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Abstract: Monotonicity formulae have frequently turned out to be usefuls tools in studying nonlinear partial differential equations. They have been helpful deriving growth estimates, analyzing asymptotic behavior, proving regularity etc.

However, most monotonicity formulae are as such only valid at interior points of the domain and can therefore not be applied directly to the analysis at the boundary of the domain. In the parabolic setting the situation is even worse, as many known monotonicity formulae do not even work at interior points in the case of non-trivial domains. Depending on the equation this difficulty may not be avoided by cut-off or perturbation arguments (cf. blow-up for the heat equation). Even in cases where a perturbation might be possible, a genuine monotonicity formula (not a perturbation) is desirable when studying qualitative properties of solutions that are neither of a microscopic nor of a very large order.

For a certain class of nonlinear elliptic and parabolic equations we present monotonicity formulae at the boundary and discuss applications to free boundary problems.