KTH Matematik

Near Optimal Joint Channel and Power Allocation Algorithms in Multicell Networks

Master Thesis within Optimization and Systems Theory

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Abstract

We examine heuristic algorithms for assigning mobile users to base stations and allocating power in a multicell system with orthogonal channels. These algorithms are based on results from the problem of assigning users to base stations, assigning channels and allocate power jointly. That problem was recently shown to be NP hard. The objective in the power allocation examined in this work is either to maximize the minimum user throughput, a fair allocation, or to maximize the total system throughput. In addition a hybrid power allocation with an objective which is a combination of the two previously stated is examined. The heuristics are implemented and tested in the orthogonal frequency-division multiplexing (OFDM) RUNE simulator for a single channel seven cell system. The four different algorithms for assigning base stations to mobiles are compared and shown that two of those result in higher throughput in the system. Comparing the completely fair and the throughput optimal power allocation schemes, we find that in the fair system the total throughput is at most 66% of the total throughput of the throughput maximizing system. Furthermore, half of the time the total system throughput in the fair allocation is less than 24% of the maximum total system throughput. Finally results for the hybrid objective power allocation are reported.

Acknowledgment

Almost two years ago I packed my bags, boarded an airplane and moved from Iceland to Stockholm, Sweden. I started my master studies in mathematics at the Royal Institute of Technology; the last two years have been a wonderful journey which this thesis is a final point to. I would never have made it here if it were not for the continuous support and confidence in my abilities from my family.
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Stockholm, June 2010

Í minningu ástkærs föður míns, Odds Eggertssonar (1949-1994),
Hildur Æsa Oddsdóttir

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Chapter 1

Introduction

Cellular phones, also called mobiles, have been developed much over the last years; they are now not only used for making phone calls (voice traffic) but also to connect to the internet (data traffic). A cellular system consists of the mobile users and base stations, which the mobile connects to in order to make calls or send/receive data. The area around a base station is called a cell. When a mobile is sending information to the base station it is called uplink, downlink is then when the mobile is receiving information. In the types of system considered here mobiles can connect to the base stations on different frequencies or channels. In this work the system considered has orthogonal channels, this is true for a variety of modern cellular systems. Orthogonal channels minimize interference within the cell, intracell interference. Examples of systems with orthogonal channels are time and/or frequency division multiple access (TDMA, FDMA), orthogonal code division multiple access (CDMA) and orthogonal frequency division multiple access (OFDMA), see for example [2]. Since the intracell interference is minimized when the channels are orthogonal only interference between the cells needs to be considered. The quality of the link or connection between a base station and a mobile is described by the path gain, shadow fading and fast fading. For assigning mobile users to base stations (links) the path gain, which is dependent on a distance component, and the shadow fading component are considered. The fast fading component is then added to the path gain and shadow fading when the send power is considered. When a mobile has connected to a base station to send information it must use some power in order to send, in downlink the base station uses power to send to the mobile. The power received at the destination is equal to the power used to send the information times the path gain (distance component, shadow fading and fast fading) on that link. Signal to interference and noise ratio (SINR) is a measure on how good the received signal is.

Mobile users need to be assigned to base stations, channels and allocated power in order for the cellular system to function. This can be done through centralized algorithms or distributed. Centralized algorithms need information on the whole system while distributed work on a local scale. In this work we will consider centralized heuristic algorithms for allocating links, channels and power jointly to users in downlink and uplink. This is considered for a system which has orthogonal channels.

1.1 Previous Work and Contribution by This Thesis

The performance of a system with orthogonal channels is typically limited by the interference between cells, intercell interference. There has been an interest in intercell interference coordination schemes ever since such systems have been deployed [10]. Such intercell interference coordination algorithms usually attempt to minimize intercell interference by implementing a controller which allocates channels to be used within each cell and power to users "wisely". The intercell interference coordination problem is often formulated as an optimization task that either maximizes the system throughput subject to a power budget constraint or minimizes the overall power subject to an overall throughput or sum-rate target [1, 14].

Because the complexity of joint channel and power allocation is often prohibiting (even for moderate size systems), most previous works develop heuristic algorithms [1, 11, 12, 14], although exact solutions have been studied for small systems [7, 9]. Classical works on multicell power control by Zander [16], Foschini and Miljanic [6], Yates [15] and others consider the case of SINR target tracking, which equalizes the signal to interference and noise ratio instead of the received power. This technique works well for transferring voice, since it aims at a constant transmission rate. Data traffic however is different from voice traffic; it can for example handle larger delays in transfer than voice but can tolerate fewer errors in the transmission. Therefore SINR target tracking may not work as well for data traffic as it does for voice. More recent works by Leung [13] and Hande [8] on the opportunistic power control algorithms, that allocates high power to users with favourable channel conditions, indicate that they are throughput optimal. Unfortunately, opportunistic power control schemes suffer from unfairness in terms of the per-user throughput and are therefore not directly applicable in practical systems without any fairness control.

Therefore, there is an interest in developing multicell power control schemes and joint power and channel allocation algorithms that are able to maximize the overall system throughput (the user sum-rate) under some predetermined fairness constraints.

In our work, previous results are extended by considering the problem of assigning mobile users to base stations, channels and allocate power jointly in a multicell system such that an overall objective function is maximized. In this model, any user can be allocated multiple channels that all contribute to that user's sum-throughput. As a prime example that inherently takes into account fairness among users is the max-min allocation, where the objective is to maximize the minimum user throughput over all users of the multi-cell system. This problem has recently been shown to be NP hard and not ρ approximable unless P is equal to NP [5] and we therefore aim at examining heuristic algorithms.

The algorithms we examine are presented in [4] and work in the following sequential way:

1. Assign mobile users to base stations; link assignment.

    a) Assume that the number of mobile users in the system is no larger than the number of base stations times the number of channels. All mobiles are assigned to a base station.

2. Assign channels to links.

    a) Assume that there are no more mobile users connected to the base station than there are channels. All mobile users are assigned a channel.

3. Allocate power to active links.

The link assignments algorithms we examine are, the direct greedy approach (DGA), the reversed greedy approach (RGA), maximize total path gain (MTGA) and maximize the minimum user path gain (MMG). The direct greedy approach and reversed greedy approach are heuristic algorithms while maximizing the total path gain and maximize the minimum path gain are optimization based routines. In this work we limit ourselves to a single channel system; hence there is no channel assignment.

A power allocation optimization problem which optimizes over all channels in the system is presented in [3]. That problem is also shown to be not approximable and not convex, hence there is no guarantee that a global optimum can be found. Therefore it is assumed that each mobile only uses one channel and another power allocation is formulated which optimizes over each channel in the system at a time. For this single channel problem it is shown that each local maximizer is also a global maximizer [3] and hence a global optimum can be found. Here we only examine a single channel system; therefore a power allocation based on the single channel optimization is developed. The objective in the power allocation optimization is either to maximize the minimum user throughput or to maximize the total system throughput. In addition a power allocation with an objective which is a combination of the two previously stated objectives, a hybrid, is examined.

Finally a base case is developed, the base case allocates mobiles to base station based on the path gain so that all mobiles are assigned to a base station, then all senders, mobiles or base stations, are set to send at maximum power.

The main objective of this work is to gain insight into the trade-off between maximizing the overall throughput and providing fairness in a system where the degree of the resource allocation freedom includes assigning mobiles to base stations (links), assigning channel and power allocation. We also want to compare the four link assignment algorithms based on how "good" the link assignment is. To this end, the four different algorithms to assign mobile users to base stations and a power allocation optimization routine are implemented in the RUNE OFDM simulator for both uplink and downlink. The system examined is a 7 cell single channel system.

1.2 Thesis Outline

In chapter 2 the algorithms for link assignment and power allocation as developed by Fallgren are introduced and explained.

Chapter 3 then moves on to introducing RUNE and discusses the implementation of the algorithms in Matlab. Finally the simulations performed are described.

Chapter 4 presents the results from where the algorithms described are examined with a simple case where the global optimum is found through exhaustive search and compared to the solution found by the algorithms implemented.

Chapter 5 contains the results from the simulations described in chapter 3. First the different link assignment algorithms are compared and one selected for presentation in the following results. Then the minimum user throughput and the total system sum throughput are compared for the base case and for different goals with the power allocation optimization.

Chapter 6 then summarizes the results from chapter 5, discusses them and gives some ideas for further investigations.

Chapter 2

Theoretical Background

2.1 The Joint Link, Channel and Power Allocation problem

The optimization problem for finding the maximum of the minimum user throughput as derived in [5] will now be presented and explained. Here the assignment of mobiles to base stations and channels along with power allocation is done jointly.

2.1.1 Notation

The sets used in the mathematical formulation are defined in table 2.1 and table 2.2 contains the variables which will be used.

S	Set of sources
D	Set of destinations
C	Set of channels per base station
M	Set of mobile users
B	Set of base stations

Table 2.1: Definition of the sets used in the formulation

Set of sources and destination can represent either mobile user or base stations. Source is where the signal originates from and destination is where it should end. Throughout this work we will be referring to links and channels, where links are the connection between a base station and mobile and on that link there is some channel used to send on.

x_{ijk}	$\begin{cases} 1, & \text{if } i \in S \text{ can send information to } j \in D \text{ on channel } k \in C \\ 0, & \text{otherwise} \end{cases}$
y_{ij}	$\begin{cases} 1, & \text{if } x_{ijk} = 1 \text{ on any channel } k \in C \\ 0, & \text{otherwise} \end{cases}$
p_{ik}	the power that source i uses to transmit information on channel $k \in C$
g_{ij}	path gain on link i to j
η_{ijk}	the throughput between source i and destination j on channel k
$P_i^{\max}$	the maximum power that sender i has
W	a positive scalar related to the size of the frequency band, a known constant

Table 2.2: Definition of the constants and variables used in the formulation

Each sender has a maximum power which limits the power it can send at. The throughput, which describes how fast the connection is, is given by (2.1) and the signal to interference and noise ratio is given by (2.2).

$$\eta_{ijk} = W \log_2(1 + SINR_{ijk}), \qquad i \in S, j \in D, k \in C \tag{2.1}$$

$$SINR_{ijk} = \frac{g_{ij} p_{ik}}{\sigma_j^2 + \sum_{m \in S \setminus \{i\}} g_{mj} p_{mk}}, \qquad i \in S, j \in D, k \in C \tag{2.2}$$

2.1.2 Mathematical Formulation

Introduce first the joint channel and power allocation. In this formulation it is assumed that all mobiles should be connected and hence all mobiles connect to one base station. The objective with the optimization is to maximize the minimum user throughput, users are assigned to base station and channels and power is allocated in combination. η is an auxiliary variable that is defined with constraint (2.3b). The downlink problem is given by (2.3) (the corresponding uplink problem is given below).

$$\begin{aligned}
\underset{\eta, p_{ik}, x_{ijk}, y_{ij}}{\text{maximize}} \quad & \eta && && (2.3a)\\
\text{subject to} \quad & y_{ij}\eta \leq \sum_{k \in C} x_{ijk}\eta_{ijk}, && i \in S, j \in D, && (2.3b)\\
& \sum_{i \in B} y_{ij} = 1, && j \in M, && (2.3c)\\
& \sum_{j \in M} x_{ijk} \leq 1, && i \in B, k \in C, && (2.3d)\\
& \sum_{k \in C} p_{ik} \leq P_i^{\max}, && i \in S, && (2.3e)\\
& x_{ijk} \leq y_{ij}, && i \in S, j \in D, k \in C, && (2.3f)\\
& 0 \leq y_{ij} \leq 1, && i \in S, j \in D, && (2.3g)\\
& x_{ijk} \in \{0, 1\}, && i \in S, j \in D, k \in C, && (2.3h)\\
& p_{ik} \geq 0, && i \in S, k \in C_i, && (2.3i)
\end{aligned}$$

Constraint (2.3b) ensures that for all active links η is less than or equal to the sum of the throughput on the active channels on that link. By then also maximizing η in the objective function (2.3a) it is ensured that η is equal to the minimum sum throughput on the active links, and that minimum is maximized. Constraint (2.3c) ensures that all mobiles are connected to exactly one base station, constraint (2.3d) ensures that all base stations have at max one mobile per channel. Constraint (2.3e) ensures that the power each source uses to send at is less than that sources maximum. Equations (2.3f) defines the connection between x and y, and finally constraints (2.3g) to (2.3i) require that all mobiles are only connected to one base station, and that there is at most one mobile per channel on each base station, and that the power is always positive.

The corresponding uplink problem is given by interchanging i and j in constraints (2.3c) and (2.3d), they become:

$$\sum_{j \in B} y_{ij} = 1, \qquad i \in M, \tag{2.4a}$$

$$\sum_{i \in M} x_{ijk} \leq 1, \qquad j \in B, k \in C, \tag{2.4b}$$

In [5] the joint channel assignment and power allocation problem is defined to more detail and shown that that decision problem is NP-hard, it is also shown to be not ρ approximable unless P is equal to NP. This means that the problem, as it is, cannot be solved in polynomial time and that it cannot be approximated by a simpler problem which can be solved in polynomial time. Therefore heuristic methods for assigning mobiles to base stations and allocating power are considered. The heuristic algorithm we will examine here is where links and power are allocated in sequence.

2.2 Separated Link Assignment and Power Allocation

The overall objective with the heuristic algorithms is to assign links, channels and power jointly. Here we will consider heuristics which work in the following sequential way:

1. Mobile users assigned to base stations (link assignment)
    a) Assume that the number of mobile users in the system is no larger than the number of base stations times the number of channels. All mobiles are assigned to a base station.
2. Mobile users are allocated to a channel
    a) Assume that there are no more mobile users connected to the base station than there are channels. All mobile users are assigned a channel.
3. Power is allocated to users

This is also illustrated in figure 2.1

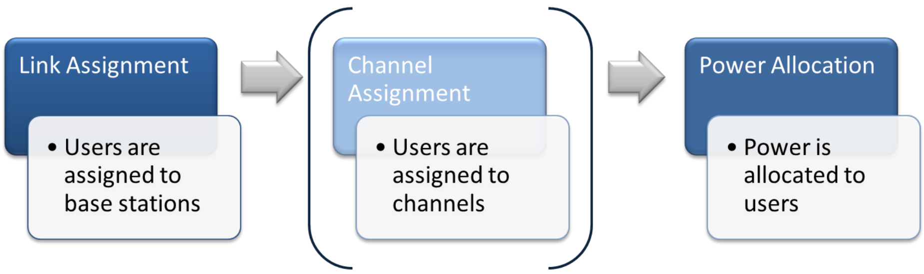


Figure 2.1: Description of how users are connected to base stations

Since all mobile users are assigned to a base station it is assumed that the system does not have more mobile users than number of base stations times channels per base station. When users are assigned to base stations it is ensured that all users are assigned to a base station and that the number of users assigned to a base station is never higher than the number of channels. In this work a single channel system is considered, therefore there is at maximum one user connected to each base station. The channel assignment is not needed in this work and will not be explained further here, more detailed information on the channel assignment can be found in [4]. Even though the channel assignment is not explained here the formulation for the link assignment algorithms and the power allocation are valid both for the single channel case and the multiple channel case.

2.2.1 Link Assignment

Four different link assignment algorithms are considered, they are derived in [4]. Two of those are heuristic algorithms, direct greedy and reversed greedy link assignment. The other two are optimization based link assignments, maximize the sum path gain over all users and maximize the minimum path gain. Link assignment uses path gain based on the distance component and shadow fading, not fast fading. In the following link assignments all users in the system are assigned to a base station.

Direct Greedy Link Assignment (DGA)

Algorithm 1 Direct Greedy Approach

Set $\tilde{g} \leftarrow g$ and $y_{ij} \leftarrow 0, \quad i \in B, j \in M$
for $\Delta = 1$ **to** $|M|$ **do**
  Let $b_j \leftarrow \arg\max_{i \in B} \tilde{g}_{ij}$, $j \in M$, {breaking ties arbitrarily}
  Consider mobile $m \leftarrow \arg\max_{j \in M} \tilde{g}_{b_j j}$
  Update $y_{b_m m} \leftarrow 1$ and let $\tilde{g}_{im} \leftarrow -1$, $i \in B$, {removes mobile m}
  if $\sum_{j \in M} y_{b_m j} = |C|$ **then**
    $\tilde{g}_{b_m j} \leftarrow -1$, $j \in M$, {which removes base station b_m}
  end if
end for

The direct greedy approach assigns links according to the largest available path gain. In algorithm 1, b_j is the desired base station for assigning mobile user j to. Then the mobile that will be assigned to its desired base station, m, is the mobile with the highest path gain among the desired base stations. Finally variables are updated in order to remove that mobile and if the base station has no more channels available the base station is removed from the algorithm as well.

Reversed Greedy Link Assignment (RGA)

Algorithm 2 Reversed Greedy Approach

Set $\tilde{g} \leftarrow g$ and $y_{ij} \leftarrow 0, \quad i \in B, j \in M$
for $\Delta = 1$ **to** $|M|$ **do**
  Let $b_j \leftarrow \arg\max_{i \in B} \tilde{g}_{ij}$, $j \in M$, {breaking ties arbitrarily}
  Consider mobile $m \leftarrow \arg\min_{j \in M} \tilde{g}_{b_j j}$
  Update $y_{b_m m} \leftarrow 1$ and let $\tilde{g}_{im} \leftarrow -1$, $i \in B$, {removes mobile m}
  if $\sum_{j \in M} y_{b_m j} = |C|$ **then**
    $\tilde{g}_{b_m j} \leftarrow -1$, $j \in M$, {which removes base station b_m}
  end if
end for

The formulation for the reversed greedy approach is similar to the direct greedy assignment but instead of $m \leftarrow \arg\max_{j\in M} \tilde{g}_{b_j j}$ there is $m \leftarrow \arg\min_{j\in M} \tilde{g}_{b_j j}$. In algorithm 2, b_j contains the desired base station for mobile j according to maximum path gain. Then the mobile that will be assigned to its desired base station, m is the mobile with the lowest path gain to its desired base station.

Maximize the Total Path Gain Link Assignment (MTGA)

In this approach the sum of g_{ij} over the transmission links is maximized, this leads to that the path gain over unused links is minimized.

$$\begin{aligned}
\underset{y_{ij}}{\text{maximize}} \quad & \sum_{i,j} g_{ij} y_{ij} && && \text{(2.5a)}\\
\text{subject to} \quad & \sum_{i\in B} y_{ij} = 1, && j \in M, && \text{(2.5b)}\\
& \sum_{j\in M} y_{ij} \leq |C|\,, && i \in B, && \text{(2.5c)}\\
& y_{ij} \in \{0,1\}\,, && i \in B, j \in M, && \text{(2.5d)}
\end{aligned}$$

By relaxing constraint (2.5d) to $y_{ij} \in [0,1]$ it becomes the transportation problem which is linear and known to give integer solutions. Constraint (2.5b) ensures that all mobiles connect to exactly one base station. Constraint (2.5c) then ensures that the base stations do not use more channels than available.

Maximize the Minimum Path Gain Link Assignment (MMG)

Maximize the smallest g_{ij} over the transmission links, using similar techniques as in the joint channel and power allocation in (2.3).

$$\begin{aligned}
\underset{\eta, y_{ij}}{\text{maximize}} \quad & \eta && && \text{(2.6a)}\\
\text{subject to} \quad & y_{ij}\eta \leq g_{ij} y_{ij}, && i \in B, j \in M && \text{(2.6b)}\\
& \sum_{i\in B} y_{ij} = 1, && j \in M, && \text{(2.6c)}\\
& \sum_{j\in M} y_{ij} \leq |C|\,, && i \in B, && \text{(2.6d)}\\
& y_{ij} \in \{0,1\}\,, && i \in B, j \in M, && \text{(2.6e)}
\end{aligned}$$

Here η is an auxiliary variable defined by (2.6b) which ensures that η is less than or equal to the minimum path gain over all active links in the system. By then maximizing η it is ensured that η is equal to the minimum path gain on each link and that minimum is maximized.

2.2.2 Power Allocation

Now users have been assigned to base stations and, if the system consists of more than one channel, channels. The next step is then to allocate power to the mobiles and base stations. Power is allocated based on the path gain which contains the distance component, shadow fading component and fast fading component. The objective with the power allocation as described in [3] is to maximize the minimum user throughput. In the following sections three variations of the power allocation will be introduced.

Before the power allocation problem is considered more notation is introduced in table 2.3.

$L =$	$\{(i,j) : y_{ij} = 1, i \in S, j \in D\},$	Set of active links
$C_{ij} =$	$\{k \in C : x_{ijk} = 1,\ (i,j) \in L\},$	Set of active channels on each links
$C_i =$	$\bigcup_{j \in D:(i,j) \in L} C_{ij},$	Set of active channels by each sender
$\eta_{ij} =$	$\sum_{k \in C_{ij}} \eta_{nijk},\ (i,j) \in L,$	Link throughput

Table 2.3: Definition of the sets used in the formulation

The Power Allocation Optimization Problem

This formulation is derived in [3] and solves over all channels at one time. The objective in (2.7) is to maximize the minimum user throughput.

$$\begin{aligned} \underset{\eta,\eta_{ij},p_{ik}}{\text{maximize}} \quad & \eta && && (2.7a) \\ \text{subject to} \quad & \eta \leq \eta_{ij}, && (i,j) \in L, && (2.7b) \\ & \sum_{k \in C_i} p_{ik} \leq P_i^{\max}, && i \in S, && (2.7c) \\ & p_{ik} \geq 0, && i \in S, k \in C_i, && (2.7d) \end{aligned}$$

Constraint (2.7b) ensures that η is less than or equal to all active links' total throughput. Finally constraint (2.7c) guarantees that the total power of each sender is less than that senders maximum.

This problem is not ρ approximable unless P is equal to NP and it is in general not convex [3]. This means that we may not be able to solve this problem towards a global optimum. Therefore an additional assumption is made, that each mobile is only allowed to use one channel.

The Single Channel on Each Link Power Allocation Optimization Problem

In this formulation each link, a connection between a base station and mobile, is only allowed to use a single channel. The formulation is similar to (2.7) however constraint (2.8b) is only defined for the active channel on the active link.

$$
\begin{aligned}
\underset{\eta, \eta_{ij}, p_{ik}}{\text{maximize}} \quad & \eta && && (2.8a)\\
\text{subject to} \quad & \eta \leq \eta_{ijk}, && (i,j) \in L, k \in C_{ij}, && (2.8b)\\
& \sum_{k \in C_i} p_{ik} \leq P_i^{\max}, && i \in S, && (2.8c)\\
& p_{ik} \geq 0, && i \in S, k \in C_i, && (2.8d)
\end{aligned}
$$

This problem is in general not convex, however each local maximizer is also a global maximizer [3] and hence problem (2.8) can be solved towards a global optimum.

