KTH ROYAL INSTITUTE OF TECHNOLOGY

Structured Model Reduction of Networks of Passive Systems

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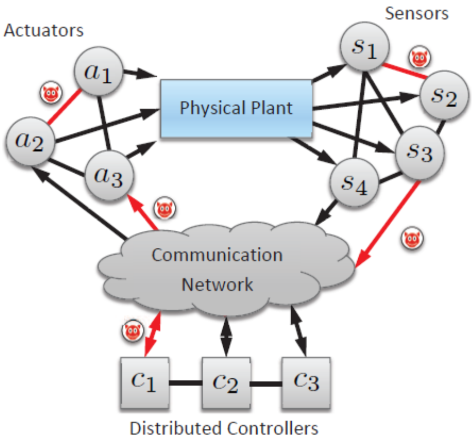
Outline

- Introduction
- Part I: Clustering-based model reduction of networked passive systems
- Part II: Coherency-independent structured model reduction of power systems
- Summary

Motivation: Networked Systems

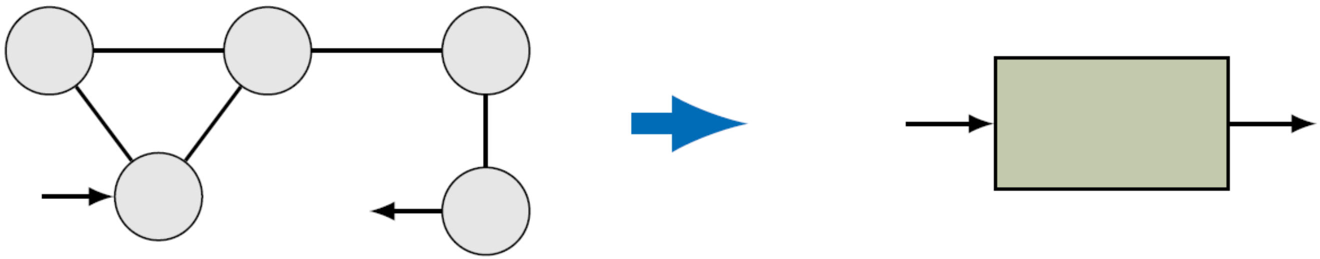


Challenges

- Dynamics dependent on subsystems and interconnection
- Large-scale interconnection complicates analysis, simulation, and synthesis

Goal. Model reduction of large-scale networked systems

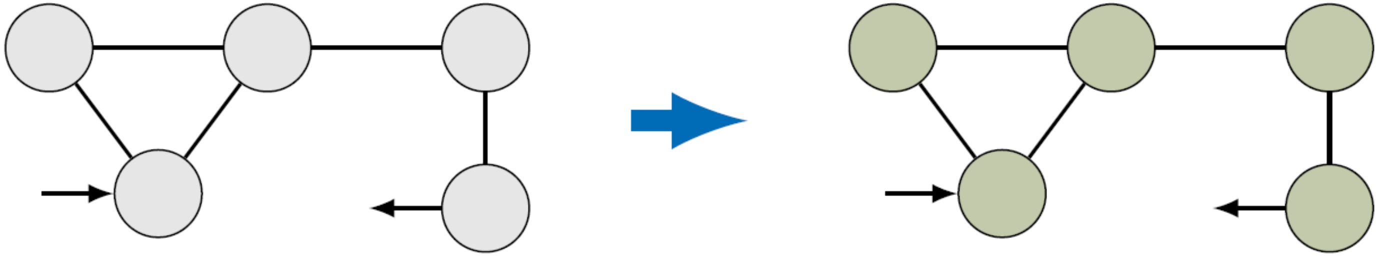
Related Work



General methods

- Balanced truncation (Moore, Glover,…)
- Hankel-norm approximation (Glover,…)
- Moment matching/Krylov-subspace methods (Antoulas, Astolfi, Benner,…)

Related Work

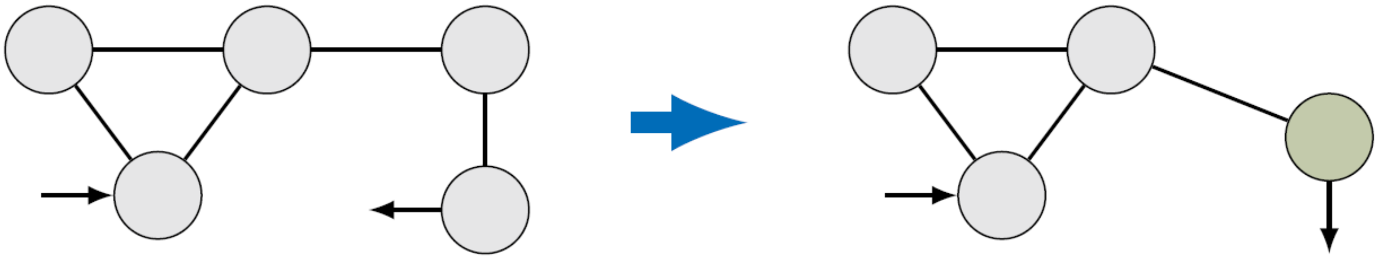


Reduction of subsystems, i.e., structured reduction

- Controller reduction/closed-loop model reduction (Anderson, Zhou, De Moor, …)
- Structured balanced truncation (Beck, Van Dooren, Sandberg,…)

- Example in Part II

Related Work

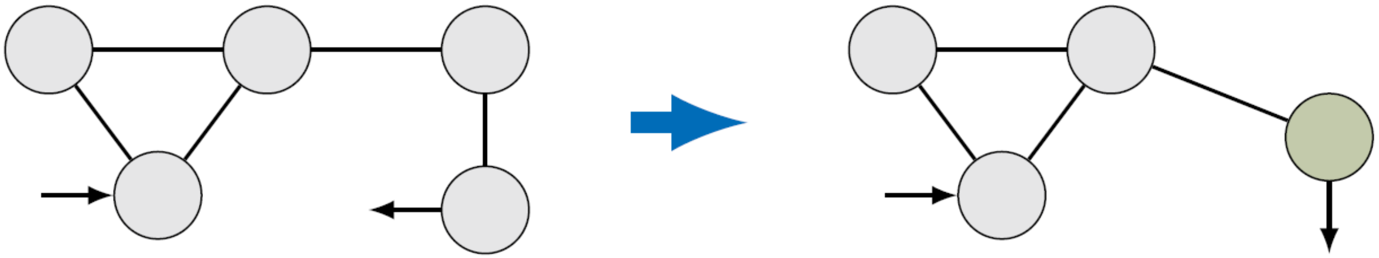


Clustering-based model reduction

- Time-scale separation (Chow, Kokotovic,…)
- Graph-based clustering (Ishizaki, Monshizadeh, Trentelman…)
- Structured balanced truncation (Besselink,…)

- Example in Part I and II

Part I: Clustering-based model reduction of networked passive systems

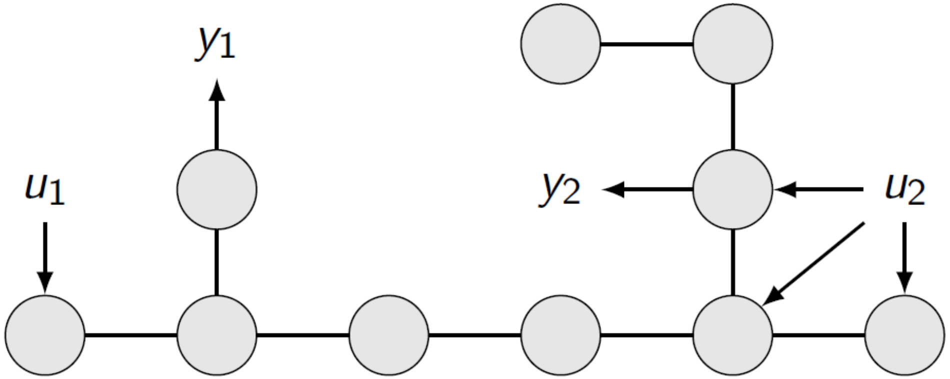


Problem and results

- Subsystems with identical higher-order dynamics
- Controllability/observability-based cluster selection
- A priori H_∞-error bound and preserved synchronization (cf. balanced truncation)

Reference. Besselink, Sandberg, Johansson: "Clustering-Based Model Reduction of Networked Passive Systems". IEEE Trans. on Automatic Control, 61:10, pp. 2958--2973, October 2016.

