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Layer solutions in a halfspace for boundary reactions.

Abstract: We consider harmonic functions in a halfspace $\mathbb{R}^n_+ = \{(x, y) \in \mathbb{R} \times \mathbb{R}^{n-1} : x > 0\}$ subject to nonlinear Neumann boundary conditions. We study bounded solutions which are monotone increasing from -1 to 1 in one of the y-variables. We call such functions *layer solutions*.

When $n = 2$, we establish that a necessary and sufficient condition for the existence of a layer solution is that the boundary energy potential (the primitive of the nonlinearity, up to a sign) has two, and only two, absolute minima in $[-1, 1]$, located at ± 1. In addition, we prove uniqueness of the layer solution up to translations in the y-variable.

When $n = 3$, we establish that every stable solution in $\mathbb{R}^3_+$ (and in particular, every layer solution, and every local minimizer) is in fact a function of only two variables.