In order to use the formulation given in (2.8) to allocate power to the whole system it can be decomposed and solved for each channel in the system at a time. This can easily be done in uplink, since each mobile is only on one channel and each mobile has its own maximum power. However in downlink there is usually only a maximum on the power that the base station has and the base station is usually sending on more than one channel, that is to more than one mobile user. Therefore in downlink the base station's power would have to be divided between the channels used before solving the power allocation with this formulation.

The Single Channel on Each Link Power Allocation Optimization Problem, weighed objective function

Here we introduce a variation from the model given in (2.8). In this formulation the objective can be changed from maximizing the minimum user throughput to maximizing the total system throughput by changing α.

$$
\begin{aligned}
\underset{\eta, \eta_{ij}, p_{ik}}{\text{maximize}} \quad & (1-\alpha)\eta + \alpha \sum_{ijk} \eta_{ijk} && && (2.9a)\\
\text{subject to} \quad & \eta \leq \eta_{ijk}, && (i,j) \in L, k \in C_{ij}, && (2.9b)\\
& \sum_{k \in C_i} p_{ik} \leq P_i^{\max}, && i \in S, && (2.9c)\\
& p_{ik} \geq 0, && i \in S, k \in C_i, && (2.9d)
\end{aligned}
$$

By setting $\alpha = 0$ the objective becomes to maximizing the minimum user throughput, which gives the same model as in (2.8), by setting $\alpha = 1$ the objective becomes to maximize the total system throughput. The formulation given by (2.9) is the one used in this report.

Chapter 3

Simulation Environment and Implementation

3.1 Simulation Environment

3.1.1 Short Description of RUNE

In order to emulate a realistic cellular system RUNE (Rudimentary Network Emulator) is used. RUNE is available with [17] which also includes some more detailed description of the simulator. RUNE is implemented within Matlab and it is widely used for network simulations. It has the possibility to set up a system with base stations, with both omni and directional antennas. RUNE can set up realistic path gain matrices both with and without fast fading. The path gain in RUNE is composed of a distance component and a shadow fading component. Fast fading can be added as well, which is simulated by Rayleigh fading. Finally it simulates how the mobile users in the system behave, that is how they move around and how mobile users end and start a cell phone call. The RUNE simulation file consists of two loops, the outer loop and the inner loop. In the outer loop users are initialized or if they are not new to the system they are moved and the path gain is calculated. In the inner loop users are moved, Rayleigh fading is calculated, links are assigned and power allocated.

3.1.2 On the Implementation in Matlab

For the power allocation the problem (2.9) is formulated in Matlab and solved using the `fmincon` optimization routine. The link assignment formulation and implementation in Matlab come from Mikael Fallgren. The algorithms are integrated with the OFDM RUNE simulator so that for each time users have moved links are assigned again and the power optimization problem solved, this is done without taking into consideration the previous setting, in other words the code has no memory. For assigning mobile users to base stations the path gain without Rayleigh fading is used. However, when allocating power path gain with Rayleigh fading is considered.

The optimization routine `fmincon` can give several outcomes. Firstly that it does not find a solution, secondly that it finds a local optimum point, thirdly it finds a point which is not the local optimum but the point is within some tolerance levels and therefore, seems to be good, for more details see the Matlab documentation. For the power allocation the problem is solved from up to nine starting points or until the solver returns that it has found a local optimum, which is shown in [3] to also be a global optimum. If a local optimum is not found the best solution found is used.

3.2 Simulation Description

3.2.1 Base Case

In order to have a base comparison with the algorithm described in section 2.2 a base case is designed. The mobile user connects to the base station they are closest

Algorithm 3 Base case link and power assignment

Let $b_j \leftarrow \arg\max_{i\in B} g_{ij},\ j \in M$,
Set $y_{b_j j} = 1, \quad \forall j \in M$,
if $\sum_{j\in M} y_{ij} > |C|$ for any $i \in B$ **then**
  Randomly select mobile users that are connected to that base station i and reallocate all but one to the available base stations,
end if
Set $p_i = P_i^{max}$ for $i \in S$, $\{p_i$ but not p_{ik} since this is the single channel case$\}$

to with respect to path gain. If another user also wants to connect to that same base station one user is randomly selected to connect to that base station and the other user(s) are connected randomly to an available base station. That means all mobiles are assigned to a base station. Finally all senders (base stations in downlink and mobiles in uplink) transmit at maximum power. This link and power assignment algorithm is run on the same system as the algorithms described in 2.2.

3.2.2 Simulations Performed

Here we will describe the simulations performed for this thesis. In the system, user generation is conditioned so that the number of users in the system is no larger than number of base stations times number of channels. In this project only the single channel case is considered, hence users in the system are never more than base stations. The system used for most simulations consists of 7 base stations and 7 users in the system. Each base station has only one channel and only one user connects to a base station at a time. The mobile users speed is presumed to be that of a person moving on foot. Parameters for the system are described in table 3.1 and table 3.2 contains the parameters for the path gain generation.

Parameter	Value
Cell Radius	500 m
Number of Channels	1
Number of Sectors per Site	1
Number of Base Stations	7
Number of Clusters per System	1
Maximum Power of Mobile	24 dB
Maximum Power of Base Station	43 dB
Carrier Frequency	2 GHz
Chunk Bandwidth	0.2 MHz

Table 3.1: The main parameters in the system

Parameter	Value
Gain at 1 meter Distance	- 28 dBm
Noise	-103 dBm
Distance Dependant Path Gain Coefficient	3.5
Standard Deviation for the log-normal Fading	6 dB
Log-normal Correlation Downlink	0.5
Correlation Distance	110 m
Fast Fading	Rayleigh

Table 3.2: Path gain specific parameters

The users can have three different distributions, uniformly distributed, close to the base station and far from the base station. These can be seen in figure 3.1.

In this thesis there are five different simulations performed, a 15 system run, a 30 second simulation, changing α, moving user towards a base station and 19 base stations. They are described in the following sections.

15 System Run

In this simulation the system is initialized and then run for 1 second (one outer loop). Then the users are given new positions, according to the uniform distribution, and again run for 1 second, this is done 15 times. Within each outer loop, 4 inner loops are calculated. It is also assumed that mobile users do not stop making calls, that is there are always 7 users in the system. The parameters can be seen in table 3.3.

30 Second Simulated Time

This system is initialized and run for 30 seconds. The time simulation specific parameters are the same as for the 15 system run and can be seen in table 3.3. This simulation is done for the three different user distributions, uniform, close and far.

Parameter	Value
Mean Hold Time	Inf
Time of Outer Loop	1 s
Time of Inner Loop	1/4 s
Number of Inner Loops per Outer Loop	4
Users Average Speed	1 m/s
Users Average Acceleration	0.2 m/s^2

Table 3.3: Simulation over time specific parameters in the system

Changing the Weights Between Maximizing Total Throughput and Maximizing the Minimum User Throughput

Here the system is initialized using the uniform user distribution. Then for the same system, that is for the same user location and path gain matrix, links are assigned and power is allocated for different values of α. Where α is the weight in the objective function in problem (2.9).

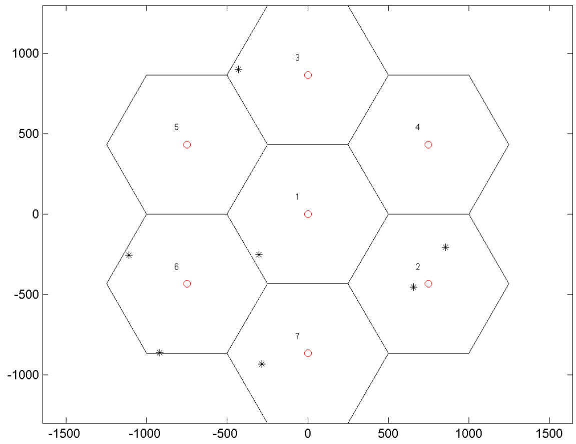
Moving User Towards a Base Station

The system is initialized with users far from the base station and then one user is selected and moved towards one base station. The remaining mobiles remained at their initial position. In this simulation there is only one inner loop. In all it took 16 loops for the user to reach the base station within a ϵ distance which is specified by the RUNE simulator.

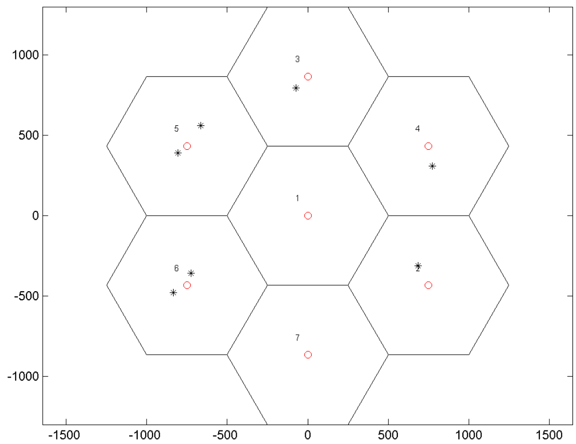
19 Base Stations

Finally the algorithm is tested on a larger system, that is a system with 19 base stations. Users are given location according to the uniform distribution and link assignment and power allocation is only calculated once. That is there is only one outer loop and only one inner loop

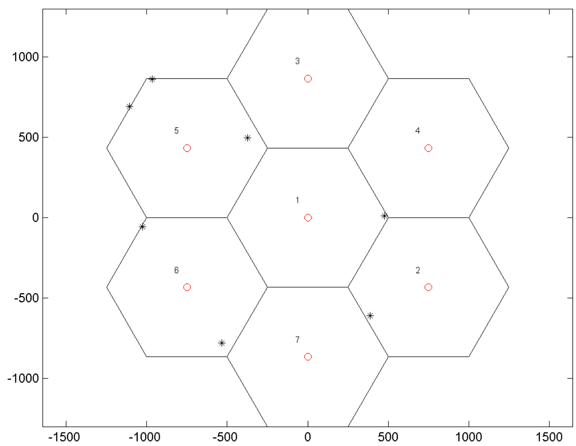
The results from the above described simulations and discussions about the results can be seen in chapter 5.



(a) Users location, uniformly distributed



(b) Users location, close to the base stations



(c) Users location, far from the base stations

Figure 3.1: Users location for users which are close to and far from the base stations and when users are uniformly distributed

Chapter 4

Algorithm Verification

4.1 Globally Optimal Link and Power Assignment by Exhaustive Search

In order to confirm that the algorithms can solve a problem correctly, the link assignments and power allocation algorithm are tested with two users. This is done through discretizing the power over the allowed interval and then calculate the user throughput for all possible link assignment combinations and all possible power combinations. For each power combination the minimum user throughput is found and then the maximum of those is the maximum of the minimum user. The path gain matrices can be found in appendix A.1.

4.1.1 Two Mobile Users and Three Base Stations

There are two mobile users and three base stations in the system. The maximum power a mobile user can send at is 24 dBm and the maximum power a base station can send at is 40 dBm. An approximate optimum for both uplink and downlink is found through exhaustive search. This is an approximation since the possible power is discretized to a certain level. The possible link assignments are given in tables 4.1.

Uplink

The minimum user throughput for each link and power combination is shown in figure 4.1. In table 4.2 the maximum of the minimum user throughput for each link can be seen, the respective power allocations are given in table 4.3.

Base station	1	2	3
Mobile 1	1	0	0
Mobile 2	0	1	0

(a) Link assignment 1

Base station	1	2	3
Mobile 1	1	0	0
Mobile 2	0	0	1

(b) Link assignment 2

Base station	1	2	3
Mobile 1	0	1	0
Mobile 2	1	0	0

(c) Link assignment 3

Base station	1	2	3
Mobile 1	0	1	0
Mobile 2	0	0	1

(d) Link assignment 4

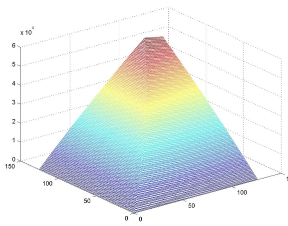
Base station	1	2	3
Mobile 1	0	0	1
Mobile 2	1	0	0

(e) Link assignment 5

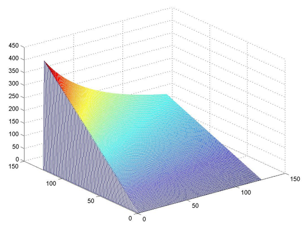
Base station	1	2	3
Mobile 1	0	0	1
Mobile 2	0	1	0

(f) Link assignment 6

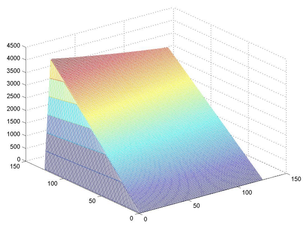
Table 4.1: Link assignments



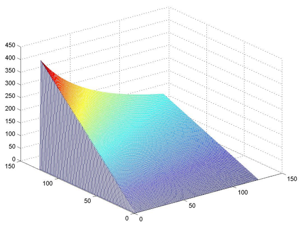
(a) Link 1



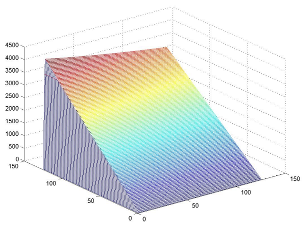
(b) Link 2



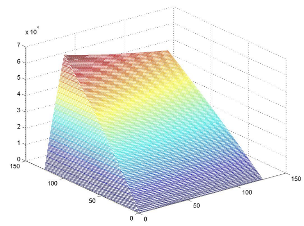
(c) Link 3



(d) Link 4



(e) Link 5



(f) Link 6

Figure 4.1: The minimum user throughput for each power combination in uplink

The shapes of the minimum user throughput indicate that there is a unique global maximizer. From table 4.2 it can be seen that link assignment 6 gives the highest throughput and hence is the globally optimal assignment. Due to the sensitivity of the algorithm the results are not exact, but give a fairly good approximation of the global optimum.

Link 1	Link 2	Link 3	Link 4	Link 5	Link 6
54.0	0.4	4.3	0.4	4.3	66.5

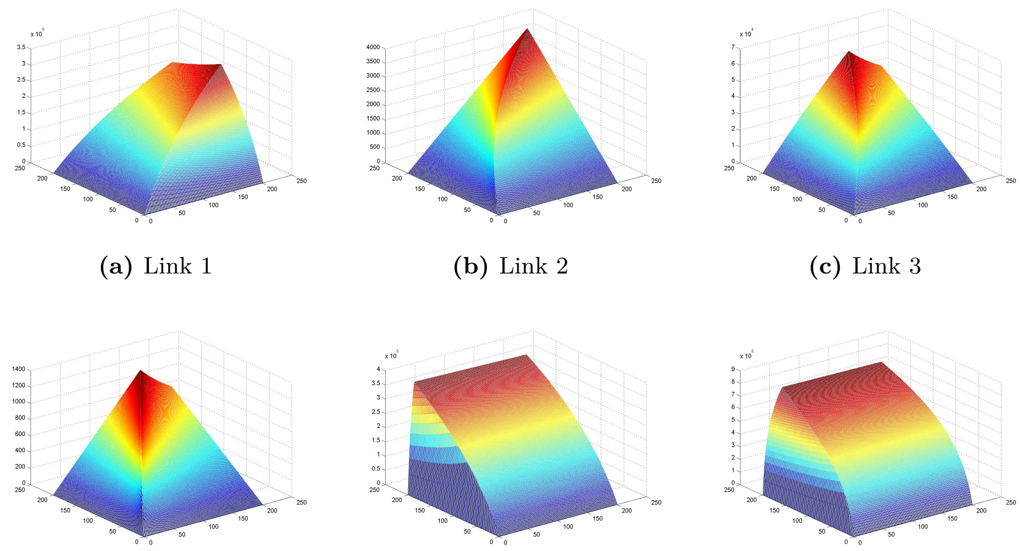
Table 4.2: The maximum of the minimum user throughput for each link in uplink, [k bits/s]

	Link 1	Link 2	Link 3	Link 4	Link 5	Link 6
Mobile 1	23.30	3.01	11.46	3.01	6.02	16.02
Mobile 2	23.98	23.98	23.98	23.98	23.98	23.98

Table 4.3: The power allocation for each maximum of the minimum user throughput for each link, in uplink [dBm]

Downlink

The minimum user throughput for each link and power combination is shown in figure 4.2. In table 4.4 the maximum of the minimum user throughput for each link can be seen, the respective power allocations are given in table 4.5.



(a) Link 1 **(b)** Link 2 **(c)** Link 3

(d) Link 4 **(e)** Link 5 **(f)** Link 6

Figure 4.2: The minimum user throughput for each power combination in downlink

The slope of the minimum user throughput for link assignment 5 and 6 is not steep as can be seen from figures 4.2e and 4.2f. However from table 4.4 it can be seen that link assignment 6 gives the highest throughput and hence is the globally optimal assignment. From table 4.4 it can be seen that the global optimum is given by link assignment 6.

Link 1	Link 2	Link 3	Link 4	Link 5	Link 6
303	4	60	1	389	808

Table 4.4: The maximum of the minimum user throughput for each link, in downlink [k bits/s]

	Link 1	Link 2	Link 3	Link 4	Link 5	Link 6
Mobile 1	40.00	40.00	38.57	38.66	27.40	32.17
Mobile 2	36.67	39.96	39.98	40.00	40.00	40.00

Table 4.5: The power allocation for each maximum of the minimum user throughput for each link, in downlink [dBm]

4.1.2 Results from the Optimization Problem With the Same Input Data

For uplink and downlink the four link assignment algorithms (direct greedy approach, reversed greedy approach, maximizing minimum path gain and maximizing total path gain) all give the same link assignment, that is link assignment 6 (see table 4.1f).

In uplink the maximum of the minimum user throughput as found through the power optimization algorithm is 66.8 k bits/s, the power allocation is given in table 4.6. This throughput is higher than the value found through exhaustive search.

	Power [dBm]
Mobile 1	16.01
Mobile 2	24.00

Table 4.6: Power allocation which gives the maximum of the minimum user throughput in uplink [dBm]

For downlink the solution found by the optimization algorithm is 807.8 k bits/s. Which is achieved with the power allocation given in table 4.7. This throughput is higher than the global optimum found through exhaustive search.

	Power [dBm]
Mobile 1	32.11
Mobile 2	40.00

Table 4.7: Power allocation which gives the maximum of the minimum user throughput in downlink [dBm]

The maximum of the minimum user throughput found through exhaustive search and the value found through the optimization algorithm are close to each other. The value found by the optimization algorithm is even better than the one found through exhaustive search in both uplink and downlink. This is since the power is discretized in the exhaustive search, while the optimization algorithm is not subject to the same discretization. From these results it can be concluded that the algorithms can find the maximum of the minimum user throughput for this case. We now move on to a more detailed analysis of the algorithms.

Chapter 5

Simulation Results

5.1 Link Assignment Algorithm Comparison

We begin by comparing the algorithm described in section 2.2.1 for both maximizing the minimum user throughput ($\alpha = 0$ in (2.9)) and for maximizing the total throughput ($\alpha = 1$ in (2.9)). That is done by examining the worst off user and the total system throughput respectively. The worst off user is the user which at each calculated time point gets the least throughput; it is not a single user which gets the least throughput through the whole simulation, but a combination of many users. The reason for examining different properties for the different objective function is that they give better information on how good the link assignment is. Here we do a comparison between the different link assignments and then based on the results select one link assignment algorithm to present for the other simulations.

5.1.1 15 System Run With All Link Assignments

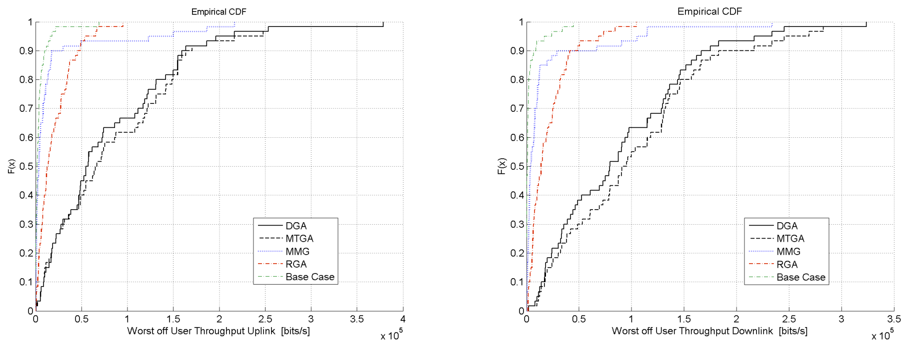
Table 5.1 lists the figures presented for each objective.

Performance measure	Minimum user throughput	Total system throughput
Objective 1: Maximize the minimum user throughput	Figure 5.1	Not given
Objective 2: Maximize total throughput	Not given	Figure 5.2

Table 5.1: Figures for the link assignment comparison in the 15 system run

The worst off user throughput for each link assignment algorithm when power is allocated to maximize the minimum user throughput can be seen in figure 5.1. It can be noted that the maximize total path gain and direct greedy link assignments are considerably better than the reversed greedy and maximize minimum path gain for maximizing the minimum user throughput in both uplink and downlink. The link assignment algorithms are also all better than the base case scenario.

When the objective is to maximize the minimum user throughput all users in the system get the same throughput when links are assigned with the direct greedy approach and maximize total path gain. For the other link assignments there are some small differences between users. This can be seen by examining the figures in appendix B.2.1.