Modeling



1. *Identical* subsystem dynamics

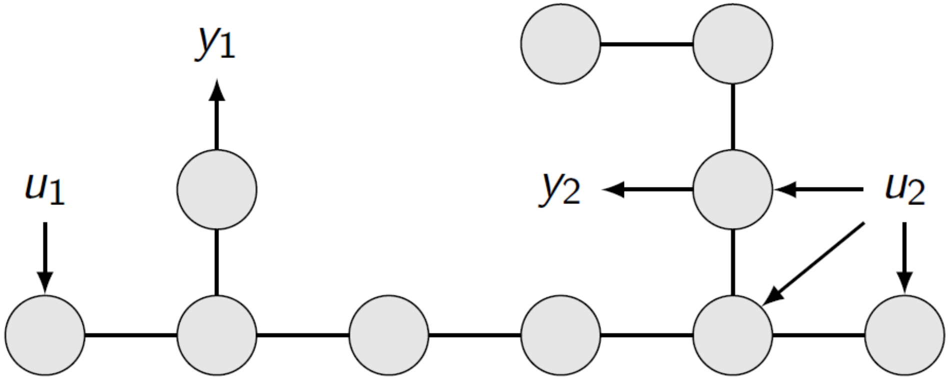
$$\Sigma_i : \ \dot{x}_i = Ax_i + Bv_i, \ \ z_i = Cx_i, \quad x_i \in \mathbb{R}^n, \ v_i, z_i \in \mathbb{R}^m$$

2. Interconnection topology with $w_{ij} \geq 0$

$$v_i = \sum_{j=1,\, j\neq i}^{\bar{n}} w_{ij}(z_j - z_i) + \sum_{j=1}^{\bar{m}} g_{ij} u_j$$

3. External outputs $\quad y_i = \sum_{j=1}^{\bar{n}} h_{ij} z_j$

Assumptions



A1. The subsystems Σ_i are *passive* with storage function $V_i(x_i) = \frac{1}{2} x_i^T Q x_i$ (supply$_i = v_i^T z_i$)

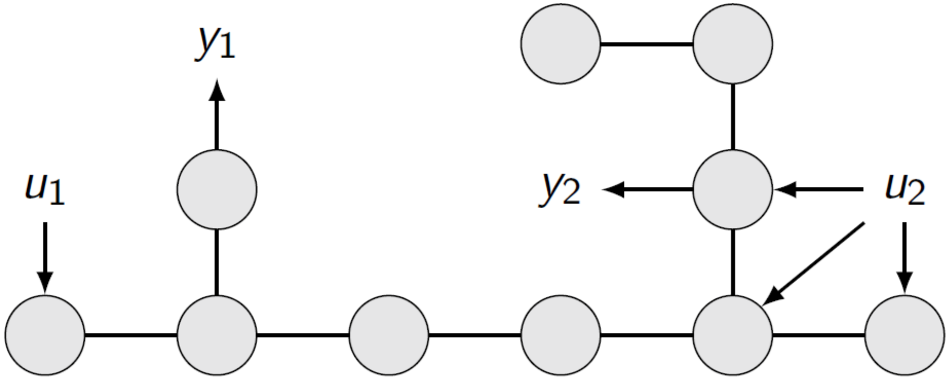
A2. The graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with graph Laplacian L is such that

a) The underlying undirected graph is a *tree*

b) $\mathcal{G}$ contains a directed rooted spanning tree

$$(L)_{ij} = \begin{cases} -w_{ij}, & i \neq j \\ \sum_{j=1, j\neq i}^{\bar{n}} w_{ij}, & i = j \end{cases}$$

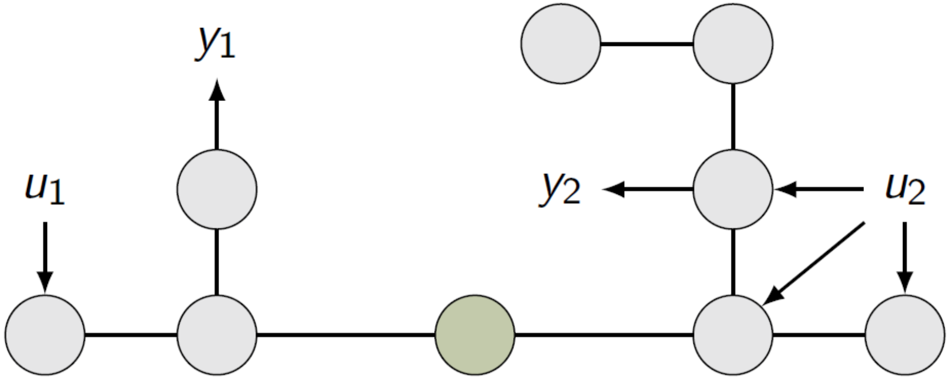
Network Synchronization



Lemma. Under **A1** and **A2**, the subsystems of Σ synchronize for $u = 0$, i.e., for all $(i, j) \in \mathcal{V} \times \mathcal{V}$,

$$\lim_{t \to \infty} \big(x_i(t) - x_j(t)\big) = 0$$

Problem and Approach

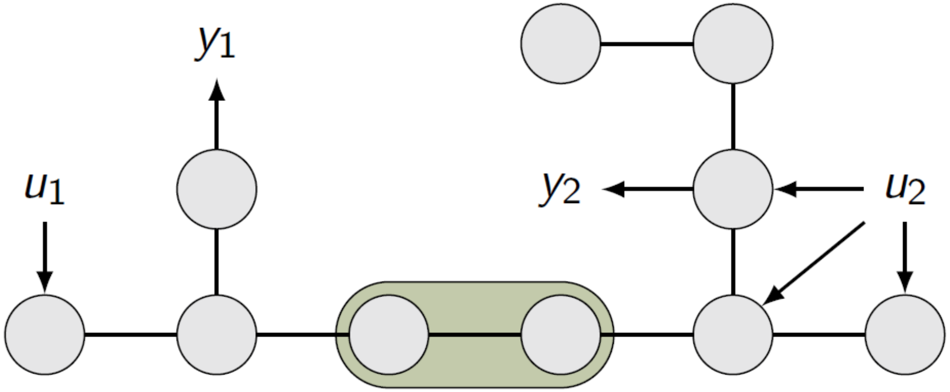


$$\Sigma : \begin{cases} \dot{x} = (I \otimes A - L \otimes BC)x + (G \otimes B)u \\ y = (H \otimes C)x \end{cases}$$

