A Note on the Asymptotic Expansion for a Regularization Approach of Higher Index Differential-Algebraic Equations*

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September 18, 1992

Abstract

In an earlier paper [12] we were able to show the existence of asymptotic expansions for the solutions of some regularization procedures fo r higher index differential-algebraic equations in integer powers of the regularization parameter. Recent numerical experiments have led to the conjecture that the asymptotic expansion does not contain the first order term for autonomous systems. In the present note it is proved that this assertion is true for index 2 Hessenberg systems, i.e. convergence of second order with respect to the regularization parameter occurs.

1 Introduction

In the present paper we consider boundary value problems for a special class of differential-algebraic equations, namely semiexplicit systems in Hessenberg form of index 2

*This paper has been written during the stay of the second author at the Humboldt-Universität zu Berlin supported by Caja de Ahorros de la Inmaculada, Ref. CB 8/91.

[4],

$$
\begin{aligned}
u' &= f(u,v,t) &\quad (1.1a)\\
0 &= g(u,t) &\quad (1.1b)
\end{aligned}
$$

for $t \in [0,1]$ subject to appropriate boundary conditions. If, in the domain of definition, $g_u(u,t)f_v(u,v,t)$ remains nonsingular, (1.1) has index 2. Equations of this type arise naturally in many applications, e. g. electrical networks, constraint mechanical systems of rigid bodies, chemical reactions kinetics, and control theory. The main point in the numerical approximation of such a system is the fact that it represents an ill-posed problem in naturally choosen topologies. Assume for a moment that (1.1) has a solution (u_0, v_0) and let (1.1b) be perturbed by a (smooth) function δ,

$$0 = g(u,t) + \delta(t). \tag{1.2}$$

Consider the system (1.1a), (1.2). Differentiation of (1.2) with respect to t gives

$$0 = g_u(u,t)f(u,v,t) + \delta'(t). \tag{1.3}$$

using now the nonsingularity of $g_u(u_0(t),t)f_v(u_0(t),v_0(t),t)$ for $t \in [0,1]$, (1.3) can be solved for v locally around (u_0, v_0), i. e.

$$v = \Phi(u,t,\delta') \tag{1.4}$$

Inserting (1.4) into (1.1a) we see that the solution (u_δ, v_δ) depends on δ', i. e. the derivative of the perturbation thus yielding a discontinuous dependence of the solution on the perturbation if the latter is measured in the maximum norm, as it is usual in the context of ordinary differential equations. Having accepted this point of view it is not surprising that usual ode methods fail (more precisely, they become unstable) when they are applied to so-called higher index equation ((1.1) being one instance of them): Integration methods are applied to differentiation problems.

The system consisting of (1.1a), (1.3) is a special instance of a so-called transferable semiexplicit differential-algebraic equation, namely,

$$
\begin{aligned}
u' &= f(u,v,t) &\quad (1.5a)\\
0 &= \bar{g}(u,v,t) &\quad (1.5b)
\end{aligned}
$$

with nonsingular Jacobian $\bar{g}_v$. The implicit function theorem shows that (1.5b) is locally solvable for v such that (1.5) is nothing else than an ordinary differential equation for u and a continuous assignment for v yielding continuous dependence of the solution on perturbation of the right-hand side. Now it is not surprising that appropriately modified ode methods applied to (1.5) work well. Such equations are called *transferable* [9] or being uniformly index 1 [8].

In a recent paper [12] we proposed a regularization procedure to overcome the numerical difficulties associated with higher index problems. We introduced a perturbation of (1.1) in such a way that the index was reduced by one thus leading to *transferable* semiexplicit systems [9] which are well-posed and, more important, which can be solved by appropriately modified numerical methods (see, e. g., [9, 8, 2, 10, 15, 5]). The idea is to replace (1.1b) by

$$0 = g(u + \varepsilon u',\, t) \tag{1.6}$$

with $\varepsilon > 0$. This special parametrization was first proposed explicitely by Knorrenschild [14], but similar approaches are known for a longer time motivated from different areas of applications (in electrical network theory cf. Chua et al. [3, **?**], in constraint mechanical systems cf. Baumgarte [1] and the overview by Eich/Hanke [6], in the theory of differential-algebraic equations cf. März [16]).