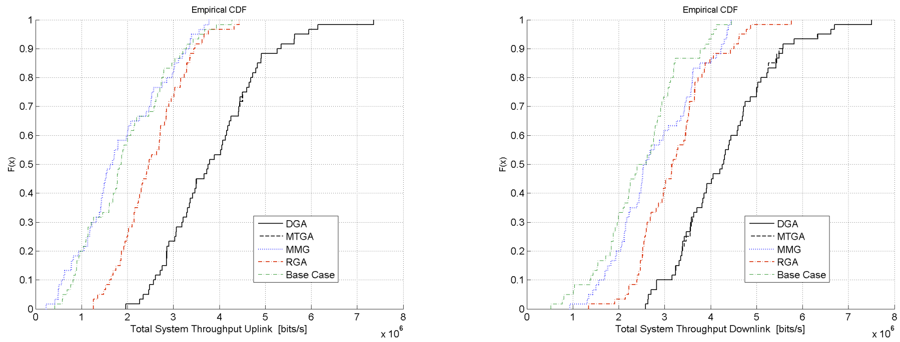


(a) Minimum user throughput, uplink **(b)** Minimum user throughput, downlink

Figure 5.1: The CDF plot for the minimum users throughput for each link assignment approach at each calculated time point for the 15 system run, power is allocated to maximize the minimum user throughput

The conclusion can therefore be reached that when the objective is to maximize the minimum user, direct greedy link assignment and maximize total path gain link assignment are better than reversed greedy link assignment and maximize minimum path gain.

Now the link assignments are compared for when the objective is to maximize the total throughput. In figure 5.2 the total system throughput is shown for the four different link assignments, power is allocated to maximizing total system throughput ($\alpha = 1$).



(a) Total system throughput, uplink **(b)** Total system throughput, downlink

Figure 5.2: The CDF plot for the total system throughput for each link assignment approach at each calculated time point over a 15 second period, power is allocated to maximize the total throughput

From figure 5.2 one can see that when maximizing total throughput direct greedy approach and maximize total path gain approach give almost exactly the same result, which is better than the other two link assignments.

In the 15 system run direct greedy approach and maximize total path gain approach are giving better results than the other link assignment. This is evident both for the worst off user when trying to maximize the minimum user throughput ($\alpha = 0$) and for total system throughput when the objective is to maximize total throughput ($\alpha = 1$). Therefore only those two link assignments are considered for the 30 second simulation.

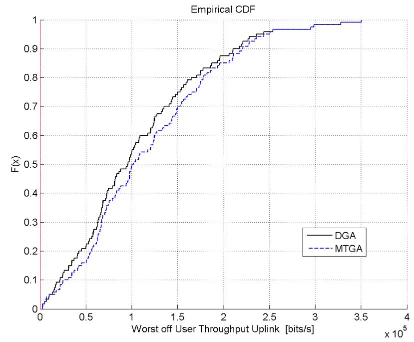
5.1.2 30 Second Simulation

Table 5.2 lists the figures presented for each objective.

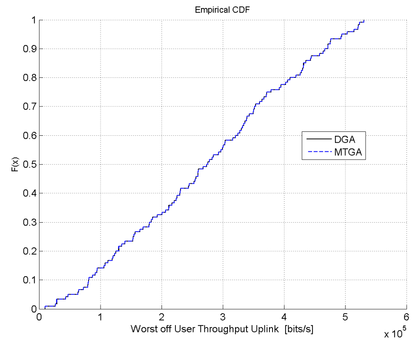
Performance measure	Minimum user throughput	Total system throughput
Objective 1: Maximize the minimum user throughput	Figures 5.3 and 5.4	Not given
Objective 2: Maximize total throughput	Not given	Figures 5.5 and 5.6

Table 5.2: Figures for the link assignment comparison in the 30 second simulation

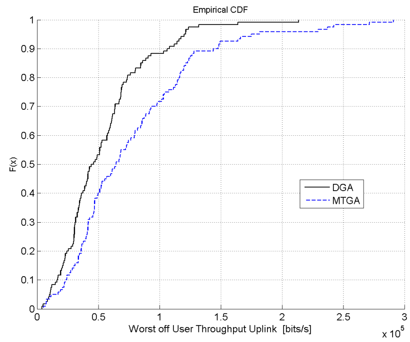
Direct greedy and maximize total path gain link assignments algorithms are compared for a system which is first initialized and then run for 30 seconds, as described in section 3.2.2. In figures 5.3 and 5.4 the worst off user throughput, when the objective is to maximize the minimum user throughput, is displayed in uplink and downlink respectively.



(a) Minimum user throughput uniform distribution

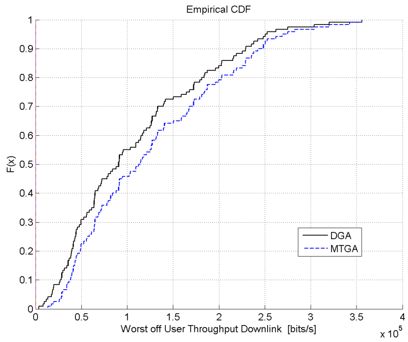


(b) Minimum user throughput users close to the base station

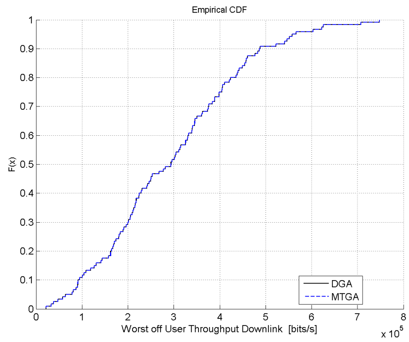


(c) Minimum user throughput users far from the base station

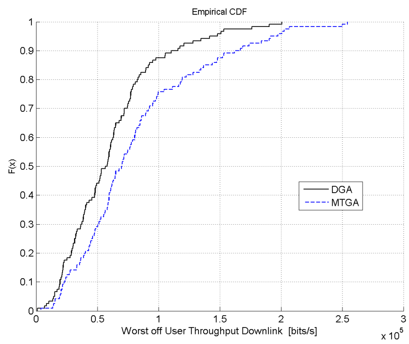
Figure 5.3: The CDF plot for the minimum users throughput in uplink at each calculated time point over a 30 second period, power is allocated to maximize the minimum user throughput



(a) Minimum user throughput uniform distribution



(b) Minimum user throughput users close to the base station

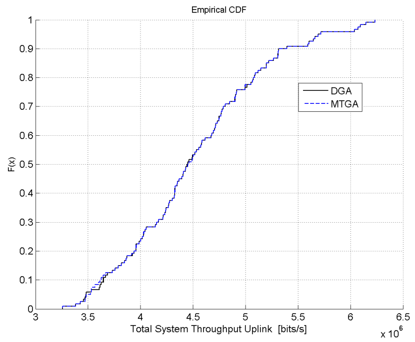


(c) Minimum user throughput users far from the base station

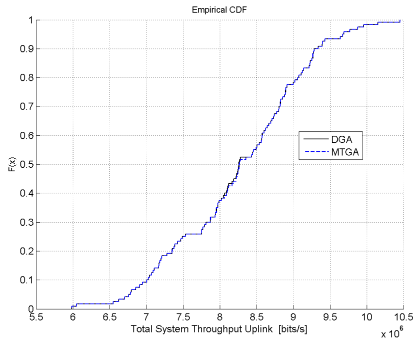
Figure 5.4: The CDF plot for the minimum users throughput in downlink at each calculated time point over a 30 second period, power is allocated to maximize the minimum user throughput

From figures 5.3 and 5.4 it can be noted that when all users are close to a base station the two link assignments give the same results. When users are far away from the base stations they differ more than when users are uniformly distributed. That maximize the total path gain link approach gives the better results is probably due to the fact that since that approach minimizes the interfering path gain and it is an optimization routine and hence takes all data in to account.

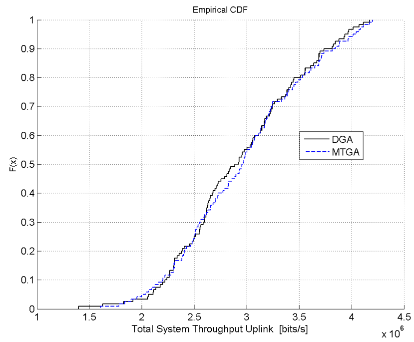
In figures 5.5 and 5.6 the total system throughput, when the objective is to maximize throughput, is displayed in uplink and downlink respectively.



(a) Total system throughput, uniform distribution



(b) Total system throughput, users close to the base station



(c) Total system throughput users far from the base station

Figure 5.5: The CDF plot for the total system throughput in downlink at each calculated time point over a 30 second period, power is allocated to maximize total throughput

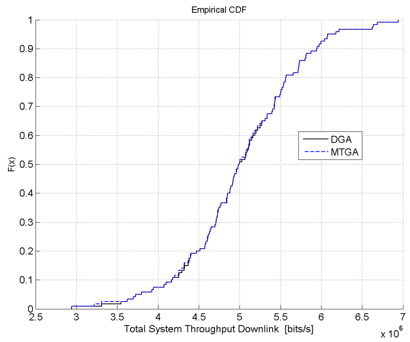
Total throughput over the whole iterated period is given in tables 5.4 and 5.3.

	Uniform	Close	Far
DGA	542.8	990.8	352.1
MTGA	542.7	991.1	355.2

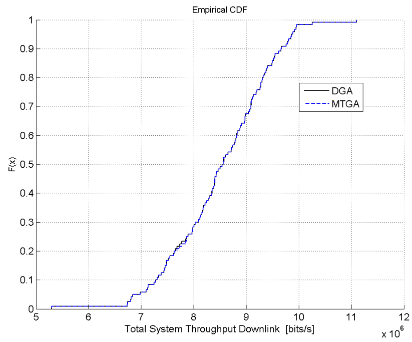
Table 5.3: Total throughput [M bits/s], uplink

	Uniform	Close	Far
DGA	604.0	1021.3	410.7
MTGA	603.2	1021.5	406.2

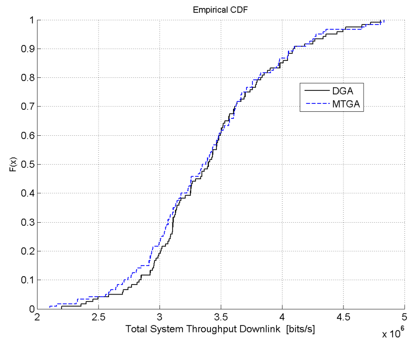
Table 5.4: Total throughput [M bits/s], downlink



(a) Total system throughput, uniform distribution



(b) Total system throughput, users close to the base station



(c) Total system throughput, users far from the base station

Figure 5.6: The CDF plot for the total system throughput in downlink at each calculated time point over a 30 second period, power is allocated to maximize total throughput

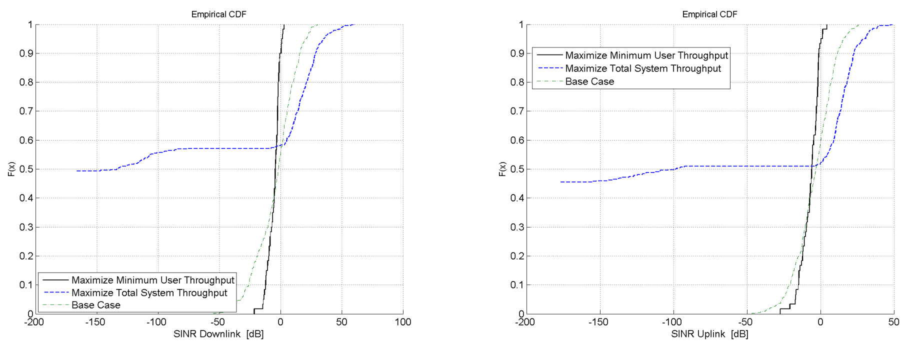
From the results for maximizing total system throughput it can be deducted that the two link assignments are similar. For some cases the direct greedy approach is better and for other cases the maximize total throughput approach is better, this can both be noted from figures 5.5 and 5.6 and from that in tables 5.3 and 5.4 the direct greedy approach gives higher throughput in some cases and maximize total path gain in other cases. However they never differ by much.

Some conclusions from comparing the link assignments algorithms include : Maximizing total path gain link assignment gives better results when the objective is to maximize the minimum user throughput. Also the difference between the direct greedy allocation and maximizing total path gain is minimal when the objective is to maximize total throughput. Therefore maximizing total path gain will be used for link assignments in the following sections.

5.2 15 System Run

In this section we will display results from the 15 system run described in 3.2.2. The following cumulative distribution functions are shown for when power is allocated to maximize the minimum user throughput, maximize the total system throughput and the base case:

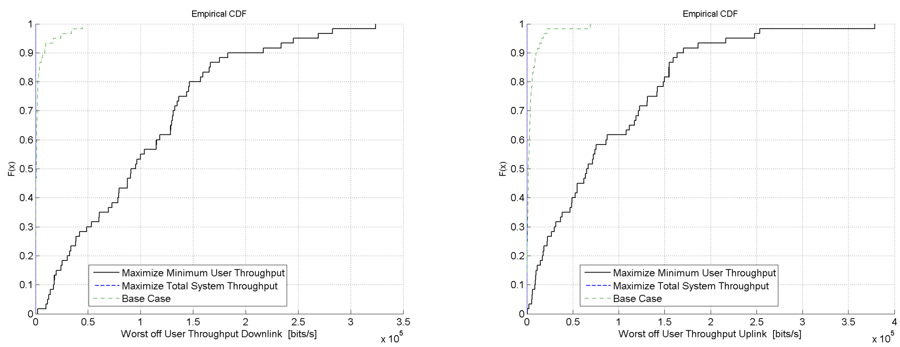
- the signal to interference and noise ratio (SINR) for all users, figure 5.7,
  - all users at all calculated times,
- the worst off user throughput, figure 5.8,
  - the user who gets the least throughput at each calculated time, a combination of many users,
- total system throughput, figure 5.9,
  - sum of all users throughput at each calculated time,
- additionally we show the throughput ratio, figure 5.10,
  - the ratio of the total system throughput when the objective is to maximize the minimum user throughput vs. when the objective is to maximize the total system throughput,



(a) SINR for all users in downlink, uniform distribution **(b)** SINR for all users in uplink, uniform distribution

Figure 5.7: The CDF plot for all users SINR in uplink and downlink at each calculated time point for 15 system simulation

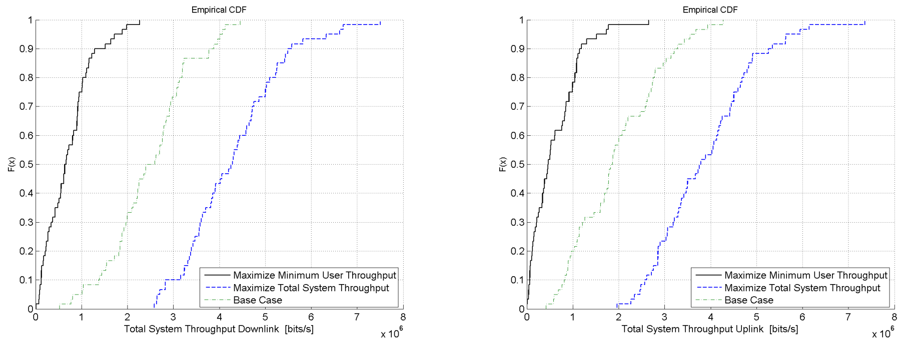
When the minimum users throughput is to be maximized it can be noted from figure 5.7 that all users have similar signal to interference and noise ratio, compared to when the total throughput is to be maximized some users get a really low signal to interference and noise ratio, while others get higher. It can even be noted that in downlink almost 50% of the users get no throughput at all, 45% in uplink, when the total system throughput is maximized. When power is allocated to maximize the minimum user throughput all users get the same throughput at each calculation step, this can be seen in table B.12 in appendix B.2.1. To maximize the minimum user throughput is therefore an extremely fair algorithm.



(a) Worst off user throughput, downlink **(b)** Worst off user throughput, uplink

Figure 5.8: The CDF plot for the worst off users throughput in uplink and downlink at each calculated time point for 15 system simulation

Figure 5.8 shows the cumulative distribution plot for the worst off user at each calculated time. The worst off user at each time point gets almost nothing when the total throughput is maximized it can hence not be seen in the figure. However when power is allocated to maximize the minimum users throughput all users get the same throughput and hence the worst off user does a lot better than when total system throughput is maximized. The difference can be almost 0.32 $M\,bits/s$ in downlink and 0.38 $M\,bits/s$ in uplink. Half of the time (the median) the worst off users throughput has increased by roughly 0.09 $M\,bits/s$ in downlink and 0.07 $M\,bits/s$ in uplink. To maximize the total throughput hence gives an unfair algorithm.



(a) Total system throughput, downlink **(b)** Total system throughput, uplink

Figure 5.9: The CDF plot for total system throughput in uplink and downlink at each calculated time point for 15 system simulation

In figure 5.9 the total system throughput can be seen and there is an evident loss in total throughput by being fair. Figure 5.10 shows better how much is lost by being fair.

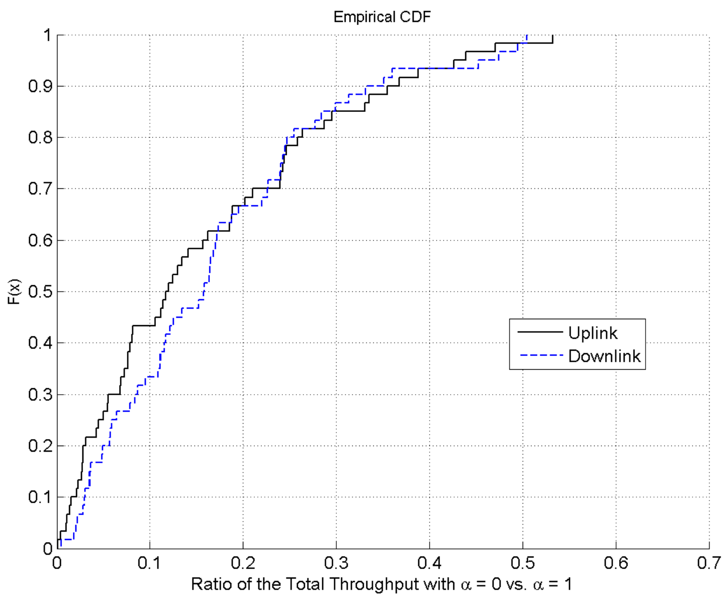


Figure 5.10: The CDF plot of the ratio of the total system throughput where the objective is to maximize the minimum user throughput divided by maximizing total throughput at each calculated time point for 15 system simulation

	$\alpha = 0/\alpha = 1$ ratio uplink	$\alpha = 0/\alpha = 1$ ratio downlink
Minimum	0.00	0.01
Maximum	0.53	0.50
Mean	0.16	0.17
Median	0.12	0.16
Standard Deviation	0.13	0.13

Table 5.5: Statistical results for the ratio of the total system throughput for $\alpha = 0$ and $\alpha = 1$ uniform user distribution, 15 second run

The fairness is reflected in the total system throughput where at each time point the total system throughput when power is fairly allocated is at most 53% of the total system throughput when power is allocated to maximize total throughput. This can be seen from figures 5.9 and 5.10. The loss can be rather high, which can be read from the fact that half of the time the total system throughput in the fair system in uplink is only 12% of the throughput in the unfair system (16% in downlink).

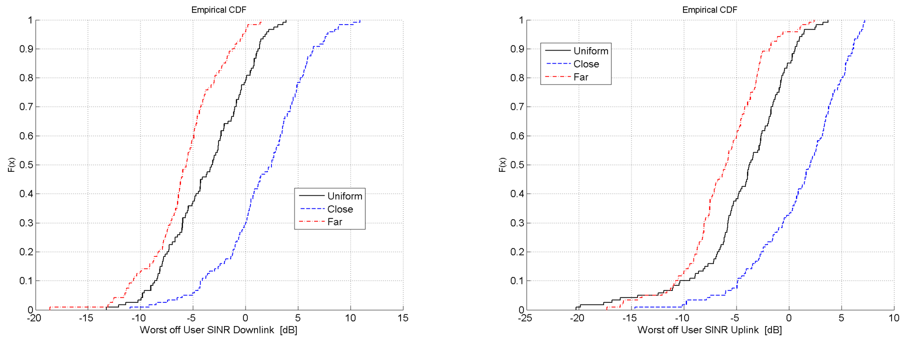
5.3 30 Second Simulation

Appendix B.1 contains more detailed results for the base case described in section 3.2.1. Here results from the 30 second simulation described in section 3.2.2 are displayed.

5.3.1 All Users Distributions Together

We begin by showing a few results for the three distributions together and comparing them. The following cumulative distribution functions are shown.

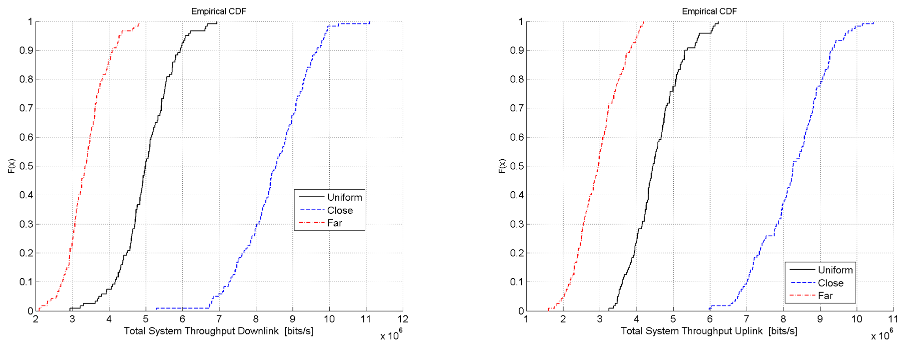
- the worst off user signal to interference ratio when power is allocated to maximize the minimum user throughput, figure 5.11,
- the total system throughput for when power is allocated to maximize total throughput, figure 5.12,
- the total system throughput ratio, figure 5.13,



(a) SINR for the minimum user in downlink, all distributions

(b) SINR for the minimum user in uplink, all distributions

Figure 5.11: The CDF plot for all users SINR in uplink and downlink at each calculated time point for a 30 second simulation, power is allocated to maximize the minimum user throughput

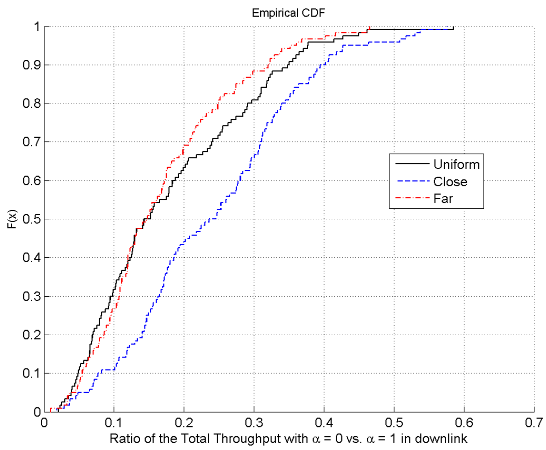


(a) Total system throughput in downlink, all distributions

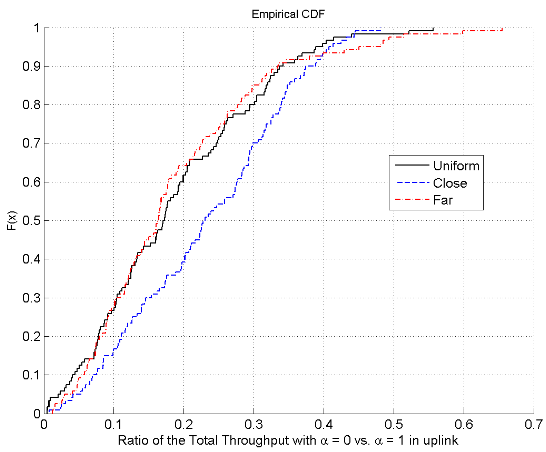
(b) Total system throughput in uplink, all distributions

Figure 5.12: The CDF plot for the total system throughput in uplink and downlink at each calculated time point for a 30 second simulation, power is allocated to maximize total throughput

Figure 5.11 shows that the signal to interference and noise ratio is lower for the worst off user when all users are far away, than when users are uniformly distributed, or when they are all close to the base station. The difference in the SINR value between when mobile users are close to a base station and far from the base stations is rather stable, roughly $7dB$. Since this is a log-normal difference this will translate in to an exponentially increasing difference in the worst off user throughput between the three distributions. The fact that users get higher throughput when they are close to a base station is also reflected in figure 5.12. It shows the total system throughput when power is allocated to maximize total throughput and it shows that the total system throughput is highest when users are all close to a base station than when they are uniformly distributed or all far away from the base stations.