Goal. Approximate the input-output behavior of Σ by a clustering-based reduced-order system $\hat{\Sigma}$

$$\hat{\Sigma} : \begin{cases} \dot{\xi} = (I \otimes A - \hat{L} \otimes BC)\xi + (\hat{G} \otimes B)u \\ \hat{y} = (\hat{H} \otimes C)\xi \end{cases}$$

Problem and Approach

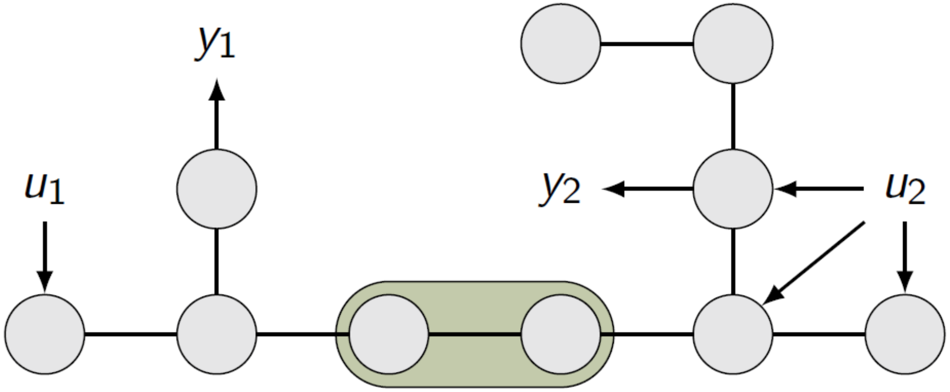


$$\Sigma : \begin{cases} \dot{x} = (I \otimes A - L \otimes BC)x + (G \otimes B)u \\ y = (H \otimes C)x \end{cases}$$

Wish list for approximation method

1. Preserve synchronization and passivity
2. Identify suitable clusters
3. Provide a priori bound on $||y - \hat{y}||$
4. Be scalable in system size (#nodes = $\bar{n}$, state dim. Σ = $n \times \bar{n}$)

Problem and Approach

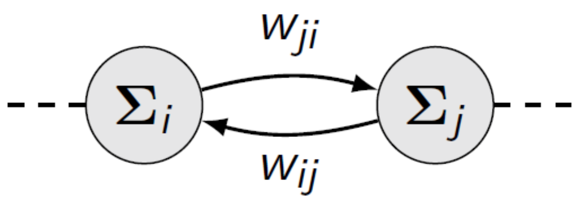


$$\Sigma : \begin{cases} \dot{x} = (I \otimes A - L \otimes BC)x + (G \otimes B)u \\ y = (H \otimes C)x \end{cases}$$

Idea. Find neighboring subsystems Σ_i that are

- hard to steer individually from the inputs
- hard to distinguish from the outputs

Edge Laplacian L_{e}



Lemma. Consider L and let E be an oriented incidence matrix of the underlying undirected graph. Then,

$$L = FE^{\mathrm{T}} \in \mathbb{R}^{\bar{n}\times\bar{n}}$$

Lemma. Under **A2**, the edge Laplacian

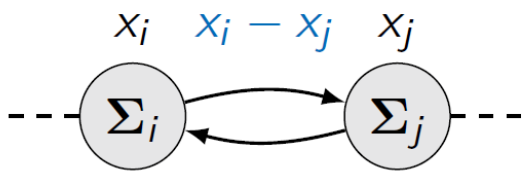
$$L_{\mathrm{e}} = E^{\mathrm{T}}F \in \mathbb{R}^{(\bar{n}-1)\times(\bar{n}-1)}$$

has all eigenvalues in the open right-half complex plane

$$E = \begin{bmatrix} * & e_i - e_j & * \end{bmatrix}, \quad F = \begin{bmatrix} * & w_{ij}e_i - w_{ji}e_j & * \end{bmatrix}$$

$$\# \text{ nodes} = \bar{n}$$

Edge Dynamics and Controllability



Edge system in coordinates $x_e = (E^T \otimes I)x$

$$\Sigma_e : \quad \dot{x}_e = (I \otimes A - L_e \otimes BC)x_e + (E^T G \otimes B)u, \quad y_e = (H_e \otimes C)x_e$$

Σ_i and Σ_j are hard to steer individually ⟷ weakly controllable coordinate in Σ_e

Edge controllability gramian P_e characterizes controllability

$$x_e^T P_e^{-1} x_e = \inf_{\{u \mid 0 \rightsquigarrow x_e\}} \int_{-\infty}^{0} |u(t)|^2 \, dt$$

Challenges

- P_e dependent on subsystems and interconnection topology
- Role of individual edges not apparent from P_e

Edge Dynamics and Controllability

Theorem. The edge controllability Gramian P_e can be bounded as

$$P_e \preccurlyeq \Pi^c \otimes Q^{-1}$$

if there exists $\Pi^c = \mathrm{diag}\{\pi_1^c, \ldots, \pi_{\bar{n}-1}^c\} \succcurlyeq 0$ such that

$$L_e \Pi^c + \Pi^c L_e^{\mathrm{T}} - E^{\mathrm{T}} G G^{\mathrm{T}} E \succcurlyeq 0$$

Properties

- Gramian can be defined as Σ_e is asymptotically stable
- $\Pi_c \in \mathbb{R}^{(\bar{n}-1)\times(\bar{n}-1)}$ only dependent on interconnection properties
- Measure of controllability for each individual edge

Lemma. $\Pi^c \succcurlyeq 0$ exists if $w_{ij} > 0 \Leftrightarrow w_{ji} > 0$

Edge Singular values

Generalized edge controllability Gramian

$$\Pi^{\mathsf{c}} = \mathrm{diag}\{\pi_1^{\mathsf{c}}, \ldots, \pi_{\bar{n}-1}^{\mathsf{c}}\}, \quad L_{\mathsf{e}}\Pi^{\mathsf{c}} + \Pi^{\mathsf{c}} L_{\mathsf{e}}^{\mathrm{T}} - E^{\mathrm{T}} G G^{\mathrm{T}} E \succcurlyeq 0$$

Generalized edge observability Gramian

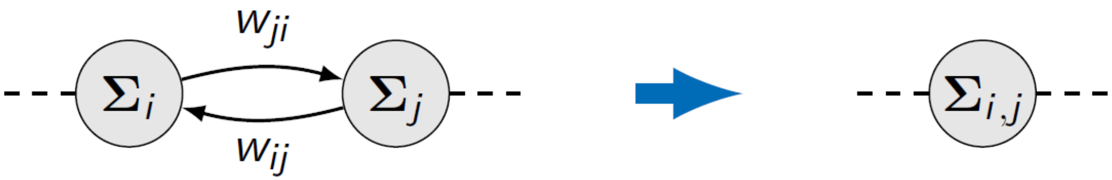
$$\Pi^{\mathsf{o}} = \mathrm{diag}\{\pi_1^{\mathsf{o}}, \ldots, \pi_{\bar{n}-1}^{\mathsf{o}}\}, \quad L_{\mathsf{e}}^{\mathrm{T}}\Pi^{\mathsf{o}} + \Pi^{\mathsf{o}} L_{\mathsf{e}} - F^{\mathrm{T}} H^{\mathrm{T}} H F \succcurlyeq 0$$