In [12] we were able to give conditions which assure the solvability of (1.1a), (1.6) for sufficiently small $\varepsilon > 0$ and the existence of asymptotic expansions in powers of ε for the perturbed solutions showing convergence of order ε of the solutions of (1.1a), (1.6) toward the solution of (1.1). Meanwhile, results of numerical experiments have led to the conjecture that the order of convergence is even ε^2. The aim of this paper is to show this claim is true provided that g of (1.1b) does not deped on t.

2 Basic Results. Example

We consider the semiexplicit index 2 system

$$u' = f(u, v, t) \tag{2.1a}$$
$$0 = g(u, t) \tag{2.1b}$$

$t \epsilon [0, 1]$ subject to the boundary condition

$$G(u(0),\, u(1)) = 0. \tag{2.2}$$

There we have taken into account that only boundary conditions for the differential components u are allowed. Writing down (2.1b) for $t = 0$ or $t = 1$ we obtain additional conditions. This fact shows that the number of boundary conditions in (2.2) must be less than the dimension of $u(t)$. The precise assumption guaranteeing the isolatedness of solutions of (2.1, 2.2) are stated below.

Assumption (NP2):

(0) [Smoothness assumptions]
$f : \mathbb{R}^n \times \mathbb{R}^k \times (0, 1) \rightarrow \mathbb{R}^n, g : \mathbb{R}^n \times [0, 1] \rightarrow \mathbb{R}^k, G : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^r$
are sufficiently smooth functions of their arguments.

(i) [Solvability assumption]
(2.1), (2.2) has a solution $(u_0,\, v_0)$.

(ii) [Index 2 assumption]
For all $t \epsilon [0, 1]$, $g_u(u_0(t), t)\, f_v(u_0(t), v_0(t), t)$ is nonsingular.

(iii) [Isolatedness assumption] $r = n - k$, the matrix

$$\begin{pmatrix} G_{u(0)}\,(u_0(0), u_0(1)) & G_{u(1)}\,(u_0(0), u_0(1)) \\ g_u(u_0(0), 0) & 0 \end{pmatrix}$$

has full rank n, and the homogeneous linearized boundary value problem

$$\begin{aligned} u' &= f_u(u_0, v_0, \cdot)\, u + f_v(u_0, v_0, \cdot)\, v, \\ 0 &= g_u(u_0, \cdot)\, u, \\ 0 &= G_{u(0)}\,(u_0(0), u_0(1))\, u(0) + G_{u(1)}\,(u_0(0), u_0(1))\, u(1) \end{aligned}$$

has the trivial solution only.

(iv) The nonlinear equation

$$g_u(u_0(0), 0)\, f(u_0(0), \beta, 0) = 0 \tag{2.3}$$

has a solution $\beta_0 \epsilon I\!R^k$ sufficiently close to $v_0(0)$.

An extensive study of this assumptions can be found in [12, 13]. At this point note that, for autonomous g, (NP2, iv) is fulfilled with $\beta_0 = v_0(0)$.

(2.1) is now parameterized:

$$u' = f(u, v, t) \tag{2.4a}$$

$$0 = g(u + \varepsilon u', t) \tag{2.4b}$$

for $t\epsilon[0, 1]$.

The boundary conditions are given by (2.2) and, additionally,

$$g(u(0), 0) = 0 \tag{2.5}$$

in order to ensure unique solvability of the parameterized boundary value problem (cf. [12]). A closer analysis shows that (2.4) has a singular perturbation behaviour (cf. [18, 20]) where a boundary layer appears only near the left-hand boundary $t = 0$. In [12] we proved the following result.

Theorem 2.1 *Let (NP2) hold. Then, for sufficiently small $\varepsilon > 0$, (2.4) is transferable, and the boundary value problem (2.4), (2.2), (2.5) has exactly one solution $(u_\varepsilon, v_\varepsilon)$ in a neighbourhood of (u_0, v_0). These solutions have asymptotic expansions*[1]

$$u_\varepsilon(t) \simeq u_0(t) + \sum_{j=1}^{N} (u_j(t) + \bar{u}_j(\tau))\varepsilon^j, \tag{2.6a}$$

$$v_\varepsilon(t) \simeq \sum_{j=0}^{N} (v_j(t) + \bar{v}_j(\tau))\varepsilon_j \tag{2.6b}$$

where $\tau = \frac{t}{\varepsilon}$, $\mid \bar{u}_j(\tau) \mid, \mid \bar{v}_j(\tau) \mid \leq C\, e^{-\alpha\tau}$, $\alpha > 0$.