(a) Ratio of the total system throughput in downlink, all distributions



(b) Ratio of the total system throughput in uplink, all distributions

Figure 5.13: The CDF plot of the ratio of the total system throughput when power is allocated to maximize the minimum user throughput divided by when power is allocated to maximize total throughput, in uplink and downlink at each calculated time point for a 30 second simulation

Statistical results for figure 5.13 can be found in appendix B.3.5.

There is some slight difference in the throughput ratio between the user distributions. From figure 5.13 it can be noted that when users are all close to the base station there is slightly less loss in throughput for being fair than when users are far from the base station or uniformly distributed. Indicates that when at least some users are close to their base station the throughput lost by being fair is less than when mobile users are far away or uniformly distributed. If we take the uplink as an example the throughput ratio is at a max 0.56-0.48-0.66 for when users are uniformly distributed, close or far respectively. At the 50% mark, the median, however the throughput ratio is 0.17-0.23-0.16 for when users are uniformly distributed, close or far respectively. This seems to indicate that for the extreme cases it does not matter how the mobile user distribution is. But in the general case mobiles lose less throughput by being fair when they are close to a base station.

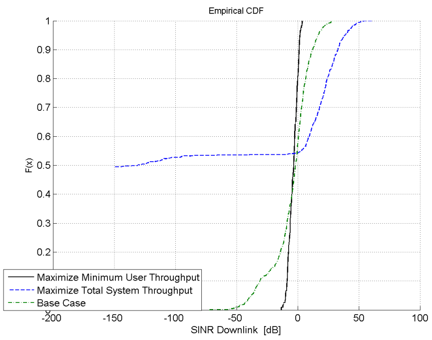
5.3.2 Users Distribution Separately

For each user distribution the following cumulative distribution functions are shown for when power is allocated to maximize the minimum user throughput, maximize the total system throughput and the base case:

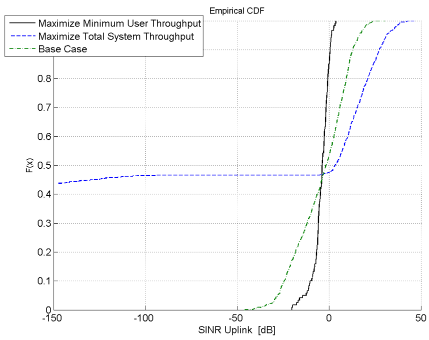
- the signal to interference and noise ratio (SINR) for all users, figures 5.14 to 5.16,
  - all users at all calculated times,
- the worst off user throughput, figures 5.17 to 5.19,
  - the user who gets the least throughput at each calculated time,
- total system throughput, figures 5.20 to 5.22,
  - sum of all users throughput at each calculated time,

The cumulative distribution functions mentioned above are shown for all mobile distributions in the following order. First when users are uniformly distributed, then when mobile users are close to a base station, and finally when the mobile users are far away from the base stations.

Figures 5.14 to 5.16 all show that maximizing minimum user throughput gives a fair system for all user distributions. When all users are close to a base station it seems that there are fewer users who are allocated no power when the total system throughput is maximized. This is consistent with previous results that mobile users get higher throughput when they are close to a base station. For maximizing total throughput there are users who do not get any signal to interference and noise ratio at all. The percentage of those users varies from roughly 25%, in uplink where users are close to a base station, to 51%, in downlink where users are far from the base station. For when power is allocated to maximize the minimum user throughput the minimum SINR is between -13 and -21 dB. The maximum SINR for the same case is between 2 and 10 dB. This is also supported by the statistical tables in B.3.3. Allocating power to maximize total system throughput gives the widest range of signal to interference and noise ratio while maximizing the minimum user

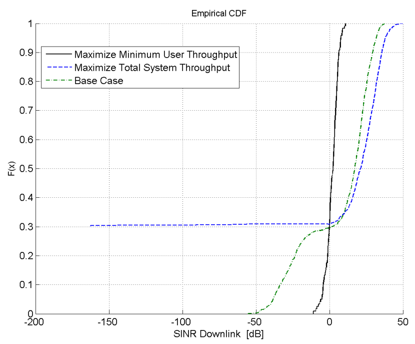


(a) SINR for all users in downlink, uniform distribution

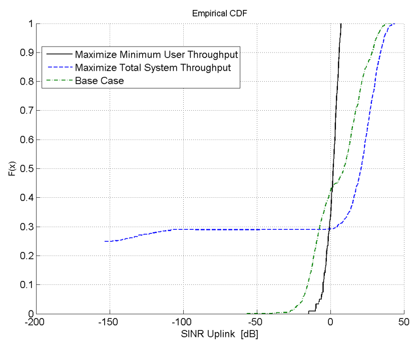


(b) SINR for all users in uplink uniform distribution

Figure 5.14: The CDF plot for all users SINR in uplink and downlink at each calculated time point for 30 second simulation, uniform user distribution

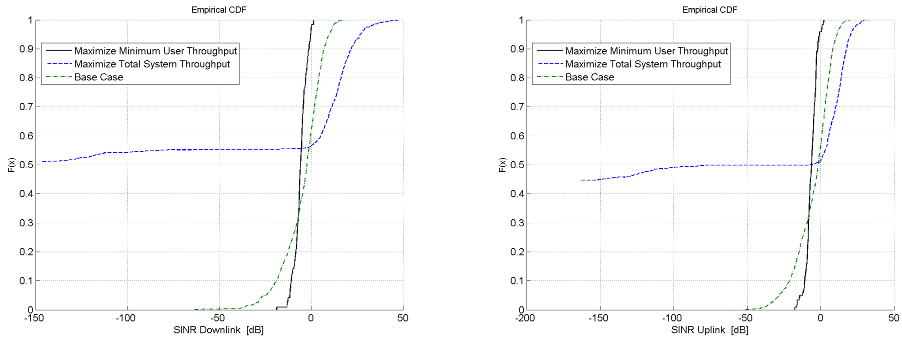


(a) SINR for all users in downlink, users are close to the base station



(b) SINR for all users in uplink, users are close to the base station

Figure 5.15: The CDF plot for all users SINR in uplink and downlink at each calculated time point for 30 second simulation, users are close to the base station

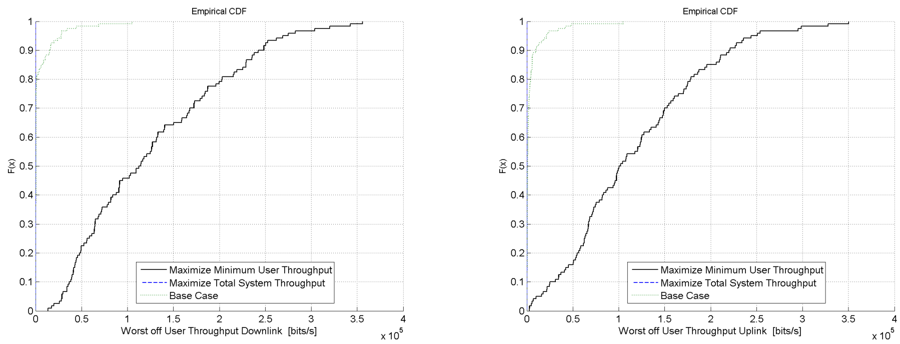


(a) SINR for all users in downlink, users far from the base station

(b) SINR for all users in uplink, users far from the base station

Figure 5.16: The CDF plot for all users SINR in uplink and downlink at each calculated time point for 30 second simulation, users far from the base station

throughput gives the narrowest spread. The base case is then between the two different power allocations.



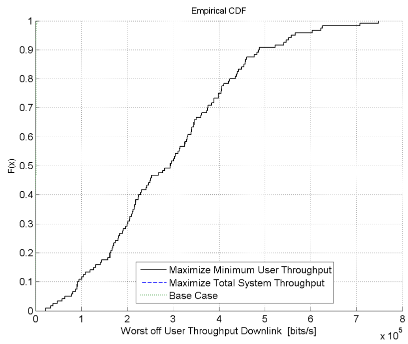
(a) Minimum user throughput downlink, uniform distribution

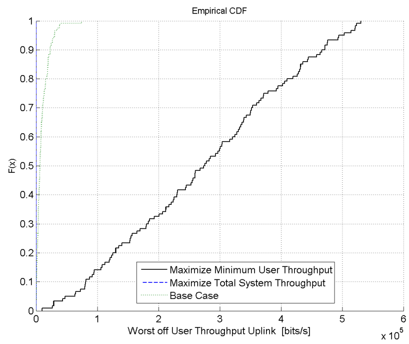
(b) Minimum user throughput uplink, uniform distribution

Figure 5.17: The CDF plot for the worst off user's throughput in uplink and downlink at each calculated time point for 30 second simulation, uniform user distribution

Figures 5.17 to 5.19 show the worst off user throughput for uniform, close and far user distribution. Similar to the 15 system run (see figure 5.8) the worst off user when power is allocated to maximize the total system throughput does not get any power and hence cannot be seen in the graphs. In the base case the worst off user does get some throughput it is however dramatically increased by a "smart" allocation of power.

When power is allocated to maximize the minimum user throughput the maximum of the worst off user throughput varies from roughly 0.25 $M\,bits/s$, in downlink

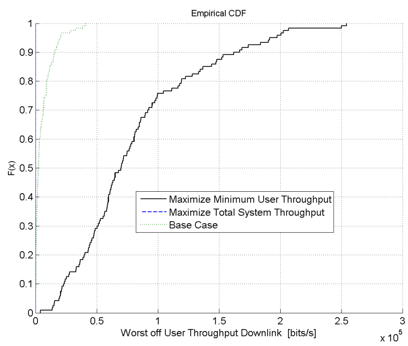


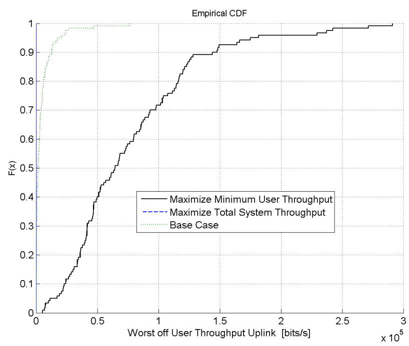


(a) Minimum user throughput downlink, users are close to the base station

(b) Minimum user throughput, uplink, users are close to the base station

Figure 5.18: The CDF plot for the worst off user's throughput in uplink and downlink at each calculated time point for 30 second simulation, users are close to the base station



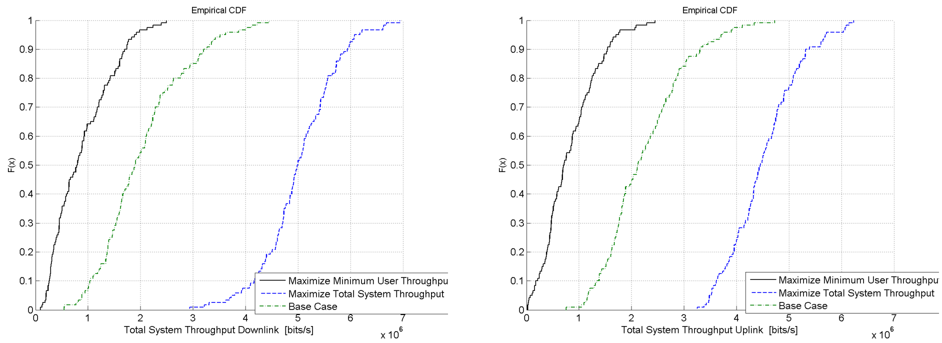


(a) Minimum user throughput in downlink, users are far from the base station

(b) Minimum user throughput uplink, users are far from the base station

Figure 5.19: The CDF plot for the worst off user's throughput in uplink and downlink at each calculated time point for 30 second simulation, users are far from the base station

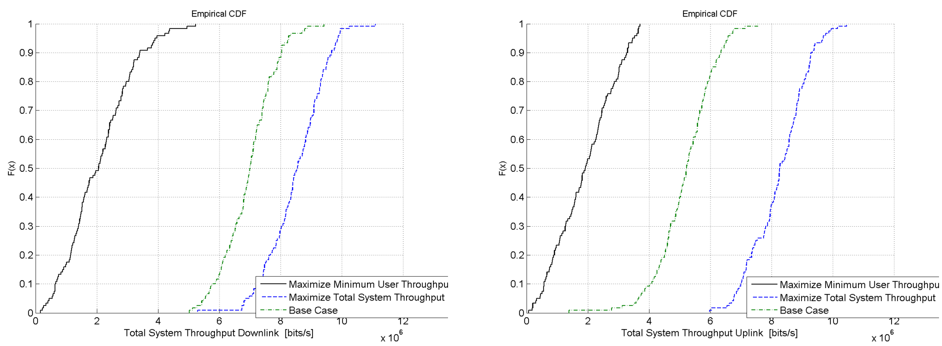
with mobile users far from the base stations to roughly 0.75e5 $M\,bits/s$ in downlink when users are close to a base station. The median of the worst off user throughput varies from roughly 0.07 $M\,bits/s$, in uplink and downlink with mobile users far from the base stations to roughly 0.3 $M\,bits/s$ in downlink when users are close to a base station.



(a) Total system throughput downlink, uniform distribution

(b) Total system throughput uplink, uniform distribution

Figure 5.20: The CDF plot for total system throughput in uplink and downlink at each calculated time point for 30 second simulation, uniform user distribution

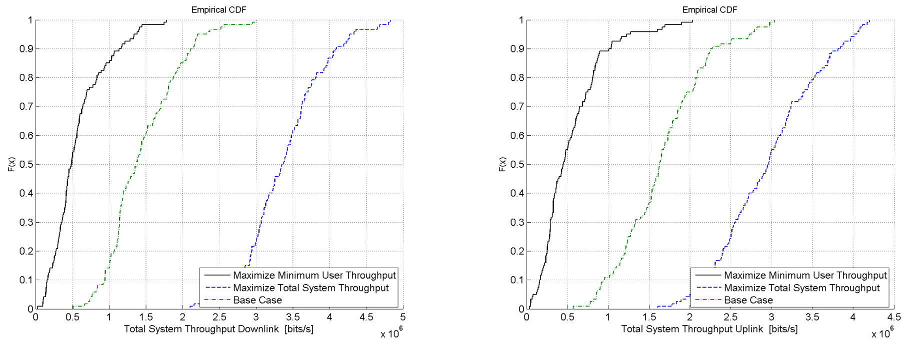


(a) Total system throughput downlink, users are close to the base station

(b) Total system throughput uplink, users are close to the base station

Figure 5.21: The CDF plot for total system throughput in uplink and downlink at each calculated time point for 30 second simulation, users are close to the base station

Figures 5.20 to 5.22 show the total system throughput for the different user distributions. The total system throughput is again considerably lower when the minimum user throughput is maximized. The base case again is now somewhere in the middle, describing that maximum throughput is not achieved nor is fairness or power saving.



(a) Total system throughput downlink, users far from the base station

(b) Total system throughput uplink, users far from the base station

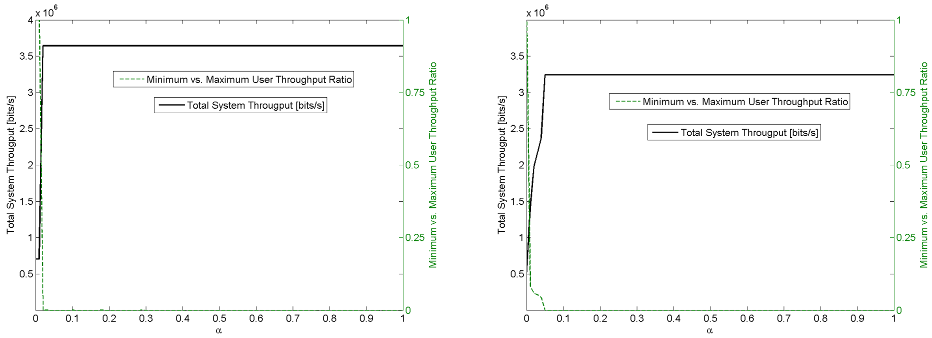
Figure 5.22: The CDF plot for total system throughput in uplink and downlink at each calculated time point for 30 second simulation, users far from the base station

When power is allocated to maximize the minimum user throughput the minimum total system throughput varies from 19 $k\,bits/s$, in uplink with mobile users uniformly distributed from the base stations to 0.15 $M\,bits/s$ in uplink when users are close to a base station. The maximum total system throughput varies from 1.8 $M\,bits/s$, in downlink with mobile users far from the base stations to 5.2 $M\,bits/s$ in downlink when users are close to a base station. The median total system throughput varies from 0.45 $M\,bits/s$, in uplink with mobile users far from the base stations to 2.1 $M\,bits/s$ in downlink when users are close to a base station.

When power is allocated to maximize the total throughput the minimum total system throughput varies from 1.6 $M\,bits/s$, in uplink with mobile users far from the base stations to 6 $M\,bits/s$ in uplink when users are close to a base station. The maximum total system throughput varies from 4.2 $M\,bits/s$, in uplink with mobile users far from the base stations to 11 $M\,bits/s$ in downlink when users are close to a base station. The median total system throughput varies from 3 $M\,bits/s$, in uplink with mobile users far from the base stations to 8.5 $M\,bits/s$ in downlink when users are close to a base station.

5.4 Changing the Weights Between Maximizing Total Throuhgput and Maximizing the Minimum User Throughput

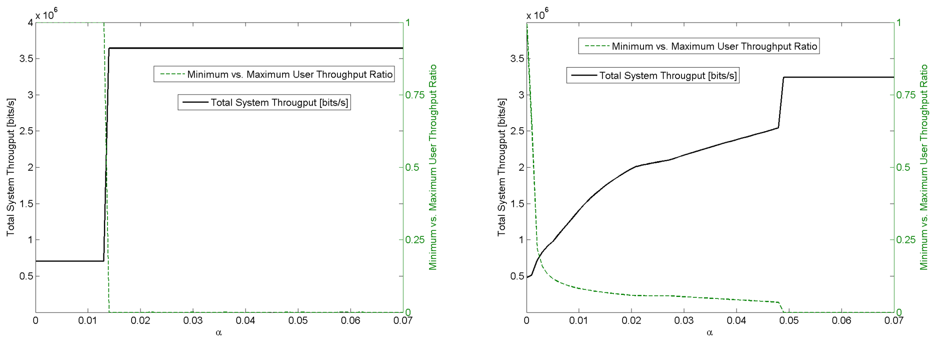
In this section the results from the simulation described in section 3.2.2 are presented. Power is allocated for when α, which describes the weights between maximizing minimum user throughput and maximizing the total system throughput in (2.9), varies from 0 (maximize minimum user throughput) to 1 (maximize total system throughput). Figure 5.23 shows the total system throughout and the throughput ratio over the whole α interval.



(a) Downlink **(b)** Uplink

Figure 5.23: The system total throughput (left axis) for all values of α along with the ratio of the minimum user throughput divided by the maximum user throughput (right axis), in uplink and downlink

From figure 5.23 one can see that over almost the whole interval, the results are equal to when total throughput is maximized and there is only a small interval on which there is any combination between fairness and maximizing total throughput. Figure 5.24 shows a more detailed view of when the solution jumps from giving the $\alpha = 0$ solution to $\alpha = 1$.



(a) Downlink **(b)** Uplink

Figure 5.24: The total system throughput (left axis) for $\alpha \leq 0.07$ along with the ratio of the minimum user throughput divided by the maximum user throughput (right axis), in uplink and downlink

In downlink it can be seen from figure 5.24a that trying to be somewhat fair and somewhat trying to maximize the total throughput is not possible. The solution is either equal to the max-min power allocation or maximize total throughput power allocation. In uplink however there is some space for trying to balance being fair and maximizing total throughput. However the values of α must then be roughly between 0 and 0.05. By examining uplink more closely it can be noted that when the minimum user is getting 50% of the maximum user the total throughput is still really low. It is not until the minimum user is gettin between 5 % and 10% of the maximum user that the throughput increases. These results are based on a single system; it would therefore be interesting to see if this behaviour is evident for different systems, for example different mobile users' location.