Generalized squared edge singular values

$$(L_{\mathsf{e}}^{-1})_{ii}^2 \pi_i^{\mathsf{c}} \pi_i^{\mathsf{o}} \geq (L_{\mathsf{e}}^{-1})_{i+1,i+1}^2 \pi_{i+1}^{\mathsf{c}} \pi_{i+1}^{\mathsf{o}} \geq 0, \quad i = 1, \ldots, \bar{n} - 1$$

Note. Minimize trace of Π_c and Π_o to obtain unique Gramians and small singular values

One-step Clustering



$$V = \begin{bmatrix} I & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad W = \begin{bmatrix} I & 0 \\ 0 & \frac{w_{ji}}{w_{ij}+w_{ji}} \\ 0 & \frac{w_{ij}}{w_{ij}+w_{ji}} \end{bmatrix}$$

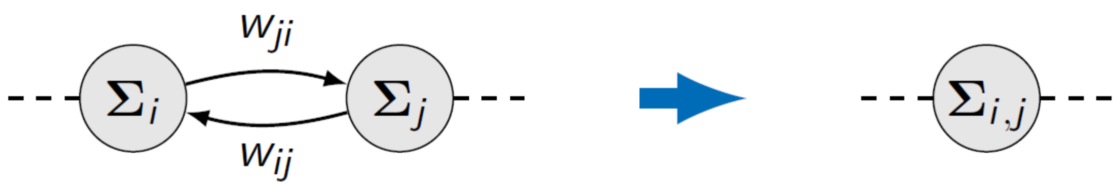
Reduced-order system

Petrov-Galerkin projection of graph Laplacian

$$\hat{\Sigma}_{\bar{n}-1}: \; \dot{\xi} = (I \otimes A - \hat{L} \otimes BC)\xi + (\hat{G} \otimes B)u, \;\; \hat{y} = (\hat{H} \otimes C)\xi$$

$$\text{with} \quad \hat{L} = W^{\mathrm{T}} L V, \quad \hat{G} = W^{\mathrm{T}} G, \quad \hat{H} = HV$$

One-step Clustering



$$V = \begin{bmatrix} I & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad W = \begin{bmatrix} I & 0 \\ 0 & \frac{w_{ji}}{w_{ij}+w_{ji}} \\ 0 & \frac{w_{ij}}{w_{ij}+w_{ji}} \end{bmatrix}$$

Theorem. Consider Σ and the one-step clustered $\hat{\Sigma}_{\bar{n}-1}$. Then,

1. The edge controllability Gramian of $\hat{\Sigma}_{\bar{n}-1}$ satisfies

$$\hat{P}_e \preccurlyeq \hat{\Pi}^c \otimes Q^{-1}, \quad \hat{\Pi}^c = \mathrm{diag}\{\pi_1^c, \ldots, \pi_{\bar{n}-2}^c\}$$

2. The edge observability Gramian of $\hat{\Sigma}_{\bar{n}-1}$ satisfies

$$\hat{Q}_e \preccurlyeq \hat{\Pi}^o \otimes Q, \quad \hat{\Pi}^o = \mathrm{diag}\{\pi_1^o, \ldots, \pi_{\bar{n}-2}^o\}$$

Opens up for repeated one-step clustering!

Performance Guarantees

Theorem. The subsystems of $\hat{\Sigma}_{\bar{k}}$ synchronize for $u = 0$, i.e.,

$$\lim_{t \to \infty} \left(\xi_i(t) - \xi_j(t) \right) = 0, \qquad (i,j) \in \hat{\mathcal{V}} \times \hat{\mathcal{V}}$$

Theorem. For trajectories $x(\cdot)$ of $\boldsymbol{\Sigma}$ and $\xi(\cdot)$ of $\hat{\Sigma}_{\bar{k}}$ for the same input $u(\cdot)$ and $x(0) = 0$, $\xi(0) = 0$, the output error is bounded as

$$\|y - \hat{y}\|_2 \leq 2 \left(\sum_{l=\bar{k}}^{\bar{n}-1} \left(L_\mathrm{e}^{-1} \right)_{ll} \sqrt{\pi_l^\mathrm{c} \pi_l^\mathrm{o}} \right) \|u\|_2$$

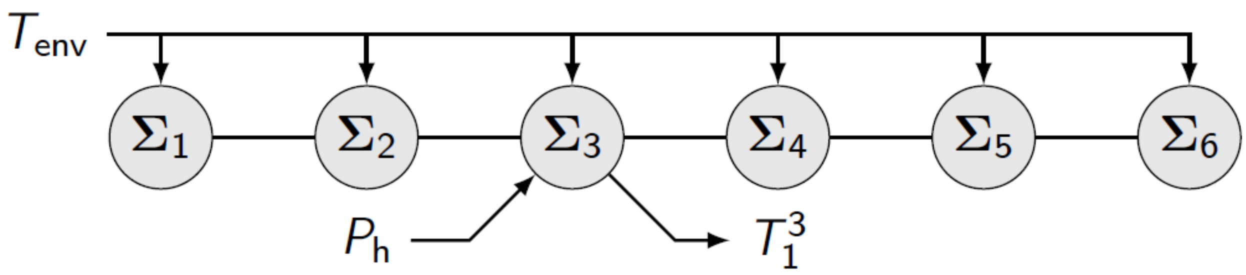
Generalized edge singular values

Summary So Far

Wish list for approximation method

1. Preserve synchronization and passivity
   - OK
2. Identify suitable clusters
   - Use generalized edge singular values
3. Provide a priori bound on $||y - \hat{y}||$
   - Generalized edge singular values provide bounds
4. Be scalable in system size (#nodes = $\bar{n}$, state dim. $\Sigma = n \times \bar{n}$)
   - Solve two LMIs of size $\bar{n}$ (independent of subsystem size n) [and possibly one Riccati equation of size n to verify passivity]

Example: Thermal Model of a Corridor of Six Rooms



Subsystems: thermal dynamics within a room

$$C_1 \dot{T}_1^i = R_{\mathrm{int}}^{-1}(T_2^i - T_1^i) - R_{\mathrm{out}}^{-1} T_1^i + P_i$$
$$C_2 \dot{T}_2^i = R_{\mathrm{int}}^{-1}(T_1^i - T_2^i)$$

Edges: thermal resistances of walls, $u_j = [\, P_{\mathrm{h}} \;\; T_{\mathrm{env}} \,]^{\mathrm{T}}$

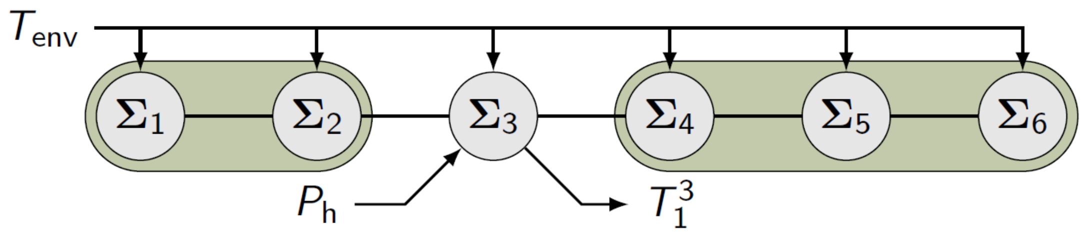
$$P_i = \sum_{j=1, j\neq i}^{\bar{n}} R_{\mathrm{wall}}^{-1}(T_1^j - T_1^j) + \sum_{j=1}^{\bar{m}} g_{ij} u_j$$

