File Fragmentation over an Unreliable Network

Bachelor Thesis

Martin Andreasson
mandreas@kth.se

Royal Institute of Technology

September 7, 2009

Martin Andreasson
mandreas@kth.se

- Imagine yourself playing an action game

- There is always a risk of getting caught, which increases by time

Martin Andreasson
mandreas@kth.se

- If you get caught, you have to start over where you last saved
- But saving very frequently would cost too much time

- If you get caught, you have to start over where you last saved
- But saving very frequently would cost too much time

Basic model

- A file is sent over an unreliable network
- Large files are more likely to be corrupted than small
- If a file is corrupted, the whole file needs to be resent

Martin Andreasson
mandreas@kth.se

Basic model

- A file is sent over an unreliable network
- Large files are more likely to be corrupted than small
- If a file is corrupted, the whole file needs to be resent

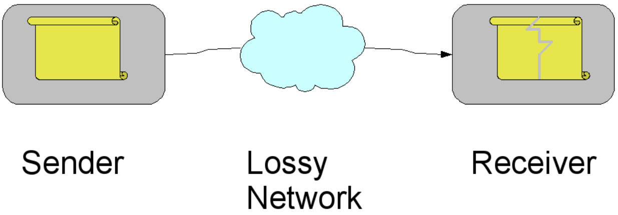
Martin Andreasson
mandreas@kth.se

Basic model

- A file is sent over an unreliable network
- Large files are more likely to be corrupted than small
- If a file is corrupted, the whole file needs to be resent

Martin Andreasson
mandreas@kth.se

Large files are likely to be corrupted



Basic model

- To prevent file losses and heavy-tailes, large files can be fragmented
- Smaller fragments are less likely to be corrupted

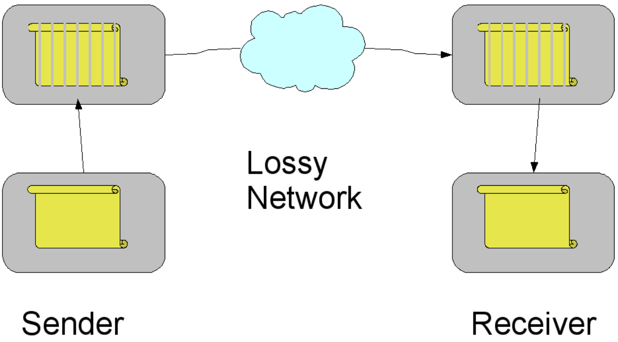
Martin Andreasson
mandreas@kth.se

Basic model

- To prevent file losses and heavy-tailes, large files can be fragmented
- Smaller fragments are less likely to be corrupted

Martin Andreasson
mandreas@kth.se

Fragmentation prevents corruption



- Packets however, carry an overhead
- This makes too small packet sizes inefficient

Thesis

- What is the optimal fragment size?

Martin Andreasson
mandreas@kth.se

Mathematical model

- $x_i + \phi$ denotes the packet size (x_i data size, ϕ overhead size)
- Transmission will succeed if $A_i \geq x_i + \phi$ (A_i random variable with distribution f)

Martin Andreasson
mandreas@kth.se

Mathematical model

- $x_i + \phi$ denotes the packet size (x_i data size, ϕ overhead size)
- Transmission will succeed if $A_i \geq x_i + \phi$ (A_i random variable with distribution f)

Martin Andreasson
mandreas@kth.se

Expected transmission time

$$J = \sum_{i=0}^{M-1} \frac{x_i + \phi}{\bar{F}(x_i + \phi)} = \sum_{i=0}^{M-1} h(x_i) \qquad \bar{F}(x) = \mathrm{P}(A_i > x) \quad (1)$$

- Want to minimize J over M and x_i

Martin Andreasson
mandreas@kth.se

Expected transmission time

$$J = \sum_{i=0}^{M-1} \frac{x_i + \phi}{\bar{F}(x_i + \phi)} = \sum_{i=0}^{M-1} h(x_i) \qquad \bar{F}(x) = \mathrm{P}(A_i > x) \quad (1)$$

- Want to minimize J over M and x_i

Martin Andreasson
mandreas@kth.se

Minimizing J

The KKT condition for (1) is

$$h'(x_i) = \frac{1}{\bar{F}(x_i+\phi)} + (x_i+\phi)\frac{f(x_i+\phi)}{\left(\bar{F}(x_i+\phi)\right)^2} = \lambda$$

$$\sum_{i=0}^{M-1} x_i = L \qquad (2)$$

Lemma

If the failure rate (failure risk) $f(x)/\bar{F}(x)$ is increasing:

- *The optimization over x_i is convex*
- *The solution is $x_i^* = x^*$ for all i (packet sizes are equal)*

