O. Savin

On a conjecture of DeGiorgi.

Abstract: We consider local minimizers of the Ginzburg-Landau energy functional

$$\int \frac{1}{2}|\nabla u|^2 + \frac{1}{4}(1-u^2)^2 dx$$

and prove that, if the 0 level set is included in a flat cylinder then, in the interior, it is included in a flatter cylinder. As a consequence we prove a conjecture of De Giorgi which states that level sets of global solutions of

$$\triangle u = u^3 - u$$

such that

$$|u| \leq 1, \quad \partial_n u > 0, \quad \lim_{x_n \to \pm\infty} u(x', x_n) = \pm 1$$

are hyperplanes in dimension $n \leq 8$.