Reduction from $\bar{n} = 6$ to $\bar{k} = 3$

Example: Thermal Model of a Corridor of Six Rooms

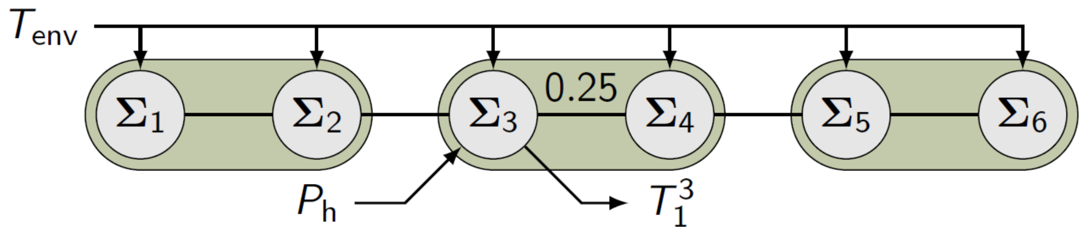
Edge singular values: $\bar{\pi}_l := (L_\mathrm{e}^{-1})_{ll}\sqrt{\pi_l^\mathrm{c}\pi_l^\mathrm{o}}$

l	1	2	3	4	5
$10^2\bar{\pi}_l$	0.145	0.951	1.267	0.337	0.092



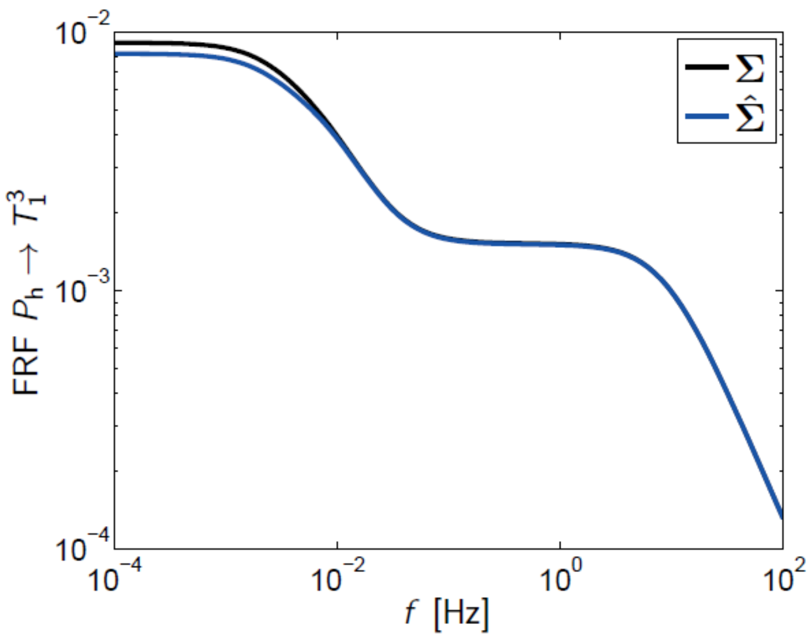
75% reduction of wall 3 resistance

l	1	2	3	4	5
$10^2\bar{\pi}_l$	0.146	0.999	0.308	0.373	0.109



Example: Thermal Model of a Corridor of Six Rooms

Frequency response function from input P_{h} to output T_1^3



Error bound: $2\sum_{l=3}^{5}\left(L_{\mathrm{e}}^{-1}\right)_{ll}\sqrt{\pi_l^{\mathrm{c}}\pi_l^{\mathrm{o}}} = 11.4\cdot 10^{-3}$

Actual error: $0.849\cdot 10^{-3}$

Summary Part I

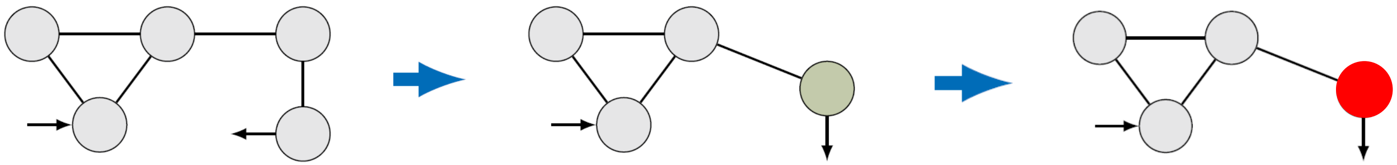
- Clustering-based reduction procedure
- Edge controllability and observability properties
- Preservation of synchronization and error bound

Possible extensions

- Arbitrary network topology
- Non-identical subsystems
- Nonlinear networked systems
- Lower bounds

Reference. Besselink, Sandberg, Johansson: "Clustering-Based Model Reduction of Networked Passive Systems". IEEE Trans. on Automatic Control, 61:10, pp. 2958--2973, October 2016

Part II: Coherency-independent structured model reduction of power systems



Problem and results

- Model reduction of nonlinear large-scale power system
- Clustering, linearization, and reduction of external area
- Application of structured balanced truncation

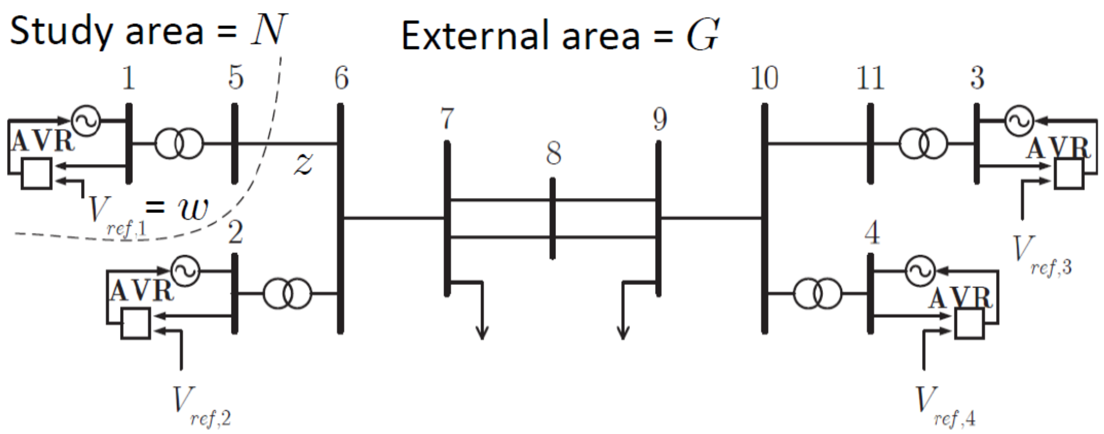
Reference. Sturk, Vanfretti, Chompoobutrgool, Sandberg: "Coherency-Independent Structured Model Reduction of Power Systems". IEEE Trans. on Power Systems, 29:5, pp. 2418--2426, September 2014.

