THE DYNAMICAL CIRCLE COVERING PROBLEM

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The classical circle covering problem introduced by Dvoretzky is the following question: Suppose that $I_1, I_2, I_3, \ldots$ are intervals of decreasing lengths $l_1, l_2, l_3, \ldots$ and that these intervals are independently and uniformly distibuted on the circle of unit circumference. It is a trivial consequence of Borel-Cantelli that any given point on the circle will a.s. be covered by infinitely many of these intervals iff the sum of the lengths is divergent, but will the whole circle be covered? Partial results on this were obtained by Dvoretzky and many others. Finally, Shepp (1972) proved the answer is a.s. yes iff $\sum_{n=1}^{\infty} \frac{e^{l_1+\ldots+l_n}}{n^2} = \infty$. Assuming that $l_n = c/n$ for a constant c, this implies that the whole circle will be covered infinitely often iff $c \geq 1$ (a special case which was known before Shepp).

We will consider a dynamical version of the problem where the intervals after having been given initial random positions move according to independent standard Brownian motions. Assume that $l_n = c/n$ for a constant c. Among other things we show that for $c < 2$ a.s. there are exceptional times when a given fixed point is covered only finitely often, whereas this is not the case when $c \geq 2$. We also show that when $c < 3$ there are a.s. exceptional times where the circle is not covered infinitely often, whereas when $c \geq 3$ the whole circle is covered all the time. This is joint work with Johan Jonasson.