K & A_l \\ -\lambda_i K_l & A_l \end{bmatrix} \begin{bmatrix} \tilde{e}^i \\ \tilde{e}_l^i \end{bmatrix},\quad i = 1,\cdots,N. \tag{13}$$

Similar to Theorem 3 of [4], one can see easily that if the adjacent graph $\bar{\mathcal{G}}$ is connected and A_l is nonsingular, the tracking problem can be solved if and only if system (13) is stabilized for $i = 1,\cdots,N$.
Note that one necessary condition that stabilizes system (13) is that A_l is nonsingular. For the case of singular A_l, we will also discuss in the next section.

4 MAIN RESULTS

Since all possible cases of matrix A_l will be considered, we transform A_l into Jordan canonical form. Let the Jordan canonical form of matrix A_l be $\bar{A}_l = SA_lS^{-1}$, where S is nonsingular matrix. Then $\bar{A}_l$ has the following forms:

1. $\bar{A}_l = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$, where σ_1 and σ_2 are two different real eigenvalues of A_l.
2. $\bar{A}_l = \begin{bmatrix} \sigma & 0 \\ 0 & \sigma \end{bmatrix}$, where σ is the real eigenvalue of A_l with geometric multiplicity 2.
3. $\bar{A}_l = \begin{bmatrix} \sigma & 1 \\ 0 & \sigma \end{bmatrix}$, where σ is the real eigenvalue of A_l with algebraic multiplicity 2.
4. $\bar{A}_l = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}$, where $\beta > 0$, $\alpha \pm j\beta$ are the complex conjugate eigenvalues of A_l.

Accordingly, under the coordinate transformation $\bar{e}^i = S\tilde{e}^i,\ \bar{e}_l^i = S\tilde{e}_l^i$, system (13) becomes

$$\begin{bmatrix} \dot{\bar{e}}^i \\ \dot{\bar{e}}_l^i \end{bmatrix} = \begin{bmatrix} -\lambda_i\bar{K} & \bar{A}_l \\ -\lambda_i\bar{K}_l & \bar{A}_l \end{bmatrix} \begin{bmatrix} \bar{e}^i \\ \bar{e}_l^i \end{bmatrix},\quad i = 1,\cdots,N \tag{14}$$

where $\bar{K} = SKS^{-1},\ \bar{K}_l = SK_lS^{-1}$.
Note that the coordinate transformation keeps the stability. Hereafter, without loss of generality, assume the matrix A_l is already the Jordan canonical form.
First, for the trivial case: $A_l = 0 \in \mathbb{R}^{2\times 2}$, $u_l = b_l$. The leader moves with the constant velocity b_l, which is the same as in [2, 7, 8], etc. In this case, we do not need to estimate u_l and just choose the control

$$u^i = -K[\sum_{j\in\mathcal{N}_i} a_{ij}(x^i - x^j) + \alpha_i(x^i - x_l)] + b_l \tag{15}$$

then the error closed-loop system is

$$\dot{e} = -(H\otimes K)e. \tag{16}$$

Obviously, if the graph $\bar{\mathcal{G}}$ is connected, system (16) is asymptotically stable for any positive definite matrix $K > 0$.
Next, assume $A_l \neq 0$. We consider the above four cases, respectively.
Denote the system matrix of (13) by

$$F_i = \begin{bmatrix} -\lambda_i K & A_l \\ -\lambda_i K_l & A_l \end{bmatrix},\quad i = 1,\cdots,N.$$

The characteristic polynomial of F_i is

$$\det(sI_4 - F_i) = \det\begin{bmatrix} sI_2 + \lambda_i K & -A_l \\ \lambda_i K_l & sI_2 - A_l \end{bmatrix} \tag{17}$$

Case 1. $A_l = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$ with $\sigma_1 < \sigma_2$. In this case, let $K = \mathrm{diag}\{k_1,k_2\}$ and $K_l = \mathrm{diag}\{k_{1l},k_{2l}\}$, then

$$\det(sI_4 - F_i) = \prod_{j=1}^{2}[s^2 + (\lambda_i k_j - \sigma_j)s + \lambda_i(k_{jl} - k_j)\sigma_j] \tag{18}$$

Obviously, F_i is stable if and only if

$$\begin{cases} \lambda_i k_j - \sigma_j > 0 \\ \lambda_i(k_{jl} - k_j)\sigma_j > 0,\quad j = 1,2,\ \ i = 1,\cdots,N. \end{cases} \tag{19}$$

(1a) If $0 < \sigma_1 < \sigma_2$, then $k_{jl} > k_j > \frac{\sigma_j}{\lambda_{min}},\ \ j = 1,2$ is sufficient to assure F_i is stable, where λ_{min} is the smallest eigenvalue of matrix H.