5.5 Moving User towards a Base Station

In this section results from the simulation described in section 3.2.2, where one user moves towards a base station are presented. The following figures are shown:

- the users' locations and movements, in figure 5.25,
- the moved user's throughput cumulative distribution function for three values of α [0, 0.013 and 1], in figure 5.26,
- the moved user's throughput for three values of α [0, 0.013 and 1], in figure 5.27,
- total system throughput cumulative distribution function for three values of α [0, 0.013 and 1], in figure 5.28,
- total system throughput at each location for three values of α [0, 0.013 and 1], in figure 5.29,

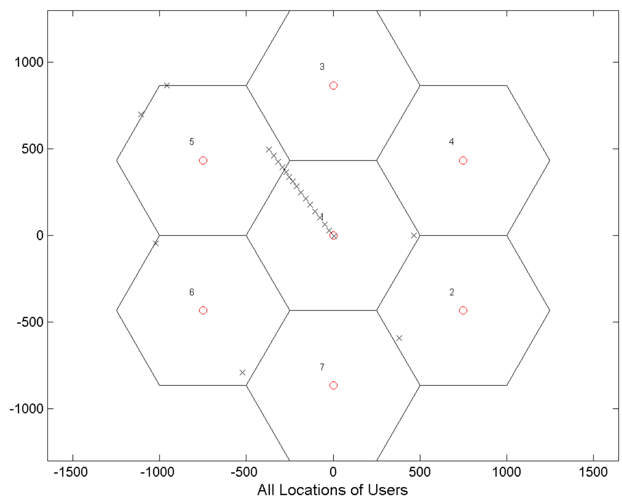
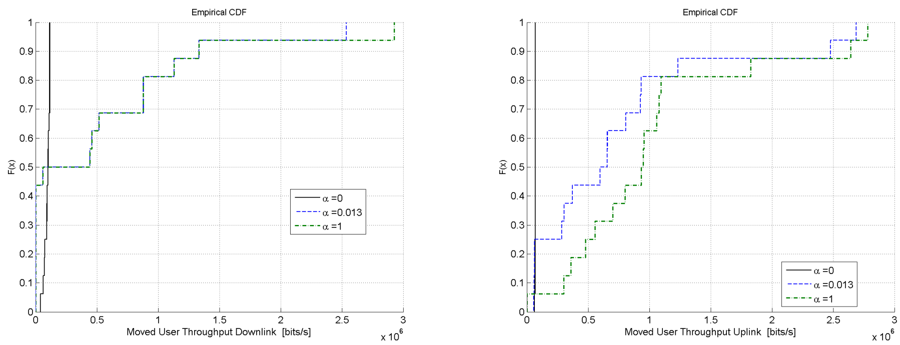


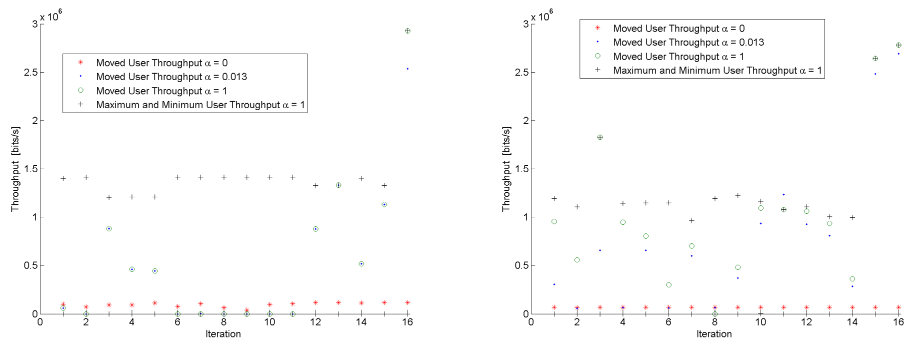
Figure 5.25: The location of all users as one is moved towards base station 1

Have three values of α that is $[0, 0.013, 1]$. $\alpha = 0.013$ is used since that is a value which in uplink is somewhere between maximizing total throughput and maximizing the minimum user throughput.



(a) Downlink **(b)** Uplink

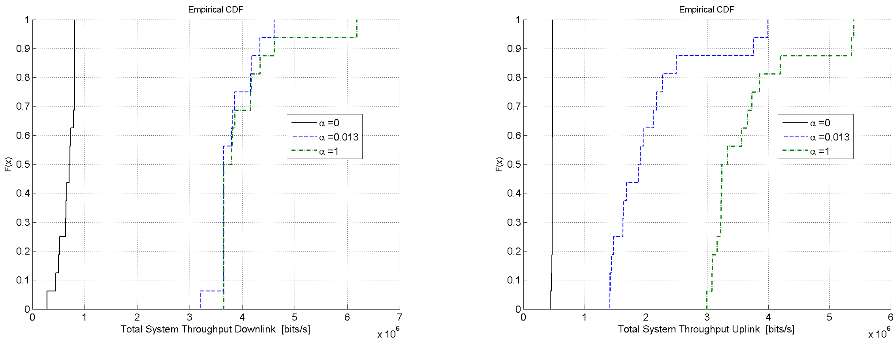
Figure 5.26: The moved user throughput cumulative distribution function for $\alpha = 0,\ 0.013$ and 1



(a) Downlink **(b)** Uplink

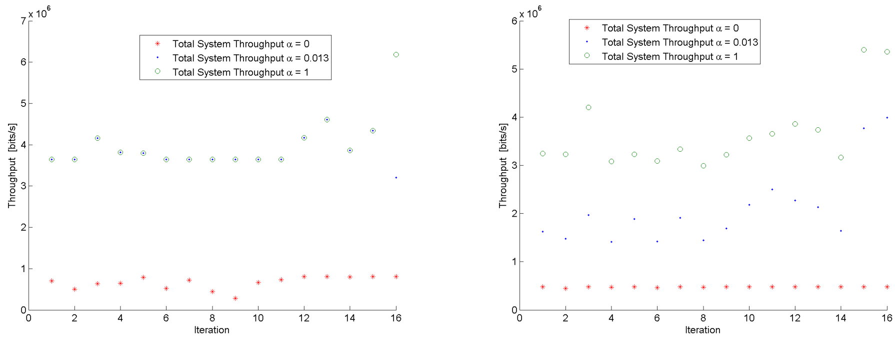
Figure 5.27: The moved user throughput at each position for $\alpha = 0,\ 0.013$ and 1

From examining figures 5.26 and 5.27 it can be noted that in downlink there is not much difference in the user's throughput between when $\alpha = 1$ and $\alpha = 0.013$. Only when the mobile user is closest to the base station does $\alpha = 1$ give higher throughput than $\alpha = 0.013$. One may also note that when the minimum user throughput is maximized the moved user's throughput does not change much. In uplink the throughput for $\alpha = 0.013$ is somewhere between $\alpha = 1$ and $\alpha = 0$ as expected.



(a) Downlink **(b)** Uplink

Figure 5.28: The total system throughput cumulative distribution function for $\alpha = 0,\ 0.013$ and 1



(a) Downlink **(b)** Uplink

Figure 5.29: The total system throughput for $\alpha = 0,\ 0.013$ and 1

Figures 5.28 and 5.29 show that the total system throughput does not increase until the moved user is close to the base station. This is similar throughput behaviour as for the moved user and seems to indicate that the small difference in path gain, in other words shadow fading and Rayleigh fading, have more impact than the distance to base station.

The results support the results from section 5.4. They do that by the fact that in downlink the results for $\alpha = 0.013$ are almost equal to the results for $\alpha = 1$ and that in uplink the results for $\alpha = 0.013$ are somewhere between $\alpha = 0$ and $\alpha = 1$.

5.6 Larger Systems

Figure 5.30 shows the location of the 15 mobile users in the system, they are uniformly distributed.

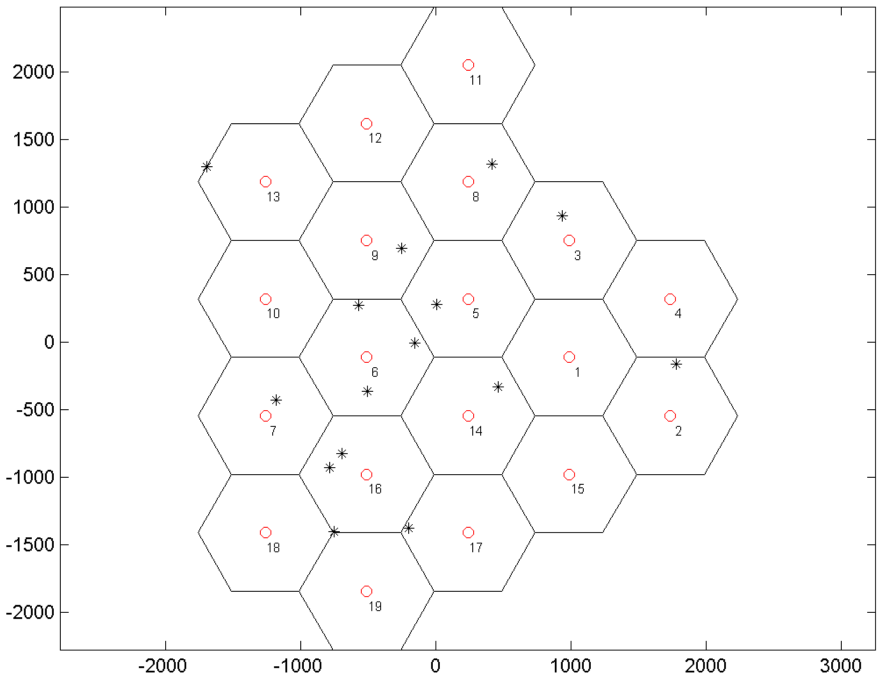
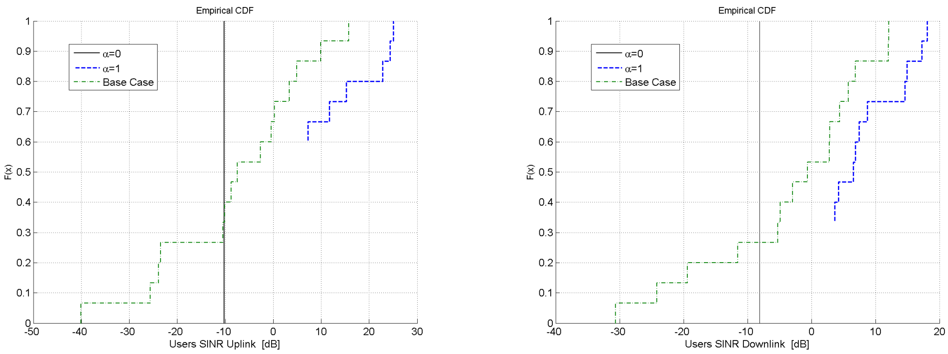


Figure 5.30: Location of users in the 19 cell case

Figure 5.31 shows the signal to interference and noise ratio for maximizing minimum user throughput ($\alpha = 0$), maximizing total throughput ($\alpha = 1$) and the base case. All users get the same signal to interference and noise ratio for when the minimum user throughput is maximized ($\alpha = 0$) and many users do not send or receive anything when the total throughput is maximized.



(a) SINR Uplink **(b)** SINR Downlink

Figure 5.31: Signal to interference and noise ratio for the 19 base stations

Total Throughput [M bits/s]	Uplink	Downlink
Maximize minimum user throughput	0.39	0.63
Maximize total system throughput	7.15	7.22
Base Case	3.15	4.08

Table 5.6: Total throughput for each power allocation

The difference in total throughput between the different power allocation algorithms is quite high; the percentage can be seen in table 5.7.

	Uplink	Downlink
Maximize minimum user throughput v.s. Maximize total system throughput [%]	5.51	8.67

Table 5.7: Total throughput ratio

These percentages are consistent with the results from the 15 system run and 30 second run, shown in section 5.2 and 5.3, where it was found that in most cases the throughput when the minimum user throughput is maximized is at most 20% of the throughput when the total throughput is maximized.

Chapter 6

Conclusions

We implemented the heuristic algorithms for allocating links and power in the single channel case. The four different algorithms for link assignment are the direct greedy approach, the reversed greedy approach, maximize the total path gain and maximize the minimum path gain. The system examined consists of 7 base stations, 7 mobile users and a single channel. A comparison between the different algorithms to assign mobiles to a base station revealed that:

- The direct greedy approach and maximize the minimum path gain result in a higher system throughput than the other two.
- When power is allocated to maximize the total throughput the two link assignments (direct greedy and maximize total path gain) are almost equal.
- When power is allocated to maximize the minimum user throughput, link assignment through maximizing total path gain gives better results than the direct greedy link assignment.

Because of these results, maximize total path gain link assignment was selected for presentation in this report.

A completely fair power allocation (maximizing the minimum user throughput) is compared to a power assignment which maximizes the total system throughput and does not consider fairness for three cases. In the first case mobile users are distributed uniformly over the coverage area, while in the second and third case mobile users are placed close to a base station and far from the base stations respectively. The throughput the system receives is different for the three different user distributions. The main results are.

- For maximizing total throughput there are mobile users in the system that do not get any throughput at all. The percentage of those users varies from roughly 25%, in uplink where users are close to a base station, to 51%, in downlink where users are far from the base station.

- The max-min allocation results in a loss in throughput compared to maximizing total throughput, more specifically,
  - At most, the system receives 66%, in uplink when users are far from the base stations, to 47%, in downlink when users are far from the base stations, of the maximum possible throughput.
  - The median of the throughput received by the system in the max-min allocation varies from 24%, in downlink when users are close to a base station, to 15%, in downlink for both the uniform user distribution and when they are far from the base stations, of the maximum possible throughput.

Roughly speaking, when power is allocated to maximize the minimum user throughput, the throughput in the system is at max about 55% of the throughput achieved in a throughput maximizing power allocation. In addition half of the time the throughput achieved by max-min allocation is less than 20% of the throughput achieved by maximizing throughput allocation.

In addition we notice that:

- For the maximize the minimum throughput power allocation all mobile users receive the same throughput, as expected.
- The algorithm works in a similar way for a larger system (19 base stations); therefore all the runs could have been done for a larger system.
- Maximizing the minimum user throughput allocates power in such a way that even when one user moves towards the base station the throughput changes are minimal.
  - When total system throughput is maximized the moved user's throughput only increases noticeably when the user is getting really close to the base station. This indicates that the distance component is not a large factor in the path gain matrix until the mobile user is close to the base station.

We examine how the throughput changes if α is changed in steps from 0 to 1. α is the weight in the objective function of how power is allocated, $\alpha = 0$ gives the max-min objective function, while $\alpha = 1$ gives the objective which maximizes total throughput. The results suggest that:

- Over a large part of the α interval the solution is stable, and always returns the same power allocation as if the objective were to maximize total throughput $\alpha = 1$.
- For downlink the solution is always either equal to the solution with $\alpha = 0$ (maximize the minimum user throughput) or $\alpha = 1$ (maximize the total throughput).

- For uplink there is a small α interval on which the total throughput is between the solutions when $\alpha = 0$ and $\alpha = 1$. The interval is roughly $[0, 0.05]$.
  - The move in the algorithm from returning the same solution as $\alpha = 0$ to $\alpha = 1$ happens rather abruptly. When the minimum user is getting 50% of the maximum user the total throughput is still really low. It is not until the minimum user is receiving between 5 % and 10% of the maximum user that the throughput increases.

The results imply that the space for weighing between the fair and the unfair allocation is rather small and in some cases power allocation can either be completely fair or completely unfair. In downlink it seems that fairness control comes at a high price, since there can exist a cellular system where there is no interval on which there can be any balance between being fair and maximizing total throughput. In uplink however there is some space for trying to balance between being fair and maximizing total throughput.

Future Work : This algorithm could have been examined further than is done in this work. Future work might include:

- Move away from the single channel case and examine how it behaves when there is more than one channel.
- Look in to how the power allocated to each user is affected by the fairness.
- Examine how the user throughput changes when a mobile user is moving closer to a base station but is taking smaller steps and begins closer to the base station than is done here.
- The weighing of the objective function was only done for a single system; it is well worth examining if the behaviour observed is the same for other systems, especially if the jump behaviour in the downlink is persists.
- The theory behind these results is developed for a snap shot in time. A future step could be to evolve the theory to work over discretized time. By for example implementing a scheduler which decides which mobile users connect to a base station in the system at each time.

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Appendix A

Detailed parameters

A.1 Data for Verification of the Algorithm

When assigning mobiles to base stations shadow fading is not considered, hence the path gain matrix for assigning mobiles to base stations and when allocation power differ. The path gain matrix used for link assignment in uplink is given in table A.1 and in downlink in table A.2.

Base station	1	2	3
Mobile 1	-136.2030	-131.8536	-130.1345
Mobile 2	-136.2256	-129.0634	-149.1755

Table A.1: Path gain matrix for link assignment in uplink [dB]

Base station	1	2	3
Mobile 1	-138.84	-140.64	-120.65
Mobile 2	-136.23	-129.06	-149.18

Table A.2: Path gain matrix for link assignment in downlink, [dB]

For the power allocation the path gain matrix in uplink and downlink is:

Uplink

For the two users the path gain matrix in uplink is given in table A.3

Base station	1	2	3
Mobile 1	-133.08	-131.03	-124.85
Mobile 2	-145.18	-132.58	-155.22

Table A.3: Path gain matrix for power allocation in uplink, in dB

Downlink

For the two users the path gain matrix in downlink is given in table A.4

Base station	1	2	3
Mobile 1	-138.27	-141.93	-119.64
Mobile 2	-138.44	-131.08	-155.70

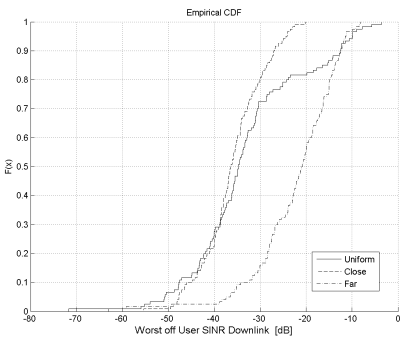
Table A.4: Path gain matrix for power allocation in downlink, [dB]

Appendix B

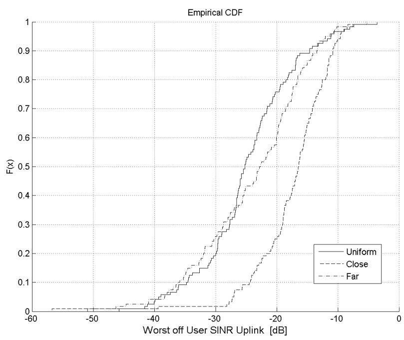
More detailed results

B.1 Base Case

For each user distribution the following cumulative distribution plots are displayed, for both uplink and downlink; the SINR for all users in the system for all calculated times, the throughput for all users in the system for all time points calculated and the SINR for each user at all time points calculated. Finally the statistics for the SINR and total system throughput distributions for all users in the system at all time points is shown in a table.



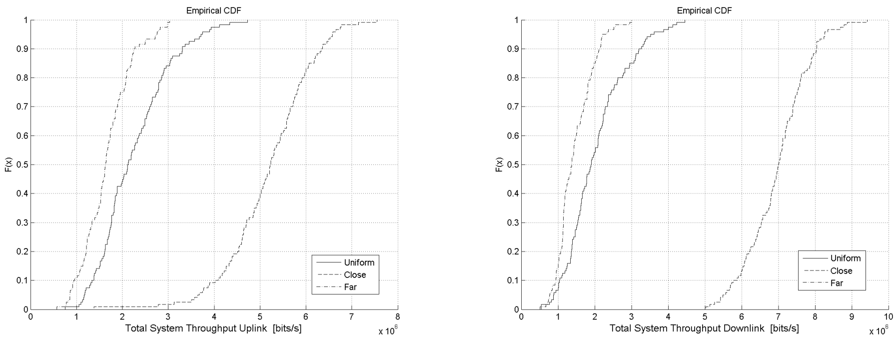
(a) SINR for the minimum user in downlink, all distributions



(b) SINR for the minimum user in uplink, all distributions

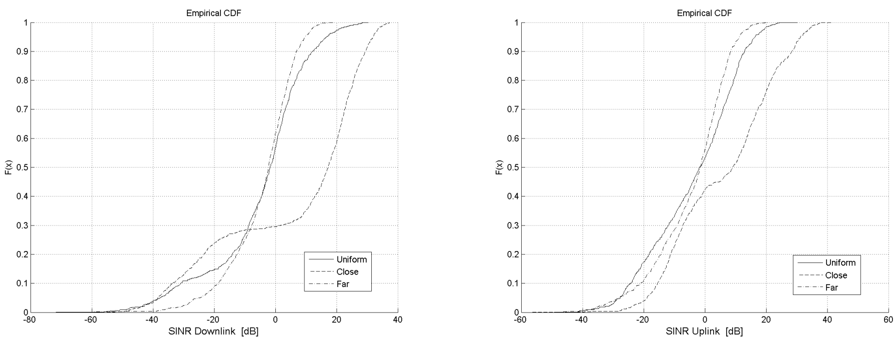
Figure B.1: The CDF plot for all users SINR in uplink and downlink at each calculated time point for a 30 second simulation using the base case link and power allocation

There is a relatively small difference in the total throughput between the uniform user distribution and the far user distribution compared to the difference between the uniform distribution and close distribution.



(a) Total system throughput in downlink, all distributions **(b)** Total system throughput in uplink, all distributions

Figure B.2: The CDF plot for the total system throughput in uplink and downlink at each calculated time point for a 30 second simulation using the base case link and power allocation



(a) SINR downlink (dB), all distributions **(b)** SINR uplink (dB), all distributions

Figure B.3: The CDF plot for all users signal to interference and noise ratio in uplink and downlink at each calculated time point for a 30 second simulation using the base case link and power allocation

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-45.77	7.540e+005	-71.71	5.440e+005
Maximum	30.24	4.727e+006	30.41	4.452e+006
Mean	-3.80	2.250e+006	-4.72	2.034e+006
Median	-1.70	2.122e+006	-1.72	1.901e+006
Standard Deviation	14.31	7.756e+005	15.71	8.275e+005

Table B.1: Statistical results for the signal to interference and noise ratio and total system throughput, power and links are assigned according to the base case scenario, uniform user distribution

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-56.71	1.384e+006	-55.37	5.028e+006
Maximum	41.27	7.551e+006	37.41	9.429e+006
Mean	5.97	5.188e+006	6.50	6.973e+006
Median	9.10	5.224e+006	17.25	7.004e+006
Standard Deviation	16.79	9.363e+005	24.07	8.449e+005

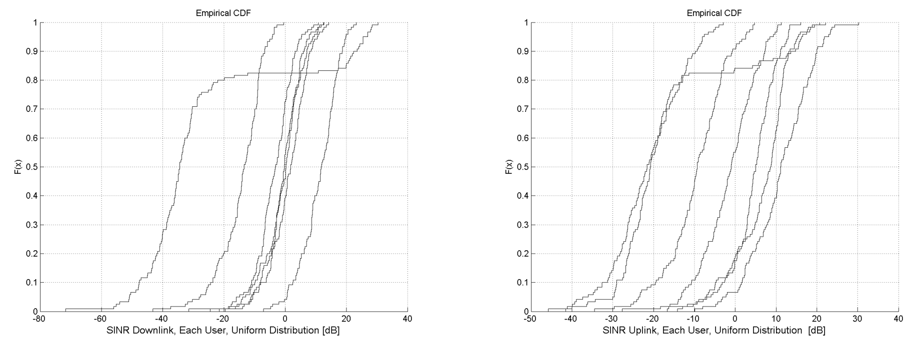
Table B.2: Statistical results for the signal to interference and noise ratio and total system throughput, power and links are assigned according to the base case scenario, user close to the base station

When all users are close to a base station some users get a higher SINR value than compared to uniform distribution. However there are also users who are close to the same base station hence there is a user who gets an even worse SINR than before. Also the standard deviation has increased since the SINR values users get spans a larger area.