[1]The sign "$\simeq$" denotes equality up to terms of order ε^{N+1} uniformly with respect to $t \in [0, 1]$.

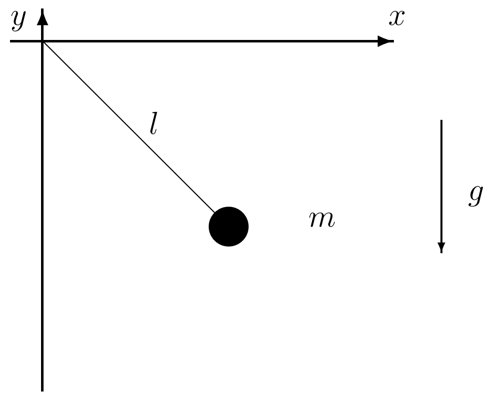

Figure 1: The mathematical pendulum

Remark 2.2 *(2.6) gives a complete characterization of the convergence behaviour of the perturbed solution. The terms which depend on the streched variable* $\tau = \frac{t}{\varepsilon}$ *vanish exponentielly with increasing* t *and decreasing* ε*. Therefore, for* $t > 0$*, they do not contribute essentially to the asymptotic behaviour for* $\varepsilon \to 0$*, i. e. (2.6) degenerates into the* regular expansion

$$u_\varepsilon(t) \simeq u_0(t) + \sum_{j=1}^{N} u_j(t)\varepsilon^j \tag{2.7a}$$

$$v_\varepsilon(t) \simeq v_0(t) + \sum_{j=1}^{N} v_\varepsilon(t)\varepsilon^j \tag{2.7b}$$

where $t \in [a, 1]$ *and* $a > 0$ *is fixed. The boundary layer part plays only an important role near the left-hand boundary* $t = 0$*.*

Example: As a simple example we consider the mathematical pendulum (Fig. 1). The Euler-Lagrange equations in Cartesian coordinates are

$$\begin{aligned} m\ddot{x} &= -x\lambda \\ m\ddot{y} &= -mg - y\lambda \\ x^2 + y^2 &= l^2 \end{aligned} \tag{2.8}$$

where m is the mass, l is the length of the rod, and g is the gravitational constant. λ describes the inner forces due to the constraint. For simplicity, let $m = 1$ and $l = 1$. In order to obtain a system in standard form, introduce $p = \dot{x}$, $q = \dot{y}$ such that (2.8) becomes

$$\begin{aligned}
\dot{x} &= p \\
\dot{y} &= q \\
\dot{p} &= -x\lambda \\
\dot{q} &= -g - y\lambda \\
x^2 + y^2 &= 1 \ .
\end{aligned} \tag{2.9}$$

(2.9) is tractable with index 3. In order to obtain an index 2 system we follow a proposal of [7]: Introduce an artificial Lagrange multiplier μ that vanishes identically in a solution:

$$\begin{aligned}
\dot{x} &= p - x\mu \\
\dot{y} &= q - y\mu \\
\dot{p} &= -x\lambda \\
\dot{q} &= -g - y\lambda \\
x^2 + y^2 &= 1 \\
2xp + 2yq &= 0 \ .
\end{aligned} \tag{2.10}$$

Equation (2.10) is an autonomous system of the form (2.1 with $u = (x, y, p, q)$, $v = (\lambda, \mu)$, and

$$g_u f_v = -\begin{pmatrix} 0 & 2x^2 + 2y^2 \\ 2x^2 + 2y^2 & 2xp + 2yq \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ -2 & 0 \end{pmatrix}$$

in a solution. Hence, (2.10) is tractable with index 2. We will compute the period of the pendulum if its initial state is the horizontal position $x = 1$, $y = 0$ with no impulse, i. e. $\dot{x} = 0$, $\dot{y} = 0$. Introducing T to be one fourth of the period we obtain the following boundary value problem

$$\begin{aligned}
\dot{x} &= T(p - x\mu) \\
\dot{y} &= T(q - y\mu) \\
\dot{p} &= -Tx\lambda \\
\dot{q} &= -T(g + y\lambda) \\
\dot{T} &= 0 \\
x^2 + y^2 &= 1 \\
2xp + 2yq &= 0 \\
y(0) &= 0 \\
q(0) &= 0 \\
x(1) &= 0 \ .
\end{aligned} \tag{2.11}$$

Equation (2.11) is again an index 2 problem of the form considered. In the following computations we have chosen $g = 13.750371636041$, which leads to $T = \frac{1}{2}$.