Background

- Increasingly interconnected power systems
- New challenges for dynamic simulation, operation, and control of large-scale power systems
- Coherency-based power system model reduction not always suitable

Approach

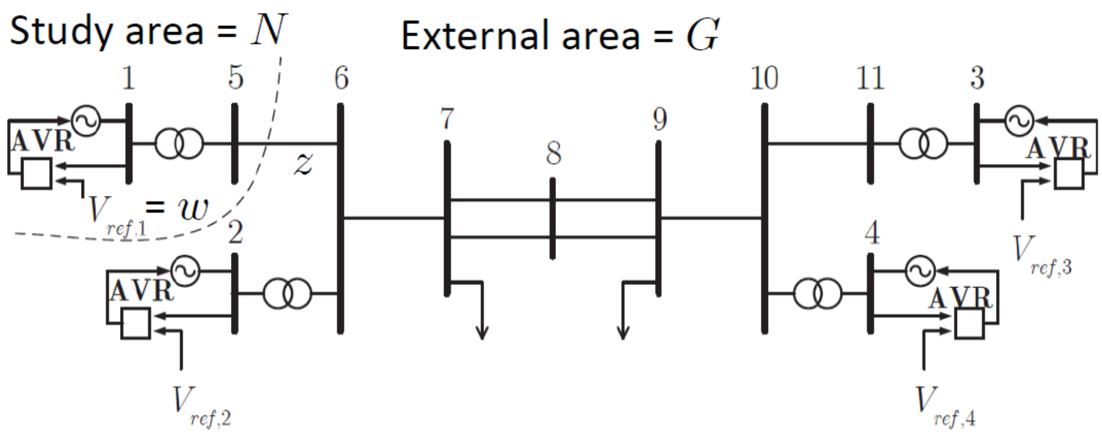
Divide system into a study area and an external area



Objective: Reduce the external area so that the effect of the approximation error in the study area is as small as possible

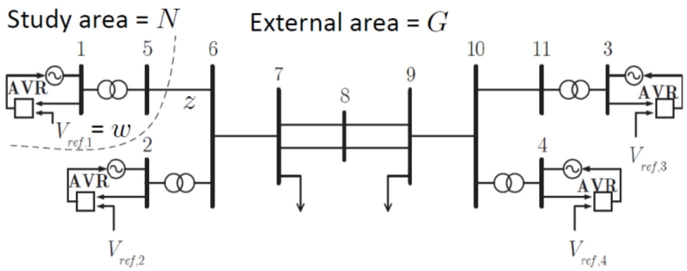
Approach

Divide system into a study area and an external area



- Study area N often set by utility ownership or market area. Nonlinear model will be retained here
- External area G denotes other utilities. Will be linearized and reduced here
- Insight from structured/closed-loop model reduction: Reduction of G should depend on N!

Four-Step Procedure



1. Define the model (DAE)

$$\dot{x} = f(x, x_{\text{alg}}, u)$$
$$0 = g(x, x_{\text{alg}}, u)$$

$$\begin{cases} \dot{x}^G = f^G\left(x^G, x^G_{\text{alg}}, u^G\right) \\ 0 = g^G\left(x^G, x^G_{\text{alg}}, u^G\right) \\ \dot{x}^N = f^N\left(x^N, x^N_{\text{alg}}, u^N_1, u^N_2\right) \\ 0 = g^N\left(x^N, x^N_{\text{alg}}, u^N_1, u^N_2\right) \end{cases}$$

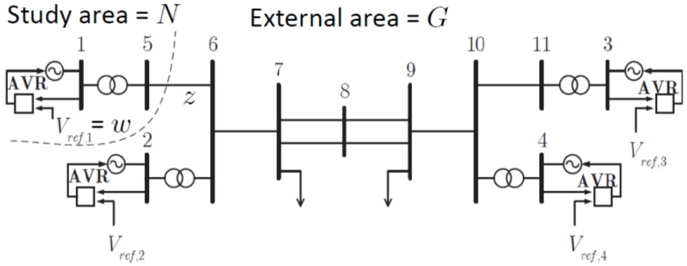
2. Linearizing

$$\begin{pmatrix} \dot{x}^G \\ 0 \end{pmatrix} = \begin{pmatrix} A^G_{11} & A^G_{12} \\ A^G_{21} & A^G_{22} \end{pmatrix} \begin{pmatrix} x^G \\ x^G_{\text{alg}} \end{pmatrix} + \begin{pmatrix} B^G_1 \\ B^G_2 \end{pmatrix} u^G$$

$$\begin{pmatrix} \dot{x}^N \\ 0 \end{pmatrix} = \begin{pmatrix} A^N_{11} & A^N_{12} \\ A^N_{21} & A^N_{22} \end{pmatrix} \begin{pmatrix} x^N \\ x^N_{\text{alg}} \end{pmatrix} + \begin{pmatrix} B^N_{11} & B^N_{12} \\ B^N_{21} & B^N_{22} \end{pmatrix} \begin{pmatrix} u^N_1 \\ u^N_2 \end{pmatrix}.$$

$$x^G_{\text{alg}} = -A^{G^{-1}}_{22}\left(A^G_{21}x^G + B^G_2 u^G\right)$$
$$x^N_{\text{alg}} = -A^{N^{-1}}_{22}\left(A^N_{21}x^N + B^N_{21}u^N_1 + B^N_{22}u^N_2\right)$$

Four-Step Procedure



3. Structured/closed-loop model reduction of external area model, $G \rightarrow \hat{G}$ (details next)

4. Nonlinear complete reduced model

$$\left.\begin{aligned} \dot{x}^{\hat{G}} &= A^{\hat{G}} x^{\hat{G}} + B^{\hat{G}} u^{\hat{G}} \\ u_2^N = y^{\hat{G}} &= C^{\hat{G}} x^{\hat{G}} + D^{\hat{G}} u^{\hat{G}} \end{aligned}\right\} \text{Reduced linear external area}$$

$$\left.\begin{aligned} \dot{x}^N &= f^N\left(x^N, x_{\text{alg}}^N, u_1^N, u_2^N\right) \\ 0 &= g^N\left(x^N, x_{\text{alg}}^N, u_1^N, u_2^N\right) \end{aligned}\right\} \text{Unreduced nonlinear study area}$$

$$u^{\hat{G}} = y_2^N = M^N x_{\text{alg}}^N.$$

Structured Model Reduction of G

(Following Schelfhout/De Moor, Vandendorpe/Van Dooren, Sandberg/Murray): $(N, G) = \Sigma(A, B, C, D)$

$$AP + PA^T + BB^T = 0, \qquad A^TQ + QA + C^TC = 0$$

$$P = \begin{bmatrix} P_N & P_{NG} \\ P_{NG}^T & P_G \end{bmatrix}, \qquad Q = \begin{bmatrix} Q_N & Q_{NG} \\ Q_{NG}^T & Q_G \end{bmatrix}$$

Local balancing of G only:

$$\Sigma_G = T_G^{-1} P_G T_G^{-T} = T_G^T Q_G T_G$$

Structured (Hankel) singular values of G:

