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On the distance function to the boundary,cut locus, and the singular set of solutions of some Hamilton-Jacobi equations.

Abastract: In a bounded smooth domain D in $\mathbb{R}^n$, we consider the closed set Z where the distance function to the boundary is not smooth.From a point y on the boundary we follow the interior normal util it first hits Z. We prove that the length of this ray i s Lipschitz continuous in y. Cor.:The singular set has finite n-1 Hausdorff measure.These results are then extended to Finsler metrics and are then used to show that the singular set of viscosity solutions of Hamilton-Jacobi equations of the form

H(x,Du)= 1 in D ,u=0 on the boundary of D, have finite n-1 dimensional Hausdorff measure.