The solution of (2.11) is approximated numerically. As a standard boundary value solver we used a simplification of Degenhardt's algorithm (a fourth order collocation code with automatic grid selection and error control) for semiexplicit systems [19].

The exact solution of (2.8) cannot be analytically expressed. Only the period can be obtained via incomplete elliptic integrals. Instead, we used a solution computed by means of the program RADAU5 [11] with a high precision (10^{-8}) in an initial value formulation as a reference solution at 10 equally spaced points in $[0, 1]$. In order to distinguish the boundary layer from the smooth part, $[0, 0.1]$ was considered as the "boundary layer region" and $[0.2, 1]$ as the "smooth region".

Table 1 contains typical results for the relative tolerance 10^{-5}. For the pendulum, the third column shows that even $u_1 = 0$ and $v_1 = 0$, and no boundary layers are present.

ε	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1.00(-2)	2.64(-4)		7.56(-4)		4.46(-3)		1.93(-2)	
5.00(-3)	6.86(-5)	3.8	1.91(-4)	3.9	1.11(-3)	4.0	4.78(-3)	4.0
2.50(-3)	1.75(-5)	3.9	4.80(-5)	4.0	2.79(-4)	4.0	1.19(-3)	4.0
1.25(-3)	4.40(-6)	4.0	1.20(-5)	4.0	6.99(-5)	4.0	3.06(-4)	3.9
6.25(-4)	1.11(-6)	4.0	3.01(-6)	4.0	1.83(-5)	3.8	1.49(-4)	2.0

height 1: Error of u – boundary layer region
2: Quotient of relative errors of u – boundary layer regi on
3: Error of u – smooth region
4: Quotient of relative errors of u – smooth region
5: Error of v – boundary layer region
6: Quotient of relative errors of v – boundary layer regi on
7: Error of v – smooth region
8: Quotient of relative errors of v – smooth region
p t

Table 1: Relative errors and convergence rates

The observation of Example is backed up by Theorem 2.3. We want to prove our main result.

Theorem 2.3 *Let g in (2.1b) be autonomous. Then it holds*

$$u_1 = 0, \;\; \bar{u}_1 = 0, \;\; \bar{v}_0 = 0 \;\; v_1 = 0 \tag{2.12}$$

in 2.6.

3 Proof of Theorem 2.3

The proof of Theorem 2.3 can be given by a straightforward but rather lengthy computation. For our purposes it is convenient to denote the remainder in (2.6) by $\mathcal{R}_n(t)$ and $\mathcal{R}_v(t)$, respectively, knowing that $\mathcal{R}_n(t) = \mathcal{O}(\varepsilon^{N+1})$ and $\mathcal{R}_v(t) = \mathcal{O}(\varepsilon^{N+1})$ uniformly with respect to t. Moreover, for $t > 0$, we assume the boundary term of (2.6) to be included in $\mathcal{R}_n$ and $\mathcal{R}_v$, respectively. $\mathcal{R}$ denotes a generic remainder of higher order. Remark that, at this stage of our investigations, (u_0, v_0) from (2.6) and (2.7), respectively, are not the same as $(u_0,\, v_0)$ from NP2. But we will see that they are identical.

Because of Theroem 2.1, $(u_\varepsilon,\, v_\varepsilon)$ fulfil (2.2) and (2.5). Inserting (2.6) we obtain

$$G\left(u_0(0) + \sum_{j=1}^{N}(u_j(0) + \bar{u}_j(0))\varepsilon^j + \mathcal{R}_n(0), u_0(1) + \sum_{j=1}^{N} u_j(1)\varepsilon^j + \mathcal{R}_u(1)\right) = 0$$

$$ \tag{3.1a} $$

$$g\left(u_0(0)+\sum_{j=1}^{N}(u_j(0)+\bar{u}_j(0))\varepsilon^j+\mathcal{R}_n(0)\right)=0\,. \tag{3.1b}$$