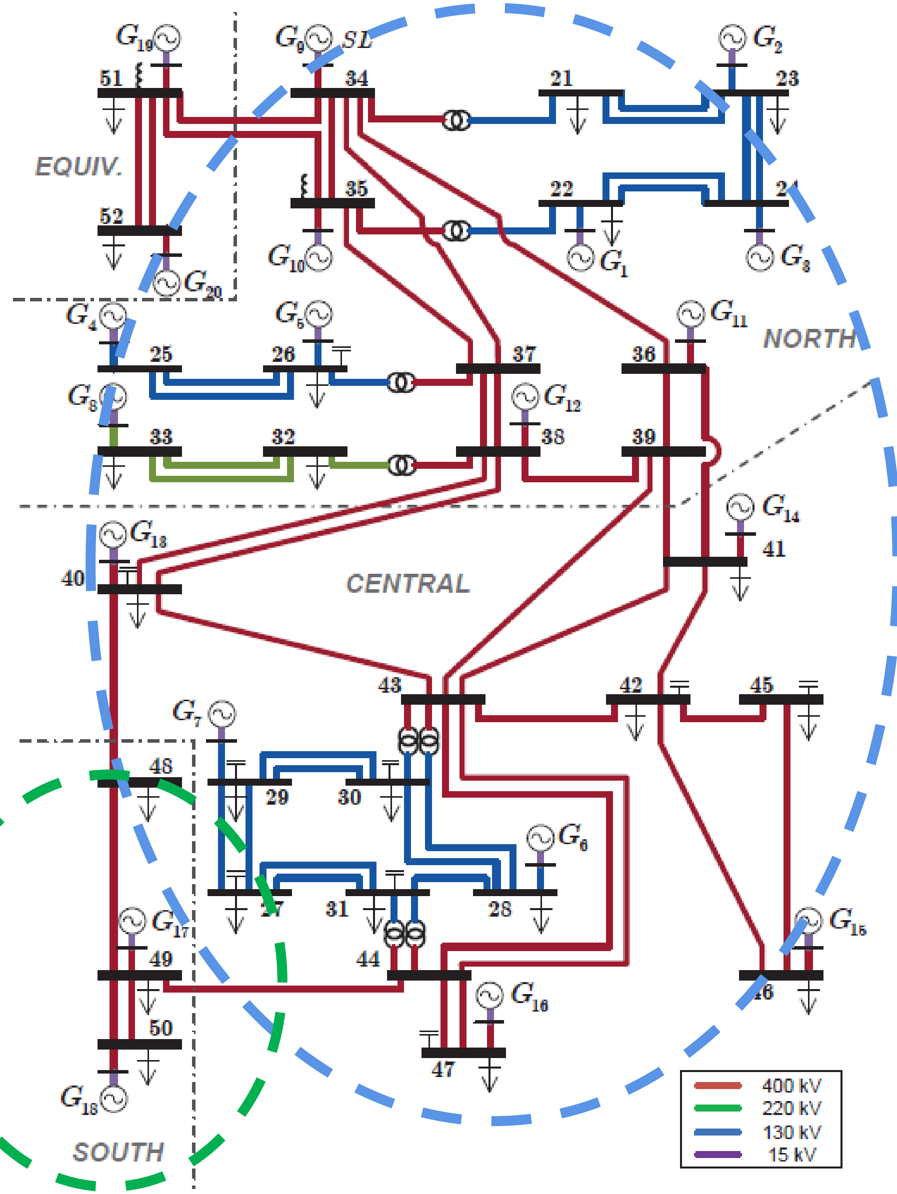
$$\Sigma_G = \operatorname{diag}\{\sigma_{G,1}, \sigma_{G,2}, \ldots, \sigma_{G,n}\}$$

Truncation or singular perturbation of G yields $\hat{G}$

Note 1. $\hat{G}$ depends on study area N

Note 2. Error bound and stability guarantee require generalized Gramians (LMIs) [Sandberg/Murray]

Model Reduction of Non-Coherent Areas: KTH-Nordic32 System



Model info:

- 52 buses
- 52 lines
- 28 transformers
- 20 generators (12 hydro gen.)

External area:
Simplify as much as possible

Study area:
Southern Sweden. Keep detailed model

Model Reduction of Non-Coherent Areas: KTH-Nordic32 System

- External area G has 246 dynamic states.
- *Reduced* external area $\hat{G}$ has 17 dynamic states

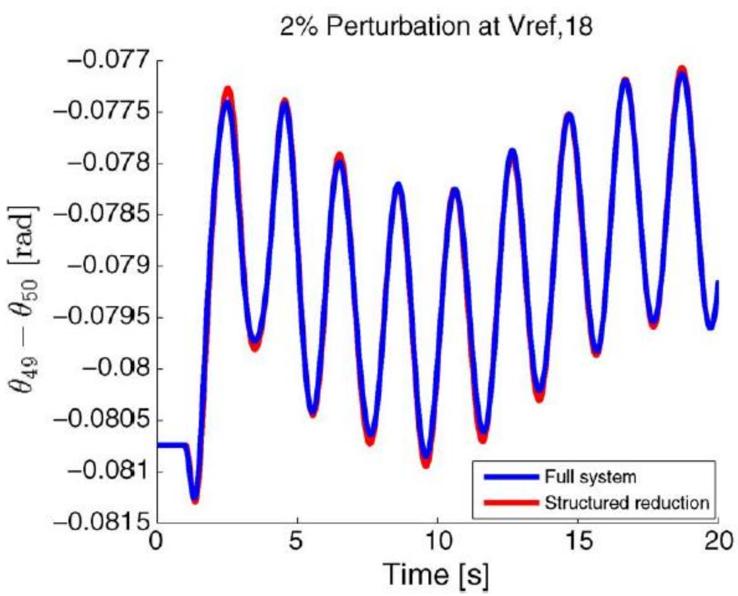


Fig. 8. Responses of $\theta_{49} - \theta_{50}$ after a 2% perturbation to $V_{ref,18}$.

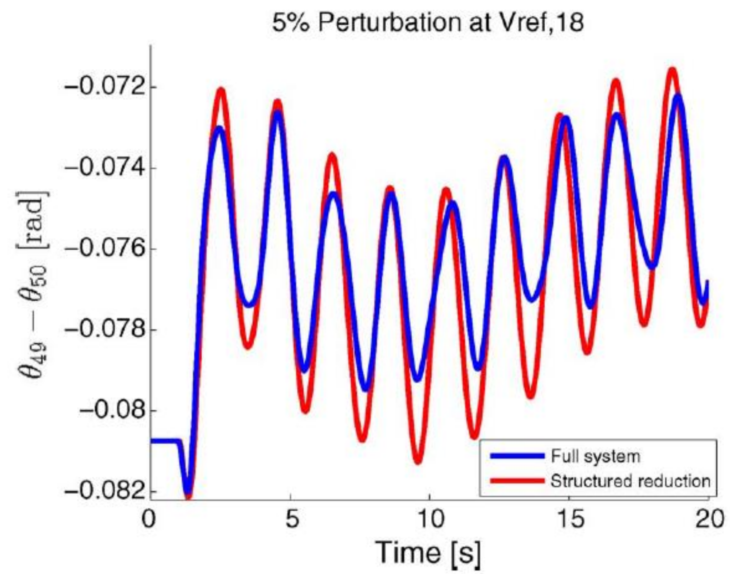


Fig. 9. Responses of $\theta_{49} - \theta_{50}$ after a 5% perturbation to $V_{ref,18}$.