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-50.92	5.730e+005	-63.12	5.124e+005
Maximum	20.64	3.035e+006	19.09	3.006e+006
Mean	-4.37	1.663e+006	-4.33	1.456e+006
Median	-1.52	1.632e+006	-2.23	1.368e+006
Standard Deviation	12.01	5.270e+005	10.73	4.853e+005

Table B.3: Statistical results for the signal to interference and noise ratio and total system throughput, power and links are assigned according to the base case scenario, user far from the base station

When all users are far from the base stations the maximum SINR has decreased compared to the uniformly distributed users or when all users are close to a base station. This is expected since no user is close to a base station. The minimum SINR has also decreased compared to uniformly distributed users but is similar compared to when all users are close to a base station.



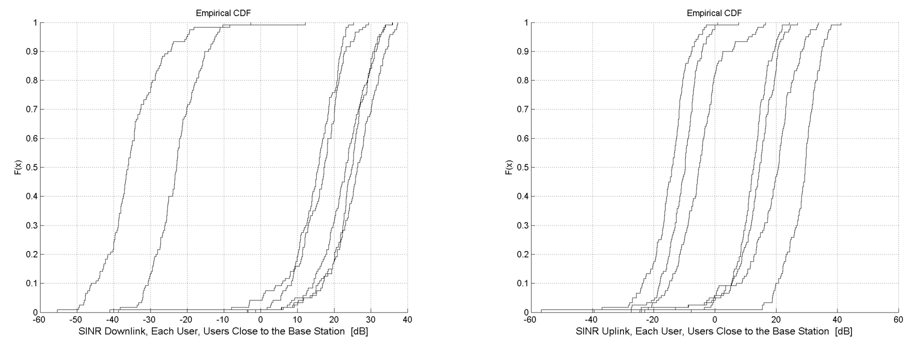
(a) SINR downlink (dB) **(b)** SINR uplink (dB)

Figure B.4: The CDF plot for each users signal to interference and noise ratio plotted separately in uplink and downlink at each calculated time point for a 30 second simulation using the base case link and power allocation and uniform distribution.

Figure B.4 shows that users do not have the same signal to interference and noise ratio at each time, in fact the variation is roughly 20 dB.

Mobile	Total User Throughput in Uplink	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink	Percentage of Total System Throughput in Downlink [%]
Mobile 1	8.152e+006	3.02	2.885e+006	1.18
Mobile 2	5.149e+007	19.07	2.886e+007	11.82
Mobile 3	6.721e+007	24.89	2.865e+007	11.73
Mobile 4	2.696e+007	9.98	3.659e+007	14.99
Mobile 5	1.189e+006	0.44	1.890e+007	7.74
Mobile 6	9.605e+007	35.57	9.535e+007	39.06
Mobile 7	1.899e+007	7.03	3.290e+007	13.48

Table B.4: Total User Throughput in Uplink And Downlink for uniform distribution, base case

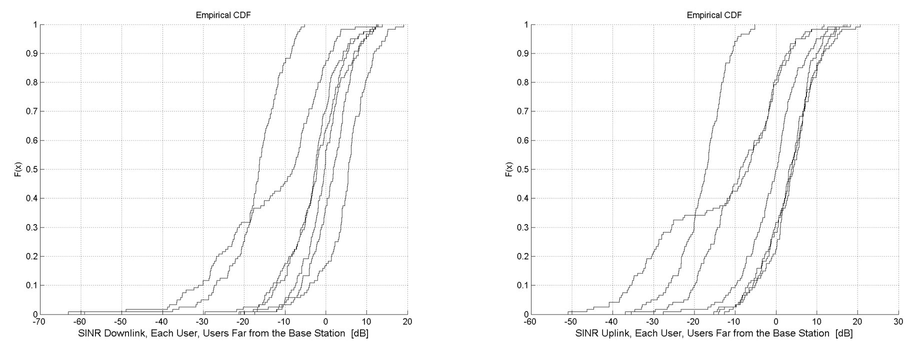


(a) SINR downlink (dB) **(b)** SINR uplink (dB)

Figure B.5: The CDF plot for each users signal to interference and noise ratio plotted separately in uplink and downlink at each calculated time point for a 30 second simulation using the base case link and power allocation and users are close to the base station

Mobile	Total User Throughput in Uplink	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink	Percentage of Total System Throughput in Downlink [%]
Mobile 1	2.404e+006	0.39	1.246e+006	0.15
Mobile 2	1.147e+008	18.43	1.827e+008	21.83
Mobile 3	1.005e+008	16.14	1.303e+008	15.58
Mobile 4	5.243e+006	0.84	1.900e+005	0.02
Mobile 5	1.536e+008	24.67	2.061e+008	24.63
Mobile 6	2.275e+008	36.54	1.933e+008	23.10
Mobile 7	1.860e+007	2.99	1.230e+008	14.69

Table B.5: Total User Throughput in Uplink And Downlink for close distribution, test case



(a) SINR downlink (dB) **(b)** SINR uplink (dB)

Figure B.6: The CDF plot for each users signal to interference and noise ratio plotted separately in uplink and downlink at each calculated time point for a 30 second simulation using the base case link and power allocation and users are far from the base station

Mobile	Total User Throughput in Uplink	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink	Percentage of Total System Throughput in Downlink [%]
Mobile 1	1.167e+006	0.59	1.496e+006	0.86
Mobile 2	4.853e+007	24.32	3.551e+007	20.33
Mobile 3	4.420e+007	22.15	5.595e+007	32.03
Mobile 4	1.401e+007	7.02	1.009e+007	5.77
Mobile 5	4.712e+007	23.62	2.871e+007	16.43
Mobile 6	3.037e+007	15.22	2.031e+007	11.62
Mobile 7	1.412e+007	7.08	2.264e+007	12.96

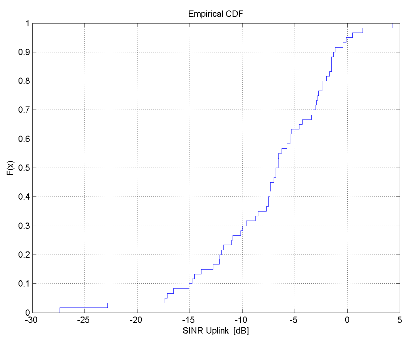
Table B.6: Total User Throughput in Uplink And Downlink for far distribution, test case

B.2 Statistical Results for the 15 System Run

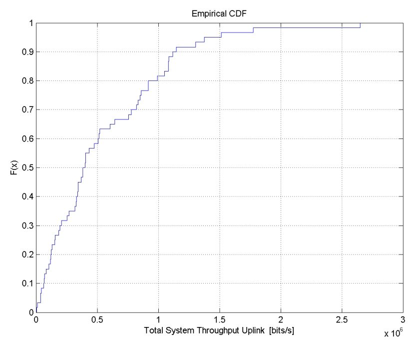
In this section more results from the 15 system run are presented. For all users SINR and total system throughput both the cumulative distribution function are shown and some statistical data, the SINR for each user over the time simulated along with a table showing the total user throughput for all mobile users. The above results are shown for both when power is allocated to maximize the minimum user throughput and maximizing total throughput, they are shown for all four link assignments.

B.2.1 Maximize the minimum user throughput

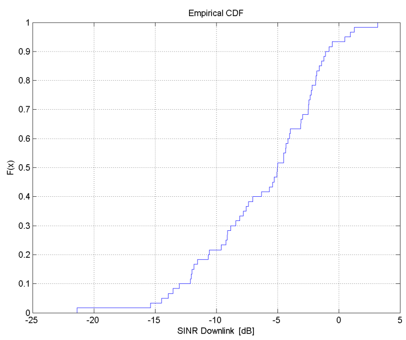
Direct Greedy Approach



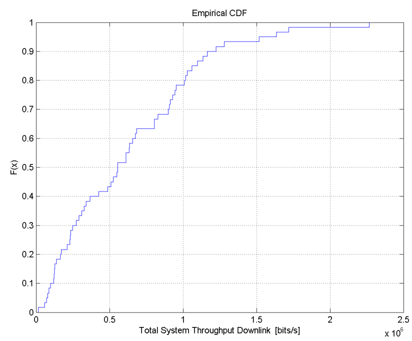
(a) SINR Uplink



(b) System Throughput Uplink

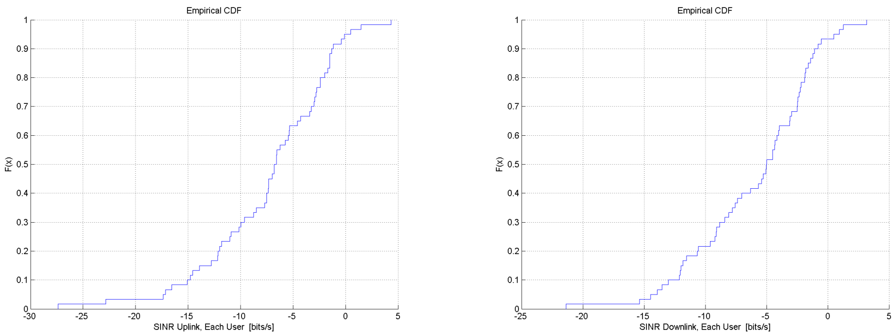


(c) SINR Downlink



(d) System Throughput Downlink

Figure B.7: The CDF Plots for all users SINR and system throughput at each calculated time point over a 15 second period, users are uniformly distributed from the base station, using direct greedy approach



(a) SINR Each User Uplink **(b)** SINR Each User Downlink

Figure B.8: The CDF Plots for the SINR of all users plotted separately over a 15 second period, users are uniformly distributed from the base station, using direct greedy approach

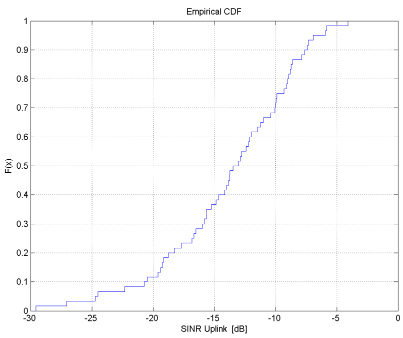
	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-27.38	3.686e+003	-21.38	1.464e+004
Maximum	4.34	2.651e+006	3.16	2.267e+006
Mean	-7.32	5.566e+005	-6.02	6.285e+005
Median	-6.69	3.921e+005	-5.00	5.552e+005
Standard Deviation	5.94	5.097e+005	4.85	4.793e+005

Table B.7: Statistical results for the signal to interference and noise ratio and total system throughput, links are assigned with the direct greedy approach, power is allocated to maximize minimum user throughput, uniform user distribution

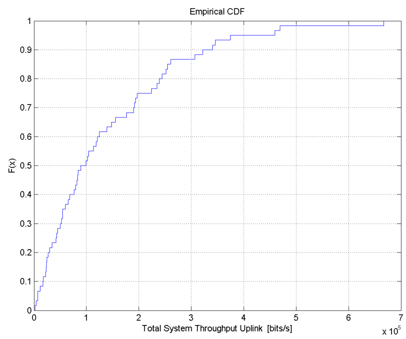
	Total User Throughput in Uplink	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink	Percentage of Total System Throughput in Downlink [%]
Mobile 1 - 7	4.771e+006	14.29	5.387e+006	14.29

Table B.8: Total user throughput in uplink and downlink for uniform distribution, DGA link assignment and power is allocated to maximize the minimum user throughput

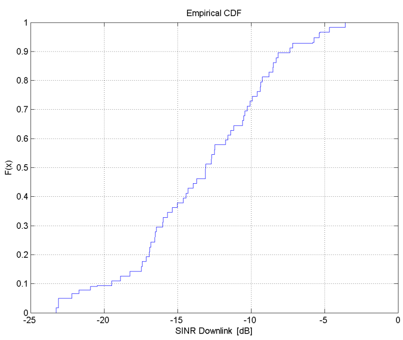
Reversed Greedy Approach



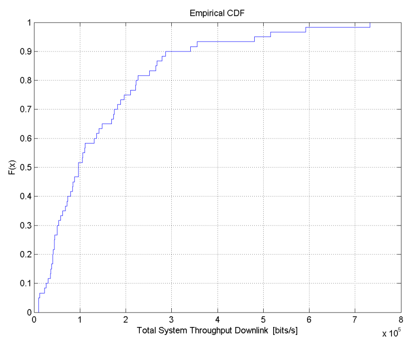
(a) SINR Uplink



(b) System Throughput Uplink

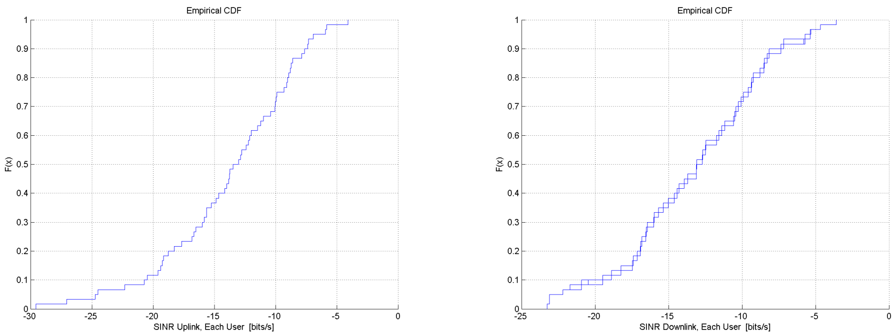


(c) SINR Downlink



(d) System Throughput Downlink

Figure B.9: The CDF plots for all users SINR and system throughput at each calculated time point over a 15 second period, users are uniformly distributed from the base station, using reversed greedy approach



(a) SINR Each User Uplink **(b)** SINR Each User Downlink

Figure B.10: The CDF Plots for the SINR of all users plotted separately over a 15 second period, users are uniformly distributed from the base station, using reversed greedy approach

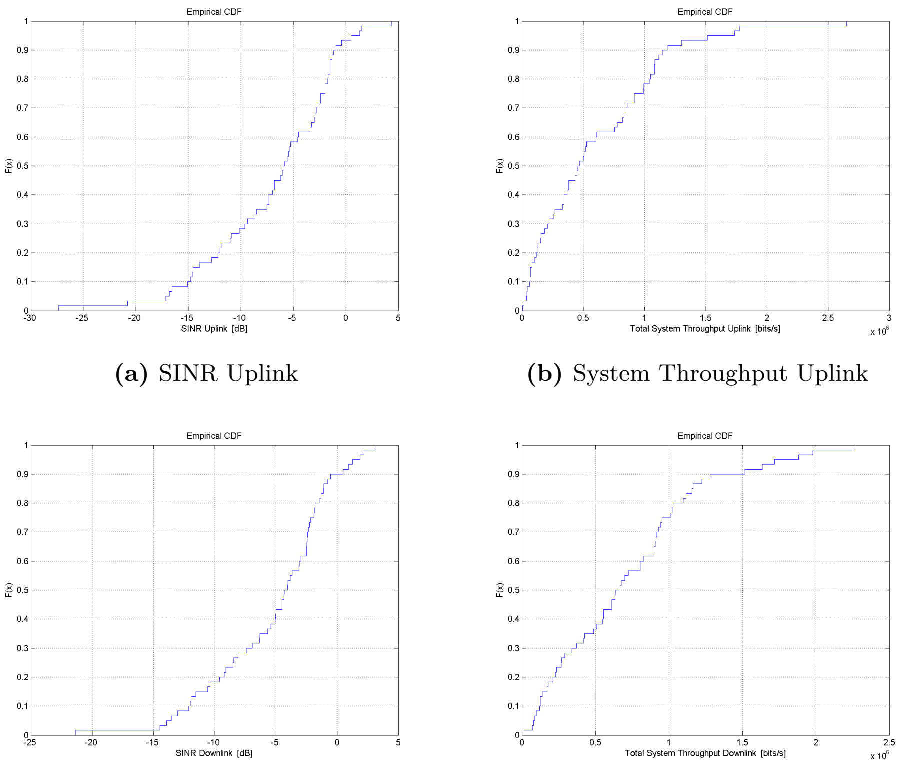
	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-29.56	2.232e+003	-23.26	9.514e+003
Maximum	-4.07	6.672e+005	-3.59	7.329e+005
Mean	-13.81	1.408e+005	-13.29	1.488e+005
Median	-13.23	9.386e+004	-13.07	9.695e+004
Standard Deviation	5.40	1.358e+005	4.74	1.475e+005

Table B.9: Statistical results for the signal to interference and noise ratio and total system throughput, links are assigned with the reversed greedy approach, power is allocated to maximize minimum user throughput, uniform user distribution

Mobile	Total User Throughput in Uplink [bits/s]	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink [bits/s]	Percentage of Total System Throughput in Downlink [%]
Mobile 1	1.207e+006	14.29	1.255e+006	14.06
Mobile 2	1.207e+006	14.29	1.260e+006	14.11
Mobile 3	1.207e+006	14.29	1.255e+006	14.06
Mobile 4	1.207e+006	14.29	1.327e+006	14.86
Mobile 5	1.207e+006	14.29	1.256e+006	14.06
Mobile 6	1.207e+006	14.29	1.321e+006	14.79
Mobile 7	1.207e+006	14.29	1.255e+006	14.06

Table B.10: Total User Throughput in Uplink And Downlink for uniform distribution, RGA link assignment and power is allocated to maximize the minimum user throughput

Maximize Total Path Gain Approach



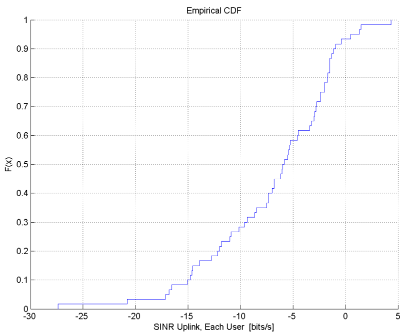
(a) SINR Uplink

(b) System Throughput Uplink

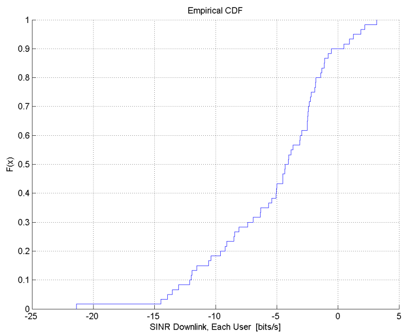
(c) SINR Downlink

(d) System Throughput Downlink

Figure B.11: The CDF plots for all users SINR and system throughput at each calculated time point over a 15 second period, users are uniformly distributed from the base station, using maximizing total gain approach



(a) SINR Each User Uplink



(b) SINR Each User Downlink

Figure B.12: The CDF plots for the SINR of all users plotted separately over a 15 second period, users are uniformly distributed from the base station, using maximizing total gain approach

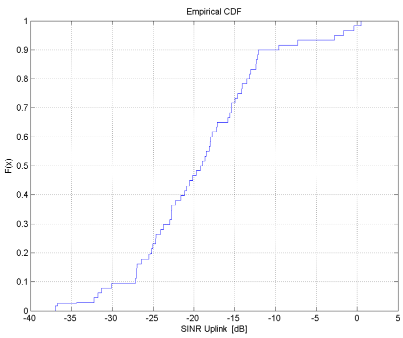
	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-27.38	3.686e+003	-21.38	1.464e+004
Maximum	4.34	2.651e+006	3.16	2.267e+006
Mean	-6.95	6.027e+005	-5.23	7.184e+005
Median	-5.89	4.627e+005	-4.19	6.525e+005
Standard Deviation	6.05	5.299e+005	4.82	5.174e+005

Table B.11: Statistical results for the signal to interference and noise ratio and total system throughput, links are assigned with the maximizing total path gain approach, power is allocated to maximize minimum user throughput, uniform user distribution

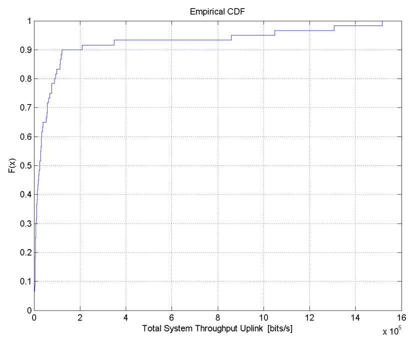
Mobile	Total User Throughput in Uplink [bits/s]	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink [bits/s]	Percentage of Total System Throughput in Downlink [%]
Mobile 1 -7	5.166e+006	14.29	6.158e+006	14.29

Table B.12: Total user throughput in uplink and downlink for uniform distribution, MTGA link assignment and power is allocated to maximize the minimum user throughput