Taylor's expansion around $(u_0(0), u_0(1))$ yields

$$\begin{aligned} G(u_0(0),u_0(1)) + G_{u(0)}(u_0(0),u_0(1)) \textstyle\sum_{j=1}^{N}(u_j(0)+\bar{u}_j(0))\varepsilon^j + \\ G_{u(1)}(u_0(0),u_0(1)) \textstyle\sum_{j=1}^{N} u_j(1)\varepsilon^j + \mathcal{R} = 0 \end{aligned} \tag{3.2a}$$

$$g(u_0(0)) + g_u(u_0(0)) \textstyle\sum_{j=1}^{N}(u_j(0)+\bar{u}_j(0))\varepsilon^j + \mathcal{R} = 0. \tag{3.2b}$$

Comparing equal powers of ε leads now to the relations searched for:

$$G(u_0(0),u_0(1)) = 0 \tag{3.3a}$$

$$g(u_0(0)) = 0 \tag{3.3b}$$

and

$$G_{u(0)}(u_0(0),u_0(1))(u_1(0)+\bar{u}_1(0)) + G_{u(1)}(u_0(0),u_0(1))u_1(1) = 0 \tag{3.4a}$$

$$g_u(u_0(0))(u_1(0)+\bar{u}_1(0)) = 0. \tag{3.4b}$$

On the other hand, for $t > 0$, (2.4) should be fulfilled uniformly with the regular expansion (2.7). Inserting this expansion into (2.4) and considering the limit $\varepsilon \longrightarrow 0$, we obtain

$$u_0'(t) = f(u_0(t),v_0(t),t) \tag{3.5a}$$

$$0 = g(u_0(t)). \tag{3.5b}$$

Because of (NP2, i) and (NP2, iii), (3.5), (3.3a) has the unique solution (u_0, v_0) from (NP2). This solution fulfils (3.3b) additionally. Using this fact (2.4) can be expanded:

$$\begin{aligned} \sum_{j=1}^{N} u_j'(t)\varepsilon^j + \mathcal{R}_u'(t) = f_u(u_0(t),v_0(t),t)\sum_{j=1}^{N} u_j(t)\varepsilon^j + \\ f_v(u_0(t),v_0(t),t)\sum_{j=1}^{N} v_j(t)\varepsilon^j + \mathcal{R} \end{aligned} \tag{3.6a}$$

$$\begin{aligned} 0 = g_u(u_0(t))\Bigg\{\sum_{j=1}^{N} u_j(t)\varepsilon^j + \\ \varepsilon\Big[f(u_0(t),v_0(t),t) + f_u(u_0(t),v_0(t),t)\sum_{j=1}^{N} u_j(t)\varepsilon^j + \\ f_v(u_0(t),v_0(t),t)\sum_{j=1}^{N} v_j(t)\varepsilon^j\Big]\Bigg\} + \mathcal{R}\,. \end{aligned} \tag{3.6b}$$

Comparing the coefficients of ε^1 gives

$$\begin{aligned} u_1'(t) &= f_u(u_0(t), v_0(t), t)u_1(t) + f_v(u_0(t), v_0(t), t)v_1(t) &\quad (3.7a)\\ 0 &= g_u(u_0(t))\left\{u_1(t) + f(u_0(t), v_0(t), t)\right\}. &\quad (3.7b)\end{aligned}$$

Differentiating (3.5b), $g_u(u_0)f(u_0, v_0, \cdot) \equiv 0$ such that (3.7b) reducs to

$$0 = g_u(u_0(t))\, u_1(t). \tag{3.8}$$

The system (3.7a), (3.8) is just the same as that of (NP2, iii). Completing it with (3.4a) gives a uniquely solvable boundary value problem whose solution depends on $\bar{u}_1(0)$. Our next aim is to show that $\bar{u}_1(0) = 0$ such that $u_1 = 0, v_1 = 0$. In order to do so we consider further the boundary layer part. It holds

$$\begin{aligned} \frac{1}{\varepsilon}\frac{d}{d\tau}\sum_{j=1}^{N} \bar{u}_j(\tau)\varepsilon^j &= \frac{1}{\varepsilon}\frac{d}{d\tau}\left(u_\varepsilon(t) - \sum_{j=0}^{N} u_j(t)\varepsilon^j - \mathcal{R}_u(t)\right)\\ &= \frac{d}{dt}\left(u_\varepsilon(t) - \sum_{j=0}^{N} u_j(t)\varepsilon^j - \mathcal{R}_u(t)\right) \end{aligned} \tag{3.9}$$

