In this talk I will present recent developments of the obstacle type problems, with various applications ranging from: Industry to Finance, local to nonlocal operators, and one to multi-phases. The theory has evolved from a single equation

$$\Delta u = \chi_{u>0}, \qquad u \geq 0$$

to embrace a more general (two-phase) form

$$\Delta u = \lambda_+ \chi_{u>0} - \lambda_- \chi_{u<0}$$

with $\lambda_\pm$ reasonably smooth functions (down to Dini continuous).

Astonishing results of Yuval Peres and his collaborators has shown remarkable relationships between obstacle problem and various forms of random walks, including Smash sum of Diaconis-Fulton (Lattice sets), and there is more to come.

The two-phase form (and its multi-phase form) has been under investigation in the last 10 years, and interesting recoveries has been made about the behavior of the free boundaries in such problems. Existing methods has so far only allowed us to consider $\lambda_\pm > 0$.

The above problem changes drastically if one allows $\lambda_\pm$ to have the incorrect sign (that appears in composite membrane problem)! In part of my talk I will focus on the simple *unstable* case

$$\Delta u = -\chi_{u>0}$$

and present very recent results (Andersson, Sh., Weiss) that classifies the set of singular points ($\{u = \nabla u = 0\}$) for the above problem. The techniques developed recently by our team also shows an unorthodox approach to such problems, as the classical technique fails.

At the end of my talk I will explain the technique in a heuristic way.