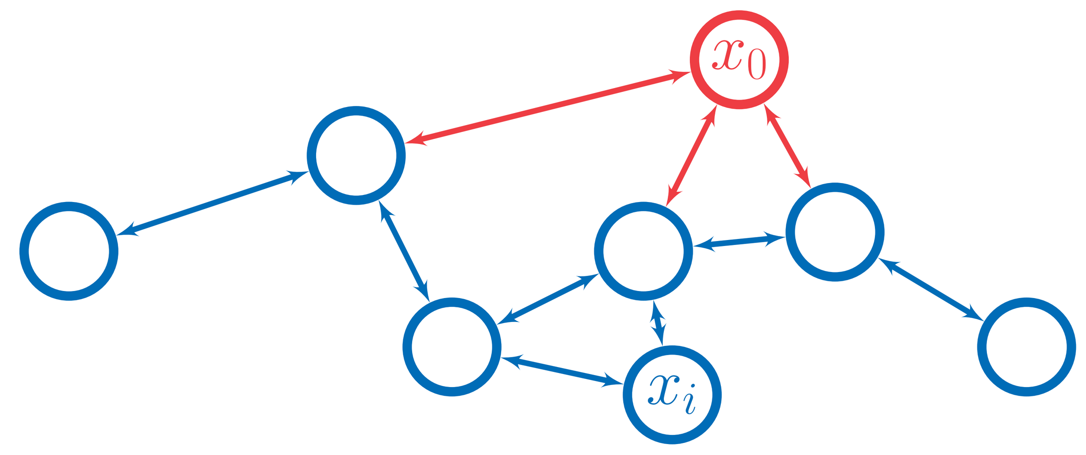
Consensus Control for Multi-agent Systems with a Faulty Node

Håkan Terelius   Guodong Shi   Karl Henrik Johansson

KTH - Royal Institute of Technology

Abstract We study consensus control for a multi-agent system with a faulty node. The node dynamics follow a continuous-time consensus protocol with negative feedback from the relative state of the neighbors, where the faulty node is instead using positive feedback from the state. Conditions for reaching consensus are established, and a fault threshold is introduced. Numerical examples investigate how the fault threshold determines the system behavior.

Problem Definition



- Multi-agent system $(\mathcal{V}, \mathcal{E}(t))$, $\mathcal{V} = \{0, \ldots, n\}$
- Each agent $i \in \mathcal{V}$ has a state $x_i(t) \in \mathbb{R}$
- One faulty agent, $i = 0$,
$$\dot{x}_0(t) = -\sum_{j \in \mathcal{N}_0(t)} \big(x_j(t) - x_0(t)\big)$$
- n follower agents, $i \in \{1, \ldots, n\}$, trying to reach consensus
$$\dot{x}_i(t) = \sum_{j \in \mathcal{N}_i(t), j>0} a_{ij}(t)\big(x_j(t) - x_i(t)\big) + b_i(t)\big(x_0(t) - x_i(t)\big)$$

Definition

- The system is *escapable* if
$$\lim_{t\to\infty} \max_{i=1,\ldots,n} \big|x_i(t) - x_0(t)\big| = \infty$$
- The system is *trackable* if
$$\lim_{t\to\infty} \max_{i=1,\ldots,n} \big|x_i(t) - x_0(t)\big| = 0$$

Fixed graph

Define the state difference $\xi_i(t) = x_i(t) - x_0(t)$, then
$$\dot{\xi} = -G\xi, \qquad G = L_F + B - E.$$
where $L_F = D_F - A_F$ is the Laplacian matrix of the follower graph and $B = \mathrm{diag}(b_1, \ldots, b_n)$, $E = \mathrm{diag}(e_1, \ldots, e_n)$, $e_i = 1$ if $i \in \mathcal{N}_0$, 0 otherwise, are the connections to and from the faulty agent.

Assumptions

The edges $\mathcal{E}$ and the weights a_{ij}, b_i are time-invariant, and the graph is connected.

Results

- The system is escapable $\Leftrightarrow \exists\, \lambda(-G) \in \mathbb{C}_+$
- The system is trackable $\Leftrightarrow \forall\, \lambda(-G) \in \mathbb{C}_-$
- The system is trackable $\Rightarrow b_* \geq |\mathcal{N}_0|$
- The system is trackable $\Leftarrow \lambda_1(-(L_F + B)) < -\sqrt{|\mathcal{N}_0|}$

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Time-varying graphs

Assumptions

- Exist bound $\tau_D > 0$ between topology switching times.
- Bounded weights, $a_* \leq a_{ij}(t) \leq a^*$
- Exist $T > 0$ such that the joint graph over T steps is connected.

Results

- The system is trackable $\Rightarrow b_* \geq 1$.
- The system is trackable $\Leftarrow$
$$0 < \left(e^{n(n+1)T_0} - w_*\left(\varrho_0 e^{-(n^2-1)a^* T_0}\right)^n\right) < 1$$

where $T_0 = T + \tau_D$, $\varrho_0 = \frac{\left(1 - e^{-((n-2)a^* + a_*)\tau_D}\right)a_*}{(n-2)a^* + a_*}$ and
$$w_* = \frac{b_* + \left(e^{\tau_D} - 1\right)\left(e^{nT_0} - 1\right)\left(e^{-b_* T_0} - 1\right)}{(n-1)a^* + b_*} \cdot e^{-(n-1)a^*(n+1)T_0}$$

Examples