Because of $t = \varepsilon\tau$ and (2.7), (3.9) is equivalent to

$$\begin{aligned} \frac{1}{\varepsilon}\frac{d}{d\tau}\sum_{j=1}^{N} \bar{u}_j(\tau)\varepsilon^j &= f(u_\varepsilon(\varepsilon\tau), v_\varepsilon(\varepsilon\tau), \varepsilon\tau) - \\ & \qquad f\left(\sum_{j=0}^{N} u_j(\varepsilon\tau)\varepsilon^j + \mathcal{R}_u(\varepsilon\tau), \sum_{j=0}^{N} v_j(\varepsilon\tau)\varepsilon^j + \mathcal{R}_v(\varepsilon\tau), \varepsilon\tau\right). \end{aligned}$$

Taking the formal limit, for $\varepsilon \to 0$, and regarding (2.6) we obtain

$$\bar{u}_1'(\tau) = f(u_0(0), v_0(0) + \bar{v}_0(\tau), 0) - f(u_0(0), v_0(0), 0). \tag{3.10}$$

A similar procedure will be applied to (2.4b). It holds

$$\begin{aligned} 0 &= g(u_\varepsilon(t) + \varepsilon f(u_\varepsilon(t), v_\varepsilon(t), t))\\ &= g\Big(\sum_{j=0}^{N} u_j(t)\varepsilon^j + \sum_{j=1}^{N} \bar{u}_j(\tau)\varepsilon^j + \mathcal{R}_u(t) + \\ &\qquad \varepsilon f(\sum_{j=0}^{N} u_j(t)\varepsilon^j + \sum_{j=1}^{N} \bar{u}_j(\tau)\varepsilon^j + \mathcal{R}_u(t), \sum_{j=0}^{N}(v_j(t) + \bar{v}_j(\tau))\varepsilon^j + \mathcal{R}_v(t), t)\Big). \end{aligned}$$

Taylor's expansion with respect to ε at $\varepsilon = 0$ yields

$$0 = g(u_0(0)) + \varepsilon g_u(u_0(0))[u_0'(0)\tau + u_1(0) + \bar{u}_1(\tau) + f(u_0(0), v_0(0) + \bar{v}_0(\tau), 0)] + \mathcal{R}\,. \tag{3.11}$$

Again, $t = \tau\varepsilon$ was used. Because of (3.5b), the first term in (3.11) vanishes. The coefficient of ε must be zero. Differentiation of (3.5b) shows that this leads to

$$g_u(u_0(0))[u_1(0) + \bar{u}_1(\tau) + f(u_0(0), v_0(0) + \bar{v}_0(\tau), 0)] = 0. \tag{3.12}$$

Setting $\tau = 0$ in (3.12), (3.4b) yields

$$g_u(u_0(0))\, f(u_0(0), \bar{\beta}, 0) = 0 \tag{3.13}$$

with $\bar{\beta} = v_0(0) + \bar{v}_0(0)$. Using now (NP2, iv) and the remark following it concerning autonomous functions g, $\beta_0 = v_0(0)$, such that

$$\bar{v}_0(0) = 0. \tag{3.14}$$

In order to determine $\bar{u}_1$ and $\bar{v}_0$ we have now the equations (3.10), (3.12), the initial condition (3.14) and the property of $\bar{u}_1$ to vanish exponentially for $\tau \to \infty$. These information are sufficient to show that both functions are identical to z ero. In fact, differentiation of (3.12) and the use of (3.10) lead to the implicit ordinary differential equation

$$g_u(u_0(0))[f(u_0(0), v_0(0)+\bar{v}_0(\tau), 0) - f(u_0(0), v_0(0), 0) + f_v(u_0(0), v_0(0)+\bar{v}_0(\tau), 0)\, \bar{v}_0'(\tau)] = 0$$

for $\bar{v}_0$. Together with (3.14), Picard-Lindelöf's theorem ensures

$$\bar{v}_0 \equiv 0. \tag{3.15}$$

(3.10) reads now $\bar{u}_1' \equiv 0$ such that

$$\bar{u}_1 \equiv 0. \tag{3.16}$$

Especially, $\bar{u}_1(0) = 0$, and (3.4) reduces to

$$G_{u(0)}(u_0(0), u_0(1))u_1(0) + G_{u(1)}(u_0(0), u_0(1))u_1(1) = 0 \tag{3.17a}$$
$$g_u(u_0(0))u_1(0) = 0. \tag{3.17b}$$

The boundary value problem (3.7a), (3.8), (3.17a) is just the same mentioned in (NP2, iii). Consequently,

$$u_1 \equiv 0, \quad v_1 \equiv 0. \tag{3.18}$$

(3.15), (3.16), (3.18) give the assertion of Theorem 2.3.