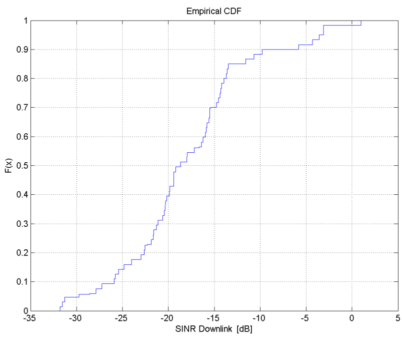
Maximize Minimum Path Gain Approach



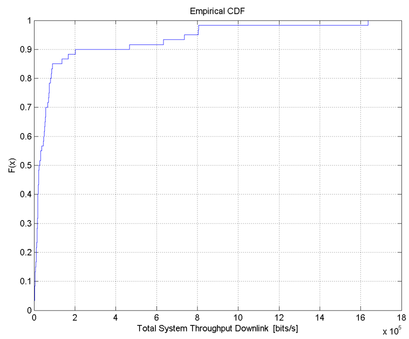
(a) SINR Uplink



(b) System Throughput Uplink

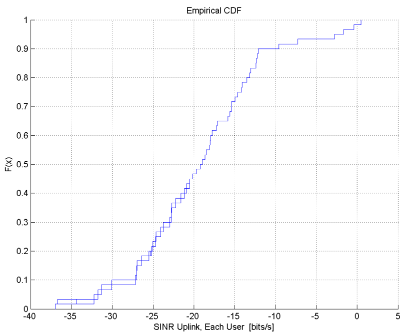


(c) SINR Downlink

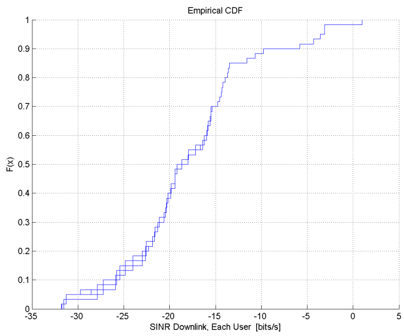


(d) System Throughput Downlink

Figure B.13: The CDF plots for all users SINR and system throughput at each calculated time point over a 15 second period, users are uniformly distributed from the base station, using maximizing minimum path gain approach



(a) SINR Each User Uplink



(b) SINR Each User Downlink

Figure B.14: The CDF plots for the SINR of all users plotted separately over a 15 second period, users are Uniformly distributed from the base station, links are assigned using maximizing minimum path gain approach

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-36.99	4.041e+002	-31.81	1.417e+003
Maximum	0.49	1.516e+006	0.96	1.637e+006
Mean	-19.18	1.182e+005	-17.85	1.204e+005
Median	-19.10	2.473e+004	-18.66	2.579e+004
Standard Deviation	7.88	2.997e+005	6.99	2.744e+005

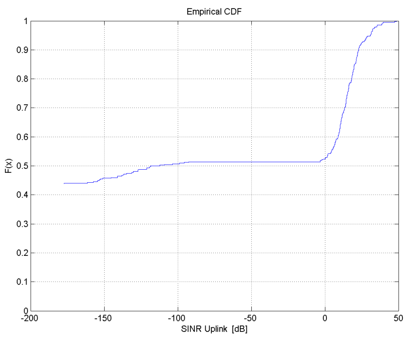
Table B.13: Statistical results for the signal to interference and noise ratio and total system throughput, links are assigned using maximizing the minimum path gain approach, power is allocated in order to maximize the minimum user throughput, users are uniformly distributed

Mobile	Total User Throughput in Uplink [bits/s]	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink [bits/s]	Percentage of Total System Throughput in Downlink [%]
Mobile 1	1.013e+006	14.28	1.030e+006	14.26
Mobile 2	1.014e+006	14.29	1.030e+006	14.26
Mobile 3	1.013e+006	14.28	1.030e+006	14.26
Mobile 4	1.013e+006	14.28	1.036e+006	14.34
Mobile 5	1.015e+006	14.31	1.039e+006	14.39
Mobile 6	1.013e+006	14.28	1.030e+006	14.26
Mobile 7	1.013e+006	14.28	1.030e+006	14.26

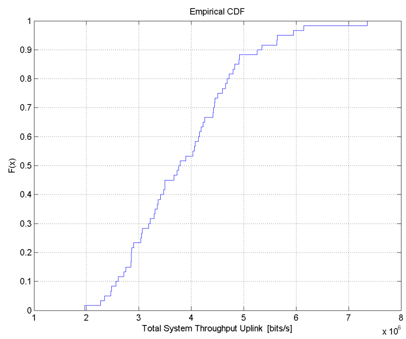
Table B.14: Total user throughput in uplink and downlink for when users are uniformly distributed, power is allocated in order to maximize the minimum user throughput, users are allocated to base stations with maximizing the minimum path gain link assignment

B.2.2 Maximizing Total Throughput

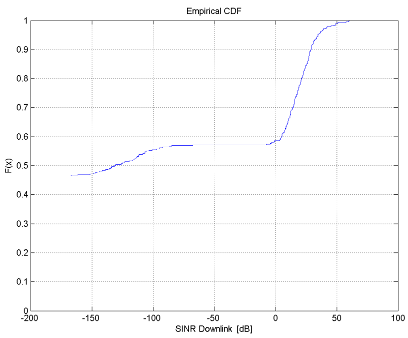
Direct Greedy Approach



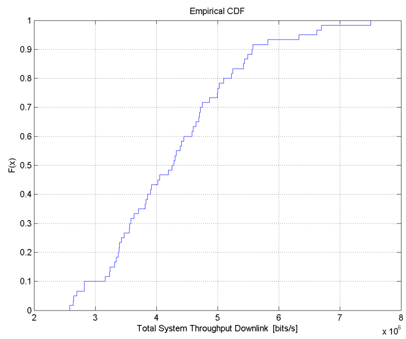
(a) SINR Uplink



(b) System Throughput Uplink

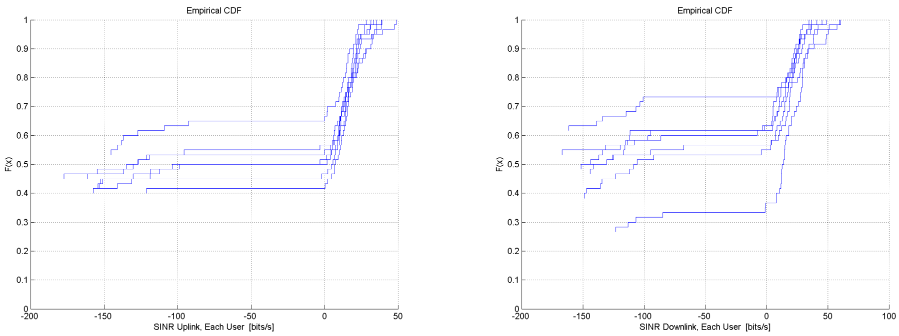


(c) SINR Downlink



(d) System Throughput Downlink

Figure B.15: The CDF Plots for all users SINR and system throughput at each calculated time point over a 15 second period, users are uniformly distributed from the base station, using direct greedy approach



(a) SINR Each User Uplink **(b)** SINR Each User Downlink

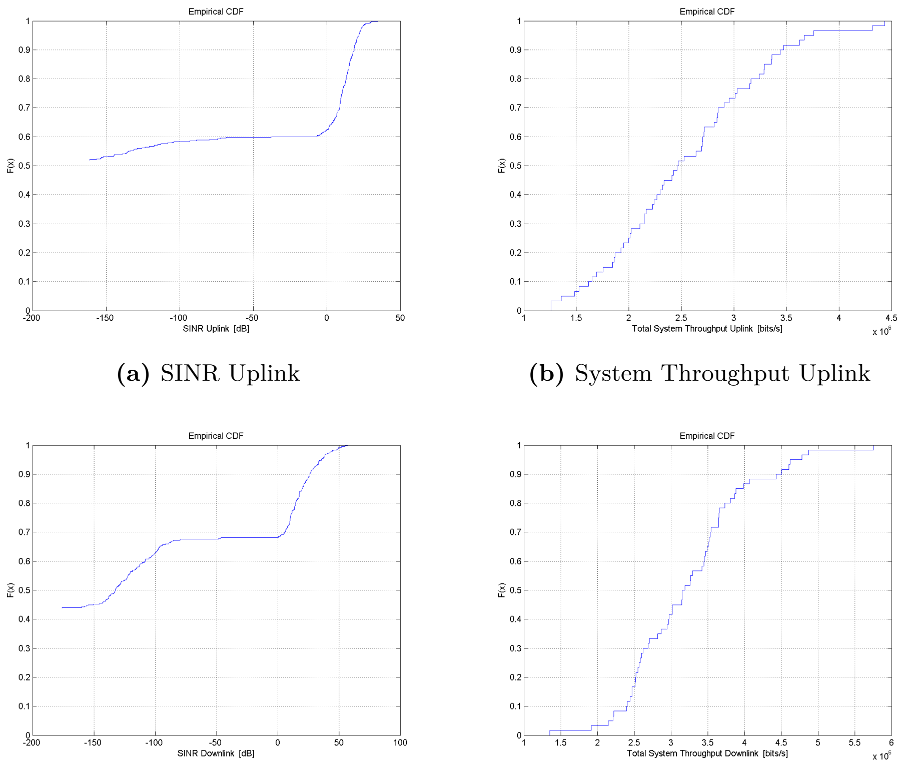
Figure B.16: The CDF Plots for the SINR of all users plotted separately over a 15 second period, users are uniformly distributed from the base station, using direct greedy approach

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-Inf	1.966e+006	-Inf	2.587e+006
Maximum	48.87	7.359e+006	60.56	7.504e+006
Mean	-Inf	3.881e+006	-Inf	4.306e+006
Median	-115.28	3.772e+006	-131.08	4.270e+006
Standard Deviation	NaN	1.081e+006	NaN	1.078e+006

Table B.15: Statistical results for the signal to interference and noise ratio and total system throughput, links are assigned with the direct greedy approach, power is allocated to maximize the total throughput , uniform user distribution

Mobile	Total User Throughput in Uplink [bits/s]	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink [bits/s]	Percentage of Total System Throughput in Downlink [%]
Mobile 1	3.070e+007	13.19	3.169e+007	12.27
Mobile 2	2.080e+007	8.93	2.421e+007	9.37
Mobile 3	3.769e+007	16.19	4.378e+007	16.95
Mobile 4	4.376e+007	18.79	6.393e+007	24.75
Mobile 5	3.442e+007	14.78	3.258e+007	12.61
Mobile 6	3.231e+007	13.88	2.799e+007	10.83
Mobile 7	3.317e+007	14.25	3.415e+007	13.22

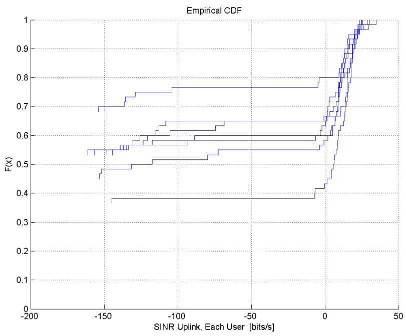
Table B.16: Total user throughput in uplink and downlink for uniform distribution, DGA link assignment and power is allocated to maximize the total throughput



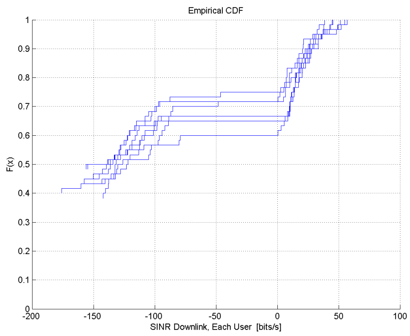
(a) SINR Uplink **(b)** System Throughput Uplink

(c) SINR Downlink **(d)** System Throughput Downlink

Figure B.17: The CDF plots for all users SINR and system throughput at each calculated time point over a 15 second period, users are uniformly distributed from the base station, using reversed greedy approach



(a) SINR Each User Uplink



(b) SINR Each User Downlink

Figure B.18: The CDF Plots for the SINR of all users plotted separately over a 15 second period, users are uniformly distributed from the base station, using reversed greedy approach

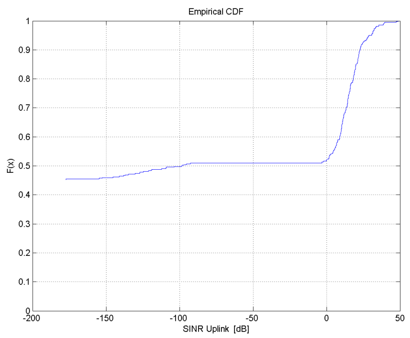
	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-Inf	1.257e+006	-Inf	1.350e+006
Maximum	34.78	4.433e+006	57.10	5.757e+006
Mean	-Inf	2.543e+006	-Inf	3.232e+006
Median	-Inf	2.464e+006	-132.51	3.171e+006
Standard Deviation	NaN	7.277e+005	NaN	8.115e+005

Table B.17: Statistical results for the signal to interference and noise ratio and total system throughput, links are assigned with the reversed greedy approach, power is allocated to maximize the total throughput, uniform user distribution

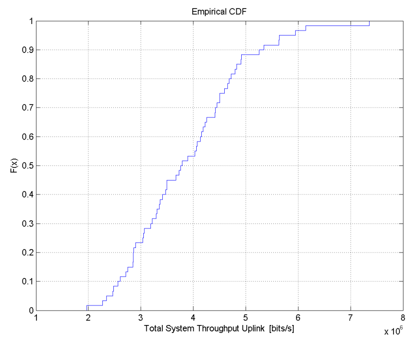
Mobile	Total User Throughput in Uplink [bits/s]	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink [bits/s]	Percentage of Total System Throughput in Downlink [%]
Mobile 1	1.285e+007	8.42	3.312e+007	17.08
Mobile 2	2.158e+007	14.14	3.072e+007	15.84
Mobile 3	1.849e+007	12.12	1.973e+007	10.17
Mobile 4	1.968e+007	12.90	2.462e+007	12.70
Mobile 5	2.649e+007	17.36	3.080e+007	15.88
Mobile 6	2.156e+007	14.13	2.595e+007	13.38
Mobile 7	3.193e+007	20.93	2.899e+007	14.95

Table B.18: Total user throughput in uplink and downlink for uniform distribution, RGA link assignment and power is allocated to maximize total throughput

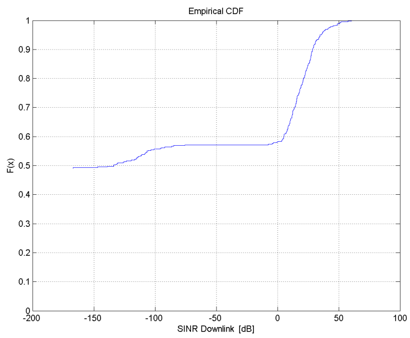
Maximize Total Path Gain Approach



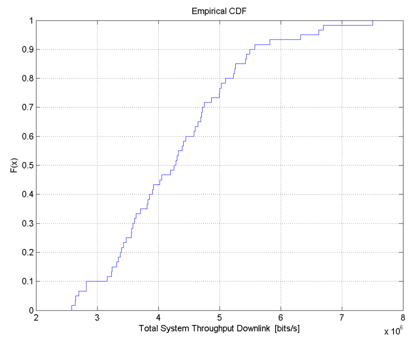
(a) SINR Uplink



(b) System Throughput Uplink

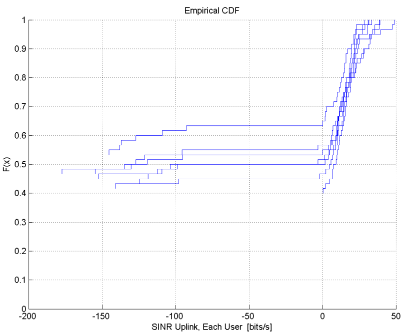


(c) SINR Downlink

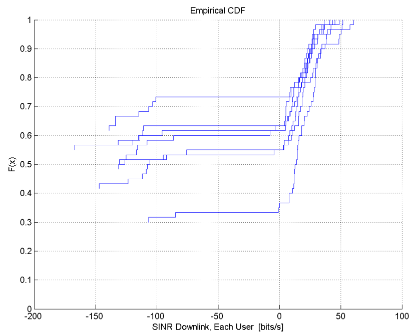


(d) System Throughput Downlink

Figure B.19: The CDF plots for all users SINR and system throughput at each calculated time point over a 15 second period, users are uniformly distributed from the base station, using maximizing total gain approach



(a) SINR Each User Uplink



(b) SINR Each User Downlink

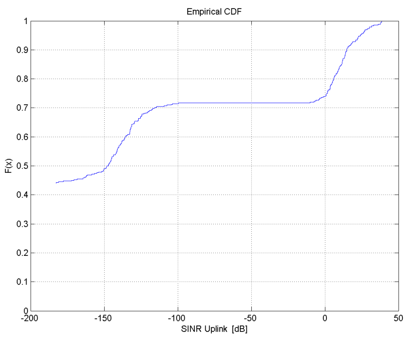
Figure B.20: The CDF plots for the SINR of all users plotted separately over a 15 second period, users are uniformly distributed from the base station, using maximizing total gain approach

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-Inf	1.966e+006	-Inf	2.587e+006
Maximum	48.87	7.359e+006	60.56	7.504e+006
Mean	-Inf	3.882e+006	-Inf	4.304e+006
Median	-98.38	3.772e+006	-133.69	4.270e+006
Standard Deviation	NaN	1.082e+006	NaN	1.069e+006

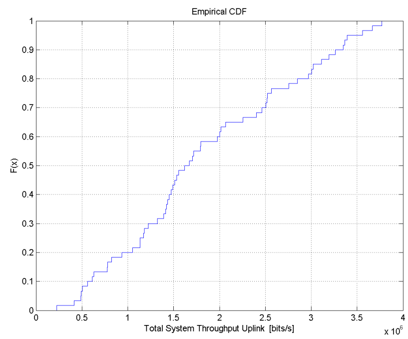
Table B.19: Some statistics for the SINR and throughput using uniform distribution, link assignment with maximizing total path gain approach, power is allocated to maximize the total system throughput

Mobile	Total User Throughput in Uplink [bits/s]	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink [bits/s]	Percentage of Total System Throughput in Downlink [%]
Mobile 1	3.070e+007	13.18	3.139e+007	12.15
Mobile 2	2.112e+007	9.07	2.489e+007	9.64
Mobile 3	3.769e+007	16.18	4.356e+007	16.87
Mobile 4	4.444e+007	19.08	6.332e+007	24.52
Mobile 5	3.442e+007	14.78	3.253e+007	12.60
Mobile 6	3.228e+007	13.86	2.799e+007	10.84
Mobile 7	3.228e+007	13.86	3.456e+007	13.38

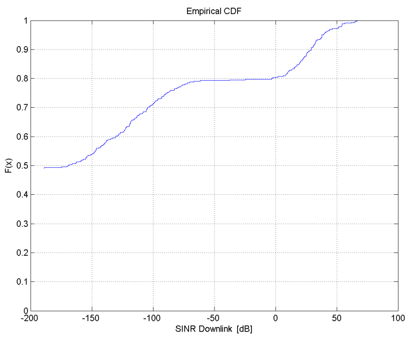
Table B.20: Total user throughput in uplink and downlink for uniform distribution, MTGA link assignment and power is allocated to maximize total system throughput



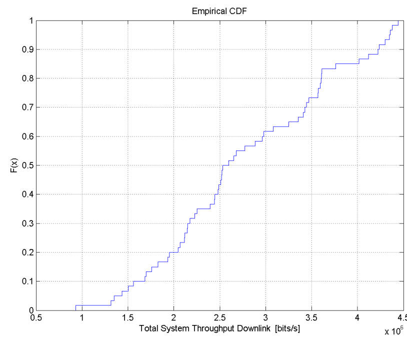
(a) SINR Uplink



(b) System Throughput Uplink

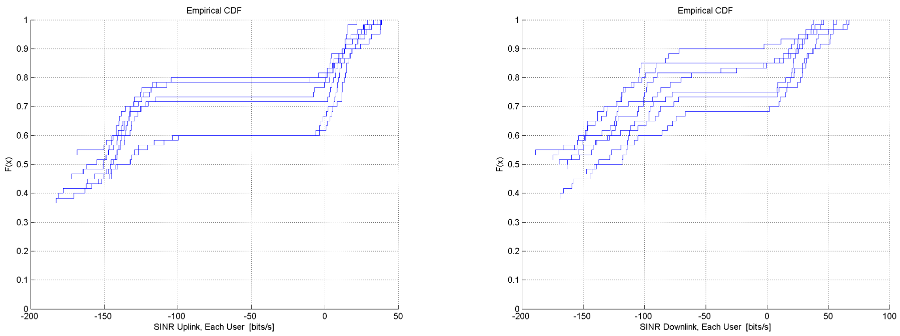


(c) SINR Downlink



(d) System Throughput Downlink

Figure B.21: The CDF plots for all users SINR and system throughput at each calculated time point over a 15 second period, users are uniformly distributed from the base station, using maximizing minimum path gain approach



(a) SINR Each User Uplink **(b)** SINR Each User Downlink

Figure B.22: The CDF plots for the SINR of all users plotted separately over a 15 second period, users are Uniformly distributed from the base station, using maximizing minimum path gain approach

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-Inf	2.262e+005	-Inf	9.325e+005
Maximum	38.85	3.768e+006	66.83	4.440e+006
Mean	-Inf	1.850e+006	-Inf	2.788e+006
Median	-147.73	1.646e+006	-169.24	2.564e+006
Standard Deviation	NaN	9.629e+005	NaN	9.230e+005

Table B.21: Statistical results for the signal to interference and noise ratio and total system throughput, links are assigned with the maximizing minimum user path gain approach, power is allocated to maximize the total throughput, uniform user distribution

Mobile	Total User Throughput in Uplink [bits/s]	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink [bits/s]	Percentage of Total System Throughput in Downlink [%]
Mobile 1	2.230e+007	20.09	1.876e+007	11.21
Mobile 2	1.270e+007	11.44	3.269e+007	19.54
Mobile 3	2.159e+007	19.45	1.457e+007	8.71
Mobile 4	1.237e+007	11.15	1.773e+007	10.60
Mobile 5	1.632e+007	14.70	3.822e+007	22.85
Mobile 6	1.325e+007	11.94	1.598e+007	9.55
Mobile 7	1.245e+007	11.22	2.933e+007	17.53

Table B.22: Total user throughput in uplink and downlink power is allocated to maximize total throughput

B.3 Statistical Tables 30 Second Run, Maximize the Minimum User Throughput

In this section more results from the 30 second run when power is allocated to maximize the minimum user throughput are presented. For all users SINR and total system throughput some statistical data are shown along with a table showing the total user throughput for all mobile users. The above results are shown for all four link assignments and all three user distributions.

