Problem: Write an equivalent LP problem for:

$$(P): \begin{cases} \text{minimize} & |x| + |y-2| \\ \text{s.t.} & x - y \leq 1 \end{cases}$$

One possible solution:

$$(P'): \begin{cases} \text{minimize} & \xi + \eta \\ \text{s.t.} & \xi \geq x \\ & \xi \geq -x \\ & \eta \geq y-2 \\ & \eta \geq -(y-2) \\ & x - y \leq 1 \end{cases}$$

Justification of the equivalence of (P) and (P'):

Claim 1: If $(\hat{\xi}, \hat{\eta}, \hat{x}, \hat{y})$ is an optimal solution to (P') then $(\hat{x}, \hat{y})$ is an optimal solution to (P).

Proof: $(\hat{x}, \hat{y})$ is feasible for (P).

$$\left.\begin{array}{l} \left.\begin{array}{l} \hat{\xi} \geq \hat{x} \\ \hat{\xi} \geq -\hat{x} \end{array}\right\} \Rightarrow \hat{\xi} \geq |\hat{x}| \\ \left.\begin{array}{l} \hat{\eta} \geq \hat{y} - 2 \\ \hat{\eta} \geq -(\hat{y} - 2) \end{array}\right\} \Rightarrow \hat{\eta} \geq |\hat{y} - 2| \end{array}\right\} \hat{\xi} + \hat{\eta} \geq |\hat{x}| + |\hat{y} - 2| \quad - ①$$

$$\left.\begin{array}{l} \xi := |\hat{x}| \\ \eta := |\hat{y} - 2| \end{array}\right\} \Rightarrow \left.\begin{array}{l} \xi \geq \hat{x} \\ \xi \geq -\hat{x} \\ \eta \geq \hat{y} - 2 \\ \eta \geq -(\hat{y} - 2) \end{array}\right\} \Rightarrow (\xi, \eta, \hat{x}, \hat{y}) \text{ is feasible for } (P')$$

So $\xi + \eta \geq \hat{\xi} + \hat{\eta}$

i.e., $|\hat{x}| + |\hat{y} - 2| \geq \hat{\xi} + \hat{\eta} \quad - ②$

$$① \ \& \ ② \Rightarrow \hat{\xi} + \hat{\eta} = |\hat{x}| + |\hat{y} - 2|$$

Let (x, y) be feasible for (P).

Define $\xi = |x|$, $\eta = |y-2|$

Then (ξ, η, x, y) is feasible for (P')

$|x| + |y-2| = \xi + \eta \geq \hat{\xi} + \hat{\eta} = |\hat{x}| + |\hat{y} - 2|$.

So $(\hat{x}, \hat{y})$ is optimal for (P). $\square$

Claim 2: If $(\hat{x}, \hat{y})$ is optimal for (P)

then $(\hat{\xi}, \hat{\eta}, \hat{x}, \hat{y})$ is optimal for (P'), where $\hat{\xi} := |\hat{x}|$,

$\hat{\eta} := |\hat{y} - 2|$.

Proof First of all $(\hat{\xi}, \hat{\eta}, \hat{x}, \hat{y})$ is feasible for (P').

Let (ξ, η, x, y) be feasible for (P')

Then
$$\left.\begin{array}{l} \xi \geq x \\ \xi \geq -x \\ \eta \geq y-2 \\ \eta \geq -(y-2) \end{array}\right\} \Rightarrow \xi + \eta \geq |x| + |y-2|,$$

So
$$\xi + \eta \geq |x| + |y-2| \geq |\hat{x}| + |\hat{y} - 2| = \hat{\xi} + \hat{\eta}.$$

since (x, y) is feasible for (P). □