Remark 3.1 *The assumption of g being independently of t was used in establishing (3.14) via (3.13) and also in the step from (3.7b) to (3.8). As the above example shows, higher order of convergence cannot be expected even if f is autonomous, too.*

References

[1] J. Baumgarte. Stabilization of constraints and integrals of motions in dynamical systems. *Comp.Methods in Appl. Mech. and Eng.*, 1:1–16, 1972.

[2] K. E. Brenan and B. E. Engquist. Backward differentiation approximations of nonlinear differential/algebraic systems. *Math. Comp.*, 51(184):659–676, 1988.

[3] L. O. Chua, T. Matsumoto, and S. Ichiraku. Geometric properties of resistive nonlinear n-ports: transversality, structural stability, reciprocity, and anti-reciprocity. *IEEE Trans. on Circuits and Systems*, CAS 27:577–603, 1980.

[4] K. D. Clark. A structural form for higher index semistate equations i: theory and application to circuit and control theory. *Linear Alg. Appl.*, 98:169–197, 1988.

[5] A. Degenhardt. A collocation method for boundary value problems of transferable differential algebraic equations. to appear in Numer. Math.

[6] E. Eich and M. Hanke. *Regularization methods for constraint mechanical multibody systems.* Preprint 91-8, Humboldt-Univ., Fachbereich Mathematik, Berlin, 1991.

[7] C. W. Gear, G. K. Gupta, and B. J. Leimkuhler. Automatic integration of euler-lagrange equations with constraints. *J. Comp. and Appl. Math.*, 12/13:77–90, 1985.

[8] C. W. Gear and L. R. Petzold. Ode methods for the solution of differential algebraic systems. *SIAM J. Numer. Anal.*, 21:716–728, 1984.

[9] E. Griepentrog and R. März. *Differential-Algebraic Equations And Their Numerical Treatment.*

Teubner-Texte zur Mathematik; 88, Teubner, Leipzig, 1986.

[10] E. Hairer, Ch. Lubich, and M. Roche. *The numerical solution of differential-algebraic systems by Runge-Kutta methods. Lecture Notes in Math.; 1409*, Springer-Verlag, Berlin, 1989.

[11] E. Hairer and G. Wanner. *Radau5 - an implicit Runge-Kutta code.* Report, Université de Genève, Dept. de mathématiques, Genève, 1988.

[12] M. Hanke. On the asymptotic representation of a regularization approach to nonlinear semiexplicit higher index differential-algebraic equations. *IMA J. Appl. Math.*, 46:225–245, 1990.

[13] M. Hanke. *Regularization methods for higher index differential-algebraic equations.* Preprint 268, Humboldt-Univ., Fachbereich Mathematik, Berlin, 1990.

[14] M. Knorrenschild. *Regularisierung von differentiell- algebraischen Systemen - theoretische und numerische Aspekte.* PhD thesis, Rhein.-Westfälische Techn. Hochschule, Aachen, 1988.

[15] R. Lamour. A well-posed shooting method for transferable dae's. To appear in Numer. Math.

[16] R. März. *On tractability with index 2.* Preprint 109, Humboldt-Univ., Sektion Mathematik, Berlin, 1986.

[17] T. Matsumoto, L. O. Chua, H. Kawakami, and S. Ichiraku. Geometric properties of dynamic nonlinear networks: transversality, local-solvability and eventual passivity. *IEEE Trans. on Circuits and Systems*, 28:406–428, 1981.

[18] R. E. O'Malley. *Introduction to Singular Perturbations.* Academic Press, New York, 1974.

[19] H. Valentini. *KOVEIS - ein Kollokationsverfahren für semiexplizite Algebro-Differentialgleichungen.* Praktikumsarbeit, Humboldt-Univ., Sektion Mathematik, Berlin, 1989.

[20] A. B. Vasil'eva and B. F. Butuzov. *Asymptotic expansions of solutions of singularly perturbed equations.* Nauka, Moskva, 1973. (in Russian).