B.3.1 Direct Greedy Link Assignment

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-20.26	1.892e+004	-19.12	2.459e+004
Maximum	3.74	2.453e+006	3.86	2.492e+006
Mean	-4.79	7.569e+005	-4.71	7.654e+005
Median	-4.06	6.689e+005	-4.33	6.349e+005
Standard Deviation	4.61	5.186e+005	4.38	5.525e+005

Table B.23: Statistical results for the signal to interference and noise ratio and total system throughput, power is allocated according to the maximize minimum user throughput, uniform user distribution

	SINR Uplink	Throughput Uplink	SINR Downlink	Throughput Downlink
Minimum	-14.66	6.786e+004	-10.98	1.552e+005
Maximum	7.24	3.716e+006	10.93	5.238e+006
Mean	1.16	1.912e+006	1.80	2.069e+006
Median	2.02	1.924e+006	2.48	2.057e+006
Standard Deviation	4.31	9.791e+005	4.16	1.083e+006

Table B.24: Statistical results for the signal to interference and noise ratio and total system throughput, power is allocated according to the maximize minimum user throughput, user are close to the base station

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-18.35	2.933e+004	-24.05	7.939e+003
Maximum	0.40	1.496e+006	0.02	1.405e+006
Mean	-7.90	3.742e+005	-7.26	4.262e+005
Median	-7.70	3.168e+005	-6.57	4.022e+005
Standard Deviation	3.41	2.471e+005	3.51	2.718e+005

Table B.25: Statistical results for the signal to interference and noise ratio and total system throughput, power is allocated according to the maximize minimum user throughput, user are far from the base station

Mobile	Total User Through-put in Uplink	Percentage of Total System Throughput in Uplink [%]	Total User Through-put in Downlink	Percentage of Total System Throughput in Downlink [%]
Mobile 1 - 7	1.297e+007	14.29	1.312e+007	14.29

Table B.26: Total user throughput in uplink and downlink for uniform distribution, links allocated with the direct greedy link assignment and power is allocated in order to maximize the minimum user throughput

Mobile	Total User Throughput in Uplink	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink	Percentage of Total System Throughput in Downlink [%]
Mobile 1 - 7	3.277e+007	14.29	3.547e+007	14.29

Table B.27: Total user throughput in uplink and downlink for when users are close to the base station, links are allocated with the direct greedy link assignment and power is allocated in order to maximize the minimum user throughput

Mobile	Total User Throughput in Uplink	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink	Percentage of Total System Throughput in Downlink [%]
Mobile 1 - 7	6.415e+006	14.29	7.305e+006	14.29

Table B.28: Total user throughput in uplink and downlink for when users are far from the base station, links are allocated with the direct greedy link assignment and power is allocated in order to maximize the minimum user throughput

B.3.2 Reversed Greedy Link Assignment

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-26.84	4.182e+003	-30.27	2.201e+003
Maximum	-6.44	4.131e+005	-4.20	6.512e+005
Mean	-15.58	8.470e+004	-16.40	7.826e+004
Median	-15.69	5.378e+004	-16.43	5.003e+004
Standard Deviation	4.35	8.245e+004	4.76	9.004e+004

Table B.29: Statistical results for the signal to interference and noise ratio and total system throughput, power is allocated according to the maximize minimum user throughput, uniform user distribution

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-26.98	4.046e+003	-28.31	6.808e+003
Maximum	-0.63	4.035e+005	2.94	5.837e+005
Mean	-16.37	7.113e+004	-14.73	1.094e+005
Median	-16.23	4.764e+004	-14.52	7.403e+004
Standard Deviation	4.29	6.828e+004	4.66	1.034e+005

Table B.30: Statistical results for the signal to interference and noise ratio and total system throughput, power is allocated according to the maximize minimum user throughput, user are close to the base station

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-29.12	2.472e+003	-21.98	1.276e+004
Maximum	-0.45	1.298e+006	2.11	1.950e+006
Mean	-9.98	2.910e+005	-9.16	3.299e+005
Median	-9.59	2.106e+005	-9.01	2.388e+005
Standard Deviation	5.03	2.490e+005	4.33	3.184e+005

Table B.31: Statistical results for the signal to interference and noise ratio and total system throughput, power is allocated according to the maximize minimum user throughput, user are far from the base station

Mobile	Total User Through-put in Uplink	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink	Percentage of Total System Throughput in Downlink [%]
Mobile 1	1.452e+006	14.29	1.275e+006	13.57
Mobile 2	1.452e+006	14.29	1.388e+006	14.78
Mobile 3	1.452e+006	14.29	1.279e+006	13.62
Mobile 4	1.452e+006	14.29	1.354e+006	14.41
Mobile 5	1.452e+006	14.29	1.326e+006	14.12
Mobile 6	1.452e+006	14.29	1.494e+006	15.91
Mobile 7	1.452e+006	14.29	1.275e+006	13.57

Table B.32: Total user throughput in uplink and downlink for uniform distribution, links allocated with the reversed greedy link assignment and power is allocated in order to maximize the minimum user throughput

Mobile	Total User Throughput in Uplink	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink	Percentage of Total System Throughput in Downlink [%]
Mobile 1	1.194e+006	13.99	1.711e+006	13.03
Mobile 2	1.373e+006	16.08	2.216e+006	16.87
Mobile 3	1.194e+006	13.99	1.778e+006	13.54
Mobile 4	1.194e+006	13.99	1.714e+006	13.05
Mobile 5	1.194e+006	13.99	1.711e+006	13.03
Mobile 6	1.194e+006	13.99	2.280e+006	17.36
Mobile 7	1.194e+006	13.99	1.724e+006	13.12

Table B.33: Total user throughput in uplink and downlink for when users are close to the base station, links are allocated with the reversed greedy link assignment and power is allocated in order to maximize the minimum user throughput

Mobile	Total User Throughput in Uplink	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink	Percentage of Total System Throughput in Downlink [%]
Mobile 1	4.989e+006	14.29	5.654e+006	14.28
Mobile 2	4.989e+006	14.29	5.654e+006	14.28
Mobile 3	4.989e+006	14.29	5.666e+006	14.31
Mobile 4	4.989e+006	14.29	5.654e+006	14.28
Mobile 5	4.989e+006	14.29	5.654e+006	14.28
Mobile 6	4.989e+006	14.29	5.654e+006	14.28
Mobile 7	4.989e+006	14.29	5.654e+006	14.28

Table B.34: Total user throughput in uplink and downlink for when users are far from the base station, links are allocated with the reversed greedy link assignment and power is allocated in order to maximize the minimum user throughput

B.3.3 Maximize Total Path Gain Link Assignment

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-20.26	1.892e+004	-13.23	9.380e+004
Maximum	3.74	2.453e+006	3.86	2.492e+006
Mean	-4.24	8.224e+005	-3.59	8.895e+005
Median	-3.75	7.100e+005	-3.17	7.939e+005
Standard Deviation	4.53	5.217e+005	3.82	5.715e+005

Table B.35: Statistical results for the signal to interference and noise ratio and total system throughput, power is allocated in order to maximize the minimum user throughput, uniform user distribution

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-14.66	6.786e+004	-10.98	1.552e+005
Maximum	7.24	3.716e+006	10.93	5.238e+006
Mean	1.16	1.912e+006	1.80	2.069e+006
Median	2.02	1.924e+006	2.48	2.057e+006
Standard Deviation	4.31	9.791e+005	4.16	1.083e+006

Table B.36: Statistical results for the signal to interference and noise ratio and total system throughput, power is allocated in order to maximize the minimum user throughput, users are close to the base station

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-17.34	3.693e+004	-18.63	2.747e+004
Maximum	2.41	2.037e+006	1.50	1.780e+006
Mean	-6.20	5.435e+005	-5.78	5.732e+005
Median	-6.01	4.518e+005	-5.61	4.906e+005
Standard Deviation	3.72	3.841e+005	3.46	3.697e+005

Table B.37: Statistical results for the signal to interference and noise ratio and total system throughput, power is allocated in order to maximize the minimum user throughput, users are far from the base station

Mobile	Total User Throughput in Uplink [bits/s]	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink [bits/s]	Percentage of Total System Throughput in Downlink [%]
Mobiles 1 - 7	1.410e+007	14.29	1.525e+007	14.29

Table B.38: Total user throughput in uplink and downlink for uniform distribution, links allocated with the maximum total path gain link assignment and power is allocated in order to maximize the minimum user throughput

Mobile	Total User Throughput in Uplink [bits/s]	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink [bits/s]	Percentage of Total System Throughput in Downlink [%]
Mobile 1 - 7	3.277e+007	14.29	3.547e+007	14.29

Table B.39: Total user throughput in uplink and downlink for when users are close to the base station, power is allocated in order to maximize the minimum user throughput, users are close to the base station

Mobile	Total User Throughput in Uplink [bits/s]	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink [bits/s]	Percentage of Total System Throughput in Downlink [%]
Mobile 1 - 7	9.318e+006	14.29	9.827e+006	14.29

Table B.40: Total user throughput in uplink and downlink for when users are far from the base station, power is allocated in order to maximize the minimum user throughput, users are allocated to base stations with maximizing total path gain link assignment

B.3.4 Maximize the Minimum Path Gain

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-41.40	1.462e+002	-38.88	4.518e+002
Maximum	-14.09	7.731e+004	-6.43	7.895e+004
Mean	-24.67	1.060e+004	-24.02	1.219e+004
Median	-24.42	7.225e+003	-23.64	9.173e+003
Standard Deviation	4.38	1.050e+004	4.17	1.140e+004

Table B.41: Statistical results for the signal to interference and noise ratio and total system throughput, power is allocated in order to maximize the minimum user throughput, users are uniformly distributed

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-51.02	4.594e+001	-53.94	2.515e+001
Maximum	-2.10	1.605e+005	-6.54	5.925e+004
Mean	-34.69	9.565e+003	-35.00	5.187e+003
Median	-35.93	2.270e+003	-36.37	1.645e+003
Standard Deviation	7.69	2.385e+004	7.53	9.446e+003

Table B.42: Statistical results for the signal to interference and noise ratio and total system throughput, power is allocated in order to maximize the minimum user throughput, users are close to the base station

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-34.78	6.711e+002	-32.16	1.229e+003
Maximum	1.19	1.696e+006	1.50	1.780e+006
Mean	-22.65	3.907e+004	-21.90	3.847e+004
Median	-22.39	1.154e+004	-21.96	1.285e+004
Standard Deviation	5.41	1.736e+005	4.64	1.704e+005

Table B.43: Statistical results for the signal to interference and noise ratio and total system throughput, power is allocated in order to maximize the minimum user throughput, users are far from the base station

Mobile	Total User Throughput in Uplink	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink	Percentage of Total System Throughput in Downlink [%]
Mobile 1	1.813e+005	14.25	1.900e+005	12.99
Mobile 2	1.813e+005	14.25	2.174e+005	14.86
Mobile 3	1.837e+005	14.44	2.508e+005	17.14
Mobile 4	1.813e+005	14.25	1.900e+005	12.99
Mobile 5	1.824e+005	14.33	2.347e+005	16.04
Mobile 6	1.813e+005	14.25	1.900e+005	12.99
Mobile 7	1.813e+005	14.25	1.903e+005	13.01

Table B.44: Total user throughput in uplink and downlink for when users are uniformly distributed, power is allocated in order to maximize the minimum user throughput, users are allocated to base stations with maximizing the minimum path gain link assignment

Mobile	Total User Throughput in Uplink	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink	Percentage of Total System Throughput in Downlink [%]
Mobile 1	7.072e+005	61.62	3.311e+005	53.19
Mobile 2	1.263e+004	1.10	1.687e+004	2.71
Mobile 3	2.610e+004	2.27	3.950e+004	6.34
Mobile 4	8.721e+003	0.76	1.355e+004	2.18
Mobile 5	4.165e+004	3.63	2.854e+004	4.59
Mobile 6	5.902e+004	5.14	1.482e+004	2.38
Mobile 7	2.924e+005	25.48	1.781e+005	28.61

Table B.45: Total user throughput in uplink and downlink for when users are close to the base station, power is allocated in order to maximize the minimum user throughput, users are allocated to base stations with maximizing the minimum path gain link assignment

Mobile	Total User Throughput in Uplink	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink	Percentage of Total System Throughput in Downlink [%]
Mobile 1	6.699e+005	14.29	6.727e+005	14.57
Mobile 2	6.690e+005	14.27	6.563e+005	14.22
Mobile 3	6.735e+005	14.37	6.563e+005	14.22
Mobile 4	6.689e+005	14.27	6.563e+005	14.22
Mobile 5	6.689e+005	14.27	6.565e+005	14.22
Mobile 6	6.689e+005	14.27	6.612e+005	14.32
Mobile 7	6.689e+005	14.27	6.569e+005	14.23

Table B.46: Total user throughput in uplink and downlink for when users are far from the base station, power is allocated in order to maximize the minimum user throughput, users are allocated to base stations with maximizing the minimum path gain link assignment

B.3.5 Ratio tables

Statistical table for the α ratio cdf plots

	$\alpha = 0/\alpha = 1$ ratio, uniform	$\alpha = 0/\alpha = 1$ ratio, close	$\alpha = 0/\alpha = 1$ ratio, far
Minimum	0.00	0.01	0.01
Maximum	0.56	0.48	0.66
Mean	0.19	0.23	0.19
Median	0.17	0.23	0.16
Standard Deviation	0.12	0.12	0.13

Table B.47: Statistical results for the ratio of the total system throughput for $\alpha = 0$ and $\alpha = 1$ all user distributions in uplink, 30 s run

	$\alpha = 0/\alpha = 1$ ratio, uniform	$\alpha = 0/\alpha = 1$ ratio, close	$\alpha = 0/\alpha = 1$ ratio, far
Minimum	0.02	0.02	0.01
Maximum	0.59	0.58	0.47
Mean	0.18	0.24	0.17
Median	0.15	0.24	0.15
Standard Deviation	0.12	0.12	0.10

Table B.48: Statistical results for the ratio of the total system throughput for $\alpha = 0$ and $\alpha = 1$ all user distributions in downlink, 30 s run

B.4 Statistical Tables 30 Second Run, Maximize Total Throughput

In this section more results from the 30 second run when power is allocated to maximize total throughput are presented. For all users SINR and total system throughput some statistical data are shown along with a table showing the total user throughput for all mobile users. The above results are shown for the direct greedy and maximize total path gain link assignments and all three user distributions.

B.4.1 Users Uniformly Distributed

Direct Greedy Link Assignment

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-Inf	3.256e+006	-Inf	2.943e+006
Maximum	46.21	6.236e+006	60.42	6.944e+006
Mean	-Inf	4.523e+006	-Inf	5.033e+006
Median	3.03	4.447e+006	-Inf	4.994e+006
Standard Deviation	NaN	6.688e+005	NaN	7.134e+005

Table B.49: Statistical results for the signal to interference and noise ratio and total system throughput, power is allocated according to the maximize the total throughput, uniform user distribution and direct greedy link assignment

Mobile	Total User Through-put in Uplink	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink	Percentage of Total System Throughput in Downlink [%]
Mobile 1	1.898e-006	0.00	4.917e-001	0.00
Mobile 2	8.964e+007	16.51	9.999e+007	16.55
Mobile 3	3.223e+007	5.94	1.583e+007	2.62
Mobile 4	4.186e+007	7.71	4.278e+007	7.08
Mobile 5	2.460e+006	0.45	1.175e+006	0.19
Mobile 6	1.748e+008	32.21	2.156e+008	35.69
Mobile 7	2.018e+008	37.17	2.287e+008	37.86

Table B.50: Total user throughput in uplink and downlink for uniform user distribution, links are allocated with the direct greedy link assignment and power is allocated in order to maximize total throughput

Maximize Total Path Gain Link Assignment

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-Inf	3.256e+006	-Inf	2.943e+006
Maximum	46.21	6.236e+006	60.42	6.944e+006
Mean	-Inf	4.522e+006	-Inf	5.027e+006
Median	3.31	4.456e+006	-130.71	4.994e+006
Standard Deviation	NaN	6.706e+005	NaN	7.210e+005

Table B.51: Statistical results for the signal to interference and noise ratio and total system throughput, power is allocated according to the maximize the total throughput, uniform user distribution and maximize total path gain link assignment

Mobile	Total User Through-put in Uplink	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink	Percentage of Total System Throughput in Downlink [%]
Mobile 1	2.006e-006	0.00	4.238e-006	0.00
Mobile 2	8.995e+007	16.58	9.944e+007	16.49
Mobile 3	3.188e+007	5.87	1.566e+007	2.60
Mobile 4	4.243e+007	7.82	4.125e+007	6.84
Mobile 5	2.412e+006	0.44	1.175e+006	0.19
Mobile 6	1.756e+008	32.36	2.180e+008	36.15
Mobile 7	2.004e+008	36.93	2.276e+008	37.74

Table B.52: Total user throughput in uplink and downlink for uniform user distribution, links are allocated with the maximize total path gain link assignment and power is allocated in order to maximize total throughput

B.4.2 Users Close to the Base Station

Direct Greedy Link Assignment

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-Inf	5.985e+006	-Inf	5.295e+006
Maximum	43.65	1.045e+007	49.88	1.110e+007
Mean	-Inf	8.256e+006	-Inf	8.511e+006
Median	20.81	8.262e+006	21.16	8.552e+006
Standard Deviation	NaN	9.150e+005	NaN	9.338e+005

Table B.53: Statistical results for the signal to interference and noise ratio and total system throughput, power is allocated to the maximize the total throughput, users are close to the base station and are allocated to base station using direct greedy link assignment

Mobile	Total User Throughput in Uplink	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink	Percentage of Total System Throughput in Downlink [%]
Mobile 1	7.292e+005	0.07	1.689e+005	0.02
Mobile 2	1.726e+008	17.42	2.178e+008	21.33
Mobile 3	1.315e+008	13.27	1.115e+008	10.92
Mobile 4	2.363e+008	23.85	2.450e+008	23.99
Mobile 5	2.259e+008	22.80	2.095e+008	20.52
Mobile 6	2.231e+008	22.51	2.373e+008	23.23
Mobile 7	6.986e+005	0.07	1.532e-001	0.00

Table B.54: Total user throughput in uplink and downlink for when users are close to the base station, links are allocated with the direct greedy link assignment and power is allocated in order to maximize total throughput

Maximize Total Path Gain Link Assignment

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-Inf	5.985e+006	-Inf	5.295e+006
Maximum	43.65	1.045e+007	49.89	1.110e+007
Mean	-Inf	8.259e+006	-Inf	8.513e+006
Median	20.85	8.268e+006	21.16	8.552e+006
Standard Deviation	NaN	9.149e+005	NaN	9.325e+005

Table B.55: Statistical results for the signal to interference and noise ratio and total system throughput, power is allocated to the maximize the total throughput, users are close to the base station and are allocated to base station using maximizing total path gain link assignment

	Total User Throughput in Uplink [bits/s]	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink [bits/s]	Percentage of Total System Throughput in Downlink [%]
Mobile 1	7.291e+005	0.07	1.688e+005	0.02
Mobile 2	1.726e+008	17.42	2.181e+008	21.35
Mobile 3	1.317e+008	13.29	1.111e+008	10.87
Mobile 4	2.372e+008	23.94	2.451e+008	23.99
Mobile 5	2.258e+008	22.78	2.096e+008	20.52
Mobile 6	2.230e+008	22.50	2.375e+008	23.25
Mobile 7	7.104e-006	0.00	0.000e+000	0.00

Table B.56: Total user throughput in uplink and downlink for when users are close to the base station, links are allocated with the maximizing total path gain link assignment and power is allocated in order to maximize total throughput

B.4.3 Users Far Away from the Base Stations

Direct Greedy Link Assignment

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-Inf	1.398e+006	-Inf	2.200e+006
Maximum	32.94	4.175e+006	49.58	4.804e+006
Mean	-Inf	2.934e+006	-Inf	3.422e+006
Median	-118.12	2.906e+006	-Inf	3.399e+006
Standard Deviation	NaN	5.941e+005	NaN	5.185e+005

Table B.57: Statistical results for the signal to interference and noise ratio and total system throughput, power is allocated to the maximize the total throughput, users are far from the base station and are allocated to base station using direct greedy link assignment

Mobile	Total User Through-put in Uplink	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink	Percentage of Total System Throughput in Downlink [%]
Mobile 1	8.054e+005	0.23	1.127e+006	0.27
Mobile 2	8.662e+007	24.60	9.570e+007	23.31
Mobile 3	7.450e+007	21.16	1.163e+008	28.32
Mobile 4	4.823e+007	13.70	6.589e+007	16.05
Mobile 5	6.056e+007	17.20	4.307e+007	10.49
Mobile 6	5.223e+007	14.83	6.459e+007	15.73
Mobile 7	2.913e+007	8.27	2.398e+007	5.84

Table B.58: Total user throughput in uplink and downlink for when users are far from the base station, links are allocated with the direct greedy link assignment and power is allocated in order to maximize total throughput

Maximize Total Path Gain Link Assignment

	User SINR Uplink [dB]	System Throughput Uplink [bits/s]	User SINR Downlink [dB]	System Throughput Downlink [bits/s]
Minimum	-Inf	1.608e+006	-Inf	2.103e+006
Maximum	32.94	4.201e+006	47.93	4.833e+006
Mean	-Inf	2.960e+006	-Inf	3.385e+006
Median	-6.71	2.962e+006	-Inf	3.365e+006
Standard Deviation	NaN	6.055e+005	NaN	5.470e+005

Table B.59: Statistical results for the signal to interference and noise ratio and total system throughput, power is allocated to the maximize the total throughput, users are far from the base station and are allocated to base station using maximizing total path gain link assignment

	Total User Throughput in Uplink [bits/s]	Percentage of Total System Throughput in Uplink [%]	Total User Throughput in Downlink [bits/s]	Percentage of Total System Throughput in Downlink [%]
Mobile 1	6.515e+006	1.83	7.054e+006	1.74
Mobile 2	7.718e+007	21.73	8.970e+007	22.08
Mobile 3	7.284e+007	20.50	1.123e+008	27.65
Mobile 4	3.585e+007	10.09	5.344e+007	13.15
Mobile 5	6.370e+007	17.93	3.498e+007	8.61
Mobile 6	5.385e+007	15.16	5.252e+007	12.93
Mobile 7	4.531e+007	12.75	5.622e+007	13.84

Table B.60: Total user throughput in uplink and downlink for when users are far from the base station, links are allocated with the maximizing total path gain link assignment and power is allocated in order to maximize total throughput