(1b) When $\sigma_1 < \sigma_2 < 0$, for any $0 < k_{jl} < k_j,\ \ j = 1, 2$, (19) holds.
(1c) When $\sigma_1 < 0 < \sigma_2$, to assure F_i is stable, we can choose $0 < k_{1l} < k_1$ and $k_{2l} > k_2 > \frac{\sigma_2}{\lambda_{min}}$.
(1d) For the case when A_l has just one zero eigenvalue, say, $\sigma_1 = 0,\ \sigma_2 > 0$ (the same for $\sigma_1 < 0,\ \sigma_2 = 0$), we have $u_l = A_l x_l + b_l = \begin{bmatrix} 0 \\ a x_{2l} \end{bmatrix} + b_l$ (for simplicity, set $\sigma_2 = a > 0$). To estimate u_l, we only need to estimate x_{2l}. Denote by $\hat{x}^i_{2l}$ the estimate of x_{2l} by agent i. Choose the control law

$$\begin{cases} u^i_1 = -k[\sum\limits_{j\in\mathcal{N}_i} a_{ij}(x^i_1 - x^j_1) + \alpha_i(x^i_1 - x_{1l})] + b_{1l} \\ u^i_2 = -k[\sum\limits_{j\in\mathcal{N}_i} a_{ij}(x^i_2 - x^j_2) + \alpha_i(x^i_2 - x_{2l})] + a\hat{x}^i_{2l} + b_{2l} \end{cases} \tag{20}$$

and the decentralized observer

$$\dot{\hat{x}}^i_{2l} = -k_l[\sum_{j\in\mathcal{N}_i} a_{ij}(x^i_2 - x^j_2) + \alpha_i(x^i_2 - x_{2l})] + a\hat{x}^i_{2l} + b_{2l} \tag{21}$$

where $k > 0, k_l > 0, u^i = [u^i_1, u^i_2]^T, x^i = [x^i_1, x^i_2]^T, x_l = [x_{1l}, x_{2l}]^T$.
Let $e^i_j = x^i_j - x_{jl},\ j = 1, 2,\ e^i_{2l} = \hat{x}^i_{2l} - x_{2l},\ i = 1, \cdots, N$, and $e_1, e_2, e_{2l} \in \mathbb{R}^N$ are the stacks of $\{e^i_1\},\ \{e^i_2\},\ \{e^i_{2l}\}$, respectively. Then we have

$$\begin{cases} \dot{e}_1 = -kHe_1 \\ \dot{e}_2 = -kHe_2 + ae_{2l} \\ \dot{e}_{2l} = -k_l He_2 + ae_{2l} \end{cases} \tag{22}$$

where $H = L + \Delta$ is defined as before. The stability of e_1 is obvious by Lemma 3.2 if the graph $\bar{\mathcal{G}}$ is connected. Now the characteristic polynomial of the last two equations of (22) is

$$\begin{aligned} &\det\left(sI_{2N} - \begin{bmatrix} -kH & aI_N \\ -k_l H & aI_N \end{bmatrix}\right) \\ =&\det\begin{bmatrix} sI_N + kH & -aI_N \\ k_l H & (s-a)I_N \end{bmatrix} \\ =&\det(s^2 I_N + (kH - aI_N)s + a(k_l - k)H) \\ =&\prod_{i=1}^{N}[s^2 + (\lambda_i k - a)s + a\lambda_i(k_l - k)] \end{aligned} \tag{23}$$

where $\lambda_i\ \ i = 1, \cdots, N$ are the eigenvalues of H.
(23) is stable if and only if

$$\begin{cases} \lambda_i k - a > 0 \\ a\lambda_i(k_l - k) > 0, \quad i = 1, \cdots, N. \end{cases} \tag{24}$$