Minimizing J

The KKT condition for (1) is

$$h'(x_i) = \frac{1}{\bar{F}(x_i+\phi)} + (x_i+\phi)\frac{f(x_i+\phi)}{\left(\bar{F}(x_i+\phi)\right)^2} = \lambda$$

$$\sum_{i=0}^{M-1} x_i = L \quad (2)$$

Lemma

If the failure rate (failure risk) $f(x)/\bar{F}(x)$ is increasing:

- *The optimization over x_i is convex*
- *The solution is $x_i^* = x^*$ for all i (packet sizes are equal)*

Minimizing J

The KKT condition for (1) is

$$h'(x_i) = \frac{1}{\bar{F}(x_i+\phi)} + (x_i+\phi)\frac{f(x_i+\phi)}{\left(\bar{F}(x_i+\phi)\right)^2} = \lambda$$

$$\sum_{i=0}^{M-1} x_i = L \qquad (2)$$

Lemma

If the failure rate (failure risk) $f(x)/\bar{F}(x)$ is increasing:

- *The optimization over x_i is convex*
- *The solution is $x_i^* = x^*$ for all i (packet sizes are equal)*

Martin Andreasson
mandreas@kth.se

Minimizing J

The KKT condition for (1) is

$$h'(x_i) = \frac{1}{\bar{F}(x_i+\phi)} + (x_i+\phi)\frac{f(x_i+\phi)}{\left(\bar{F}(x_i+\phi)\right)^2} = \lambda$$

$$\sum_{i=0}^{M-1} x_i = L \qquad (2)$$

Lemma

If the failure rate (failure risk) $f(x)/\bar{F}(x)$ is increasing:

- *The optimization over x_i is convex*
- *The solution is $x_i^* = x^*$ for all i (packet sizes are equal)*

Minimizing J

- Seek minimizer M^* by inserting $x^* = L/M$ into (1):

$$J = M \frac{\frac{L}{M} + \phi}{\bar{F}\left(\frac{L}{M} + \phi\right)} = L \frac{\frac{L}{M} + \phi}{\frac{L}{M}\bar{F}\left(\frac{L}{M} + \phi\right)} := L \cdot g\left(\frac{L}{M}\right) \quad (3)$$

Lemma

Under the assumption that the failure rate $f(x)/\bar{F}(x)$ is increasing, $g(x)$ is unimodal

Martin Andreasson
mandreas@kth.se

Minimizing J

- Seek minimizer M^* by inserting $x^* = L/M$ into (1):

$$J = M \frac{\frac{L}{M} + \phi}{\bar{F}\left(\frac{L}{M} + \phi\right)} = L \frac{\frac{L}{M} + \phi}{\frac{L}{M}\bar{F}\left(\frac{L}{M} + \phi\right)} := L \cdot g\left(\frac{L}{M}\right) \quad (3)$$

Lemma

Under the assumption that the failure rate $f(x)/\bar{F}(x)$ is increasing, $g(x)$ is unimodal

Martin Andreasson
mandreas@kth.se

Minimizing J

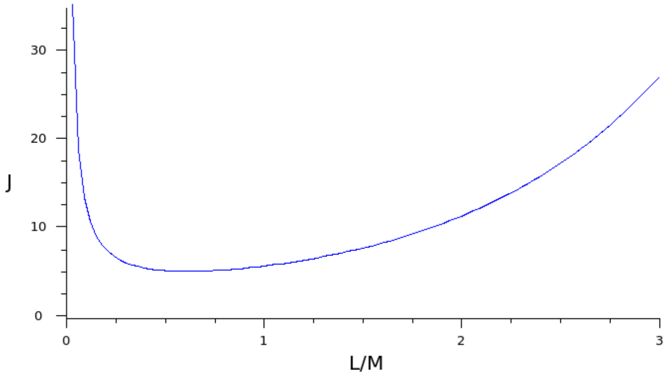
- Seek minimizer M^* by inserting $x^* = L/M$ into (1):

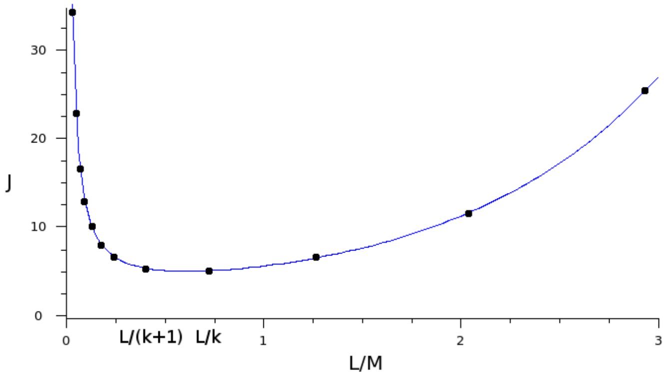
$$J = M\frac{\frac{L}{M}+\phi}{\bar{F}\left(\frac{L}{M}+\phi\right)} = L\frac{\frac{L}{M}+\phi}{\frac{L}{M}\bar{F}\left(\frac{L}{M}+\phi\right)} := L\cdot g\left(\frac{L}{M}\right) \quad (3)$$