Model Reduction of Non-Coherent Areas: KTH-Nordic32 System

- External area G has 246 dynamic states.
- *Reduced* external area $\hat{G}$ has 17 dynamic states

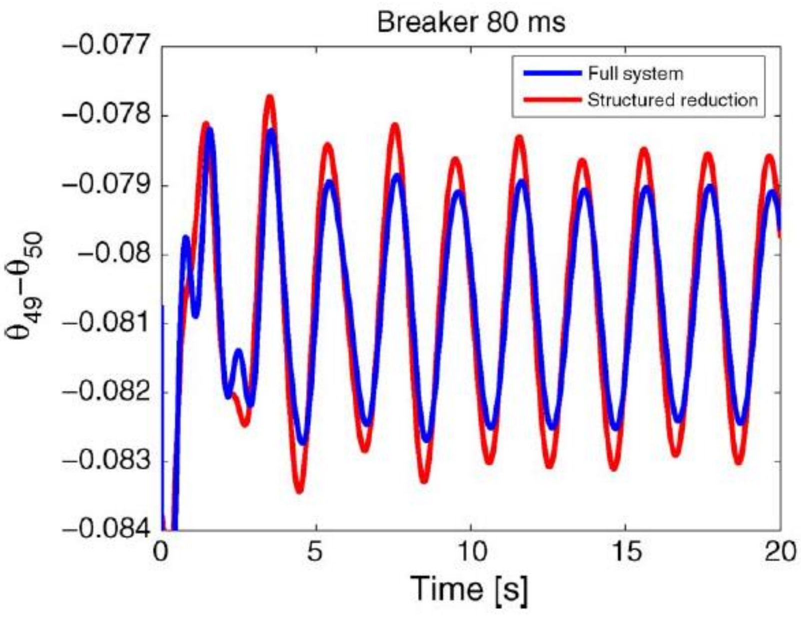


Fig. 12. Responses of $\theta_{49} - \theta_{50}$ after opening a line for 80 ms.

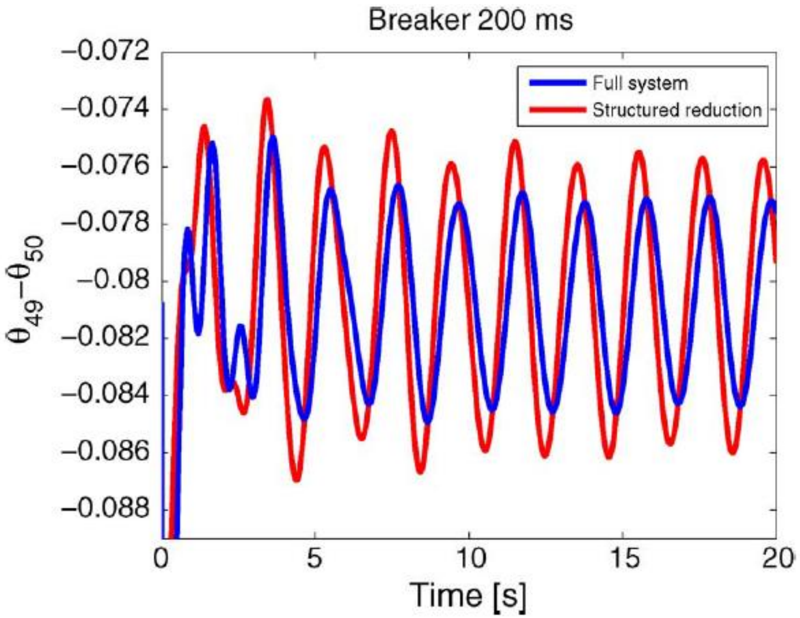


Fig. 13. Responses of $\theta_{49} - \theta_{50}$ after opening a line for 200 ms.

What If Open-Loop Reduction Used to Simplify External Area G?

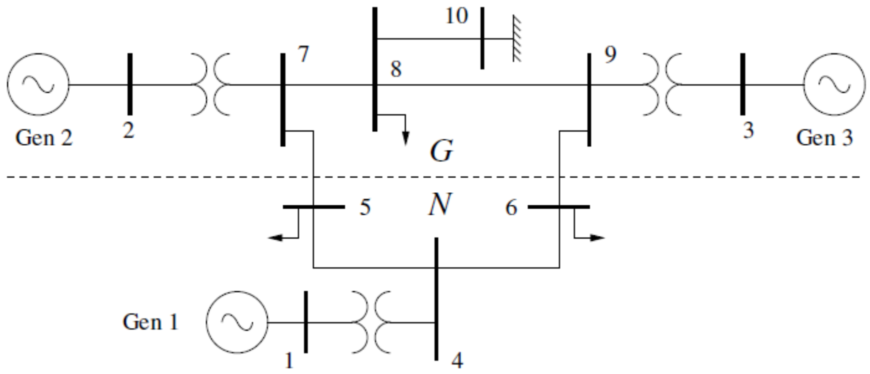


Fig. 3. The WSCC 3-machine, 9-bus system with an infinite bus.

[Sturk *et al.:*
"Structured Model Reduction
of Power Systems", ACC 2012]

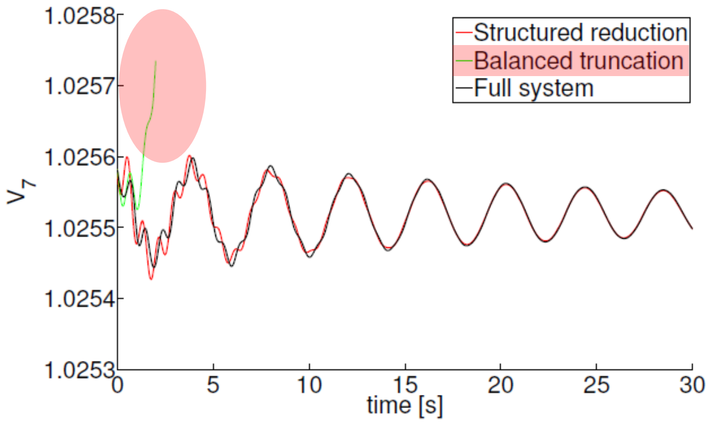


Fig. 5. Transients of V_7 at the tie-line bus with a third order system $\hat{G}$.

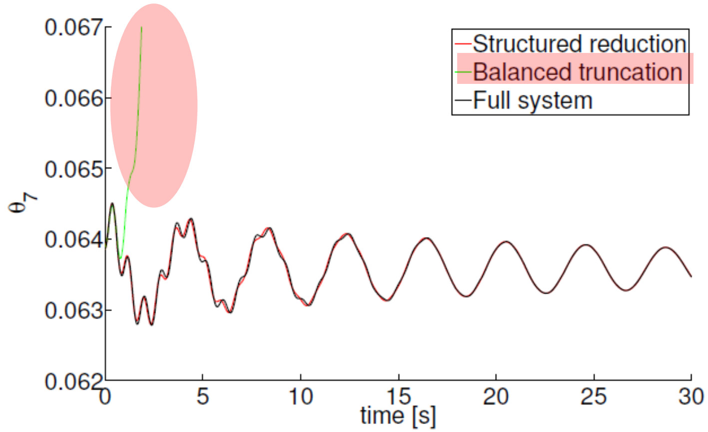


Fig. 6. Transients of θ_7 at the tie-line bus with a third order system $\hat{G}$. Structured model reduction and ordinary balanced truncation are compared with the full system.

Summary Part II

- Clustering, linearization, and reduction of external power system area
- Application of structured balanced truncation: Closed-loop behavior matters!
- Verification on a model of the Nordic grid

Possible extensions

- Nonlinear model reduction with error bounds and stability guarantees

Reference. Sturk, Vanfretti, Chompoobutrgool, Sandberg: "Coherency-Independent Structured Model Reduction of Power Systems". IEEE Trans. on Power Systems, 29:5, pp. 2418--2426, September 2014.

Concluding Remarks

- Model reduction of networked systems. Dynamics dependent on subsystems and interconnection. **Many applications!**

- Model reduction methods could reduce topology and/or dynamics

Challenge. Many heuristics possible. We want rigorous scalable methods with performance guarantees.

- Balanced truncation and Hankel-norm approximation do not preserve network structures very well
- LMIs are very expensive to solve [$\sim \mathcal{O}(n^{5.5})$]

Thank You!

Sponsors

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Knut och Alice Wallenbergs Stiftelse

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