So when $k_l > k > \frac{a}{\lambda_{min}} > 0$, system (22) is asymptotically stable.
Case 2. $A_l = \begin{bmatrix} \sigma & 0 \\ 0 & \sigma \end{bmatrix}$. This is the same discussion as in Case 1.(1a) and (1b).
Case 3. $A_l = \begin{bmatrix} \sigma & 1 \\ 0 & \sigma \end{bmatrix}$. Since A_l is upper triangular, the case of $\sigma \neq 0$ is similar to Case 1. (1a) and (1b). When $\sigma = 0$, $u_l = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_l + b_l = \begin{bmatrix} x_{2l} \\ 0 \end{bmatrix} + b_l$. So we just need to estimate x_{2l}. Choose the control law

$$\begin{cases} u^i_1 = -k[\sum\limits_{j\in\mathcal{N}_i} a_{ij}(x^i_1 - x^j_1) + \alpha_i(x^i_1 - x_{1l})] + \hat{x}^i_{2l} + b_{1l} \\ u^i_2 = -k[\sum\limits_{j\in\mathcal{N}_i} a_{ij}(x^i_2 - x^j_2) + \alpha_i(x^i_2 - x_{2l})] + b_{2l} \end{cases} \tag{25}$$

and the observer

$$\dot{\hat{x}}^i_{2l} = -k_l[\sum_{j\in\mathcal{N}_i} a_{ij}(x^i_1 - x^j_1) + \alpha_i(x^i_1 - x_{1l})] + b_{2l} \tag{26}$$

where $k > 0,\ k_l > 0$, $\hat{x}^i_{2l}$ is the estimate of x_{2l} by agent i.
Using the same notations as before, we obtain the error system

$$\begin{cases} \dot{e}_1 = -kHe_1 + e_{2l} \\ \dot{e}_2 = -kHe_2 \\ \dot{e}_{2l} = -k_l He_1. \end{cases} \tag{27}$$

The characteristic polynomial of the first and the third equations of (27) is

$$\begin{aligned} \det\begin{bmatrix} sI_N + kH & -I_N \\ k_l H & sI_N \end{bmatrix} &= \det(s^2 I_N + skH + k_l H) \\ &= \prod_{i=1}^{N}[s^2 + \lambda_i ks + k_l\lambda_i] \end{aligned} \tag{28}$$

where $\lambda_i,\ i = 1, \cdots, N$ are the eigenvalues of H.
One can see that for any $k > 0,\ k_l > 0$, (27) is asymptotically stable if the graph $\bar{\mathcal{G}}$ is connected.
Case 4. $A_l = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}$ with $\beta > 0$.
(4a) First assume $\alpha = 0$. Set $K = kI_2,\ K_l = k_l I_2$. The characteristic polynomial of system (13) is

$$\begin{aligned} \det(sI_4 - F_i) =& s^4 + 2\lambda_i ks^3 + (\lambda_i^2 k^2 + \beta^2)s^2 \\ &+ 2\lambda_i\beta^2(k - k_l)s + \lambda_i^2\beta^2(k - k_l)^2. \end{aligned} \tag{29}$$

The Hurwitz matrix of (29) is

$$H_4 = \begin{bmatrix} 2\lambda_i k & 1 & 0 & 0 \\ 2\lambda_i\beta^2(k - k_l) & \lambda_i^2 k^2 + \beta^2 & 2\lambda_i k & 1 \\ 0 & \lambda_i^2\beta^2(k - k_l)^2 & 2\lambda_i\beta^2(k - k_l) & \lambda_i^2 k^2 + \beta^2 \\ 0 & 0 & 0 & \lambda_i^2\beta^2(k - k_l)^2 \end{bmatrix} \tag{30}$$

By Lemma 2.1, (29) is stable if and only if all the leading principle minors of H_4 are positive, that is,

$$\begin{aligned} \Delta_1 =& 2\lambda_i k > 0 \\ \Delta_2 =& 2\lambda_i^3 k^3 + 2\lambda_i\beta^2 k_l > 0 \\ \Delta_3 =& 4\lambda_i^2\beta^2 k_l(k - k_l)(\beta^2 + \lambda_i^2 k^2) > 0 \\ \Delta_4 =& \lambda_i^2\beta^2(k - k_l)^2\Delta_3 > 0. \end{aligned}$$

It is easy to see that for any $k > k_l > 0$, if the adjacent graph $\bar{\mathcal{G}}$ is connected, (29) is stable.