Lemma

Under the assumption that the failure rate $f(x)/\bar{F}(x)$ is increasing, $g(x)$ is unimodal

Martin Andreasson
mandreas@kth.se


30
20
10
0
J
0
1
2
3
L/M


J
30
20
10
0
0
L/(k+1)
L/k
1
2
3
L/M

Bounding x^*

Definition

$$a := \underset{x}{\operatorname{argmin}} \left(g(x) \right)$$

Theorem

If $L > a$:

$$\frac{a}{2} \leq \frac{a}{1 + a/L} \leq x^*(L) \leq \min\left(2a, \frac{a}{1 - a/L}\right)$$

Corollary

$$\lim_{L \to \infty} x^*(L) = a$$

Bounding x^*

Definition

$$a := \underset{x}{\operatorname{argmin}}\,(g(x))$$

Theorem

If $L > a$:

$$\frac{a}{2} \leq \frac{a}{1 + a/L} \leq x^*(L) \leq \min\left(2a, \frac{a}{1 - a/L}\right)$$

Corollary

$$\lim_{L \to \infty} x^*(L) = a$$

Martin Andreasson
mandreas@kth.se

Bounding x^*

Definition

$$a := \operatorname*{argmin}_x \left(g(x)\right)$$

Theorem

If $L > a$:

$$\frac{a}{2} \leq \frac{a}{1 + a/L} \leq x^*(L) \leq \min\left(2a, \frac{a}{1 - a/L}\right)$$

Corollary

$$\lim_{L \to \infty} x^*(L) = a$$

Bounding x^*

Definition

$$a := \operatorname*{argmin}_x \left(g(x)\right)$$

Theorem

If $L > a$:

$$\frac{a}{2} \leq \frac{a}{1 + a/L} \leq x^*(L) \leq \min\left(2a, \frac{a}{1 - a/L}\right)$$

Corollary

$$\lim_{L \to \infty} x^*(L) = a$$

Martin Andreasson
mandreas@kth.se

Conclusion

Under the assumption that the failure rate $f(x)/\bar{F}(x)$ is increasing,

- Constant fragmentation is optimal
- The fragment sizes are unique
- If the file size and distribution f are known, the file can be pre-fragmented
- For large filesizes L, the optimal fragment size is independent of L

Martin Andreasson
mandreas@kth.se

Conclusion

Under the assumption that the failure rate $f(x)/\bar{F}(x)$ is increasing,

- Constant fragmentation is optimal
- The fragment sizes are unique
- If the file size and distribution f are known, the file can be pre-fragmented
- For large filesizes L, the optimal fragment size is independent of L

Martin Andreasson
mandreas@kth.se

Conclusion

Martin Andreasson
mandreas@kth.se

Under the assumption that the failure rate $f(x)/\bar{F}(x)$ is increasing,

- Constant fragmentation is optimal
- The fragment sizes are unique
- If the file size and distribution f are known, the file can be pre-fragmented
- For large filesizes L, the optimal fragment size is independent of L

Conclusion

Under the assumption that the failure rate $f(x)/\bar{F}(x)$ is increasing,

- Constant fragmentation is optimal
- The fragment sizes are unique
- If the file size and distribution f are known, the file can be pre-fragmented
- For large filesizes L, the optimal fragment size is independent of L

Martin Andreasson
mandreas@kth.se

Conclusion

Under the assumption that the failure rate $f(x)/\bar{F}(x)$ is increasing,

- Constant fragmentation is optimal
- The fragment sizes are unique
- If the file size and distribution f are known, the file can be pre-fragmented
- For large filesizes L, the optimal fragment size is independent of L

Martin Andreasson
mandreas@kth.se

Acknowledgments

- Steven H. Low, Jayakrishnan Nair (Caltech) and Lachlan Andrew (Swinburne)
- Henrik Sandberg and Karl Henrik Johansson (KTH)

Acknowledgments

- Steven H. Low, Jayakrishnan Nair (Caltech) and Lachlan Andrew (Swinburne)
- Henrik Sandberg and Karl Henrik Johansson (KTH)

Martin Andreasson
mandreas@kth.se

Acknowledgments

- Steven H. Low, Jayakrishnan Nair (Caltech) and Lachlan Andrew (Swinburne)
- Henrik Sandberg and Karl Henrik Johansson (KTH)