(4b) Assume $\alpha > 0$ (the same for $\alpha < 0$). Set $K = \mu A_l$ and $K_l = \mu\gamma A_l$, $\mu, \gamma \in \mathbb{R}$. Then the system matrix of (13) becomes

$$F_i = \begin{bmatrix} -\lambda_i \mu A_l & A_l \\ -\lambda_i \mu\gamma A_l & A_l \end{bmatrix} = \begin{bmatrix} -\lambda_i \mu & 1 \\ -\lambda_i \mu\gamma & 1 \end{bmatrix} \otimes A_l,\ i = 1, \cdots, N.$$

Since the eigenvalues of a Kronecker product are the products of the eigenvalues of the factors, if the eigenvalues of $M_i := \begin{bmatrix} -\lambda_i \mu & 1 \\ -\lambda_i \mu\gamma & 1 \end{bmatrix}$ are both negative, we are done. Note that

$$\det(sI_4 - M_i) = s^2 + (\lambda_i \mu - 1)s + \lambda_i \mu(\gamma - 1).$$

The eigenvalues of M_i are both negative if and only if

$$\begin{cases} \lambda_i \mu - 1 > 0 \\ \lambda_i \mu(\gamma - 1) > 0 \\ (\lambda_i \mu - 1)^2 - 4\lambda_i \mu(\gamma - 1) \geq 0 \end{cases} \tag{31}$$

A straightforward computation gets

$$\begin{cases} \mu \geq \frac{2\gamma - 1 + 2\sqrt{\gamma^2 - \gamma}}{\lambda_{min}} \\ \gamma > 1. \end{cases} \tag{32}$$

Remark 4.1 *In fact, the choice of K and K_l in (4b) can also be used in Case 1. (1a), (1b) and Case 2. That is, (32) holds for the case that the eigenvalues of A_l are both positive (or positive real part). For the both negative (real part) case, we can do similarly. Note that the method in (4b) is independent of the dimension of A_l.*

Based on the above discussion, we have the following main result:

Theorem 4.2 *Consider the leader-follower multi-agent system (2) and (3). Assume the adjacent graph $\bar{\mathcal{G}}$ composed of N followers and one leader is connected, then for any matrix A_l and b_l, there exist K and K_l such that all the followers can track the leader asymptotically via the decentralized feedback control (5) and the observer (6). Specifically,*
(i) for $A_l = 0$, the controller is degenerated to (15) with any positive definite matrix $K > 0$;
(ii) if the eigenvalues of A_l are both positive (real part) (the same for negative), choose $K = \mu A_l, K_l = \mu\gamma A_l$ with μ and γ satisfying (32);
(iii) for $A_l = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$ with $\sigma_1 < 0, \sigma_2 > 0$, choose $K = \mathrm{diag}\{k_1, k_2\}, K_l = \mathrm{diag}\{k_{1l}, k_{2l}\}$ satisfying $k_1 > k_{1l} > 0$ and $k_{2l} > k_2 > \frac{\sigma_2}{\lambda_{min}}$;
(iv) for $A_l = \begin{bmatrix} 0 & \beta \\ -\beta & 0 \end{bmatrix}$, choose $K = kI_2, K_l = k_l I_2$ with $k > k_l > 0$;
(v) for $A_l = \begin{bmatrix} 0 & 0 \\ 0 & a \end{bmatrix}$ with $a > 0$, choose the controller (20) and the observer (21) with $k_l > k > \frac{a}{\lambda_{min}} > 0$ (for $a < 0$, choose $k > k_l > 0$);
(vi) for $A_l = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, choose the controller (25) and the observer (26) with $k > 0, k_l > 0$.

From the above analysis, we can generalize the result to the higher dimension case.

Corollary 4.3 *Assume $A_l \in \mathbb{R}^{n \times n}$, if the real parts of the eigenvalues of A_l are all positive (or negative) and the adjacent graph $\bar{\mathcal{G}}$ is connected, then the result of Theorem 4.2 is also true by choosing $K = \mu A_l,\ K_l = \mu\gamma A_l$ with $\mu,\ \gamma$ satisfying (32).*

5 ILLUSTRATIVE EXAMPLE

In this section, we present an example to validate the theoretical results in this paper.

Example 5.1 *Consider a leader-follower multi-agent system moving in the plane with 4 followers satisfying*

$$\dot{x}^i = u^i \in \mathbb{R}^2,\ i = 1, 2, 3, 4 \tag{33}$$

and one leader satisfying

$$\dot{x}_l = A_l x_l \in \mathbb{R}^2 \tag{34}$$

where $A_l = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.
Assume the adjacent graph $\bar{\mathcal{G}}$ (with leader labeled 0) is given by Fig. 1. The Laplacian of the graph $\mathcal{G}$ (without leader) is

$$L = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

and the leader adjacent matrix $\Delta = \mathrm{diag}\{1, 0, 0, 1\}$.
Take $K = 2I_2$ and $K_l = I_2$, then by the observer-based feedback control (5) and (6), all the followers can track the leader asymptotically under any initial values. Refer to Fig. 2 for the simulation result with the initial values

$$\begin{aligned} x^1(0) =& [5, 3]^T,\ x^2(0) = [-4, -1]^T, \\ x^3(0) =& [8, -6]^T,\ x^4(0) = [-3, 9]^T,\ x_l(0) = [2, 2]^T. \end{aligned}$$

All four trajectories converge to a common circle which is the trajectory of the leader. Fig. 3 is the tracking errors $x^i - x_l,\ i = 1, 2, 3, 4$, which shows that all the followers can track the leader eventually. Fig. 4 is the estimate errors $\hat{x}_l^i - x_l,\ i = 1, 2, 3, 4$ which shows that the estimated value $\hat{x}_l^i$ by agent i will converge to x_l. □

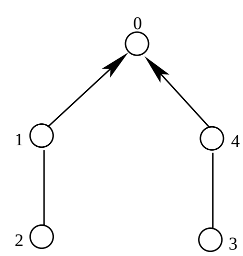

Fig. 1: The adjacent graph $\bar{\mathcal{G}}$

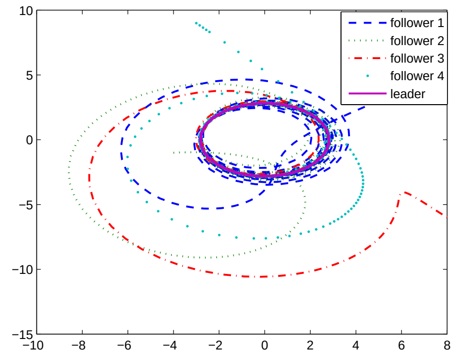

Fig. 2: The trajectories of four followers and one leader

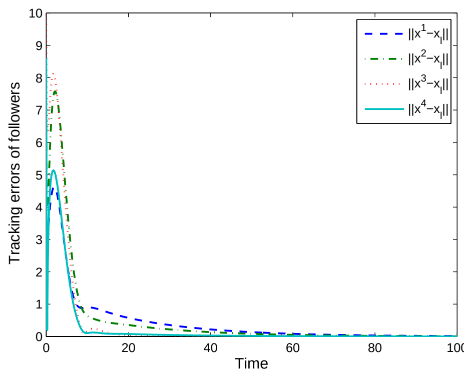

Fig. 3: The tracking errors of four followers

6 CONCLUSIONS

In this paper, the tracking problem for leader-follower multi-agent systems was considered. All the agents were assumed to move in the plane. In our model, the state of the leader is coupled and the control input u_l of the leader is unknown to any follower even if it is connected to the leader. To track such a leader, we constructed a neighbor-based observer to estimate the control u_l, based on which a local feedback controller was designed for each follower. We proved that if the adjacent graph of the system is connected, with the proposed control law, all the followers will converge to the leader. Finally, we generalize the result to higher dimension case for some special system matrix of the leader.

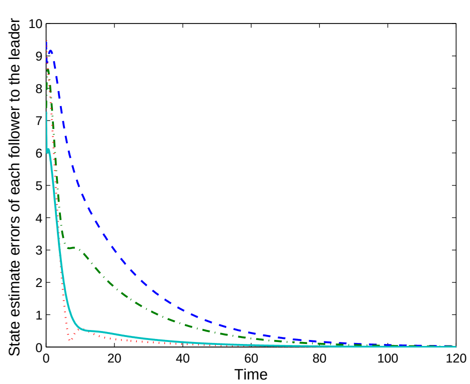

Fig. 4: The estimate errors of four followers to the leader: $\hat{x}_l^i - x_l,\ i = 1, 2, 3, 4$

This paper just considered the case that all the agents converge to one point, regardless of the collision problem. Further research will focus on the true platooning problem. By designing appropriate controller, all the agents move along the same trajectories but keep certain distance to avoid collision.

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