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ERRATUM TO "TRANSLATES OF FUNCTIONS OF TWO VARIABLES"

HÅKAN HEDENMALM

It has come to my attention that the argument on pages 288–290 of [1] is flawed. The problem is that $\bar{\partial}\mathbf{\Phi} \wedge \bar{\partial}\mathbf{\Phi}$ need not be 0.

Here we correct the argument, following the steps of [2] more closely. Let us write $\Lambda^s_{(0,r)}$ for the space of forms $\Lambda^s(\mathscr{R}_{(0,r)})$ appearing in [1], where $\mathscr{R}$ is an appropriate ring of continuous functions on Π^2. Let $\mathbf{g}' = \chi_\lambda \wedge \mathbf{\Phi} \in \Lambda^1_{(0,0)}$, and note that $P_\mathbf{f}\mathbf{g}' = \chi_\lambda$ and $\bar{\partial}\mathbf{g}' = \bar{\partial}\chi_\lambda \wedge \mathbf{\Phi} + \chi_\lambda \wedge \bar{\partial}\mathbf{\Phi}$. Set

$$\mathbf{h}' = \mathbf{\Phi} \wedge \bar{\partial}\mathbf{g}' = \bar{\partial}\chi_\lambda \wedge \mathbf{\Phi} \wedge \mathbf{\Phi} + \chi_\lambda \wedge \mathbf{\Phi} \wedge \bar{\partial}\mathbf{\Phi} = \chi_\lambda \wedge \mathbf{\Phi} \wedge \bar{\partial}\mathbf{\Phi} \in \Lambda^2_{(0,1)},$$

and observe that $P_\mathbf{f}\mathbf{h}' = \chi_\lambda \wedge \bar{\partial}\mathbf{\Phi}$. Let

$$\mathbf{h}'' = \mathbf{\Phi} \wedge \bar{\partial}\mathbf{h}' = \chi_\lambda \wedge \mathbf{\Phi} \wedge \bar{\partial}\mathbf{\Phi} \wedge \bar{\partial}\mathbf{\Phi} \in \Lambda^3_{(0,2)},$$

and note that $P_\mathbf{f}\mathbf{h}'' = \chi_\lambda \wedge \bar{\partial}\mathbf{\Phi} \wedge \bar{\partial}\mathbf{\Phi}$. As a differential form, $\mathbf{h}''$ has order $(0,2)$, and since the complex dimension of the region is 2, it follows that $\bar{\partial}\mathbf{h}'' = 0$. We find an $\mathbf{h}''' \in \Lambda^3_{(0,1)}$ such that $\bar{\partial}\mathbf{h}''' = \mathbf{h}''$, and set $\mathbf{h} = \mathbf{h}' - P_\mathbf{f}\mathbf{h}''' \in \Lambda^2_{(0,1)}$, for then $P_\mathbf{f}\mathbf{h} = \chi_\lambda \wedge \bar{\partial}\mathbf{\Phi}$, and $\bar{\partial}\mathbf{h} = -\bar{\partial}\chi_\lambda \wedge \bar{\partial}\mathbf{\Phi} \wedge \mathbf{\Phi}$. Let $\mathbf{y} \in \Lambda^2_{(0,1)}$ solve

$$\bar{\partial}\mathbf{y} = (1 - (\lambda + 4)\widehat{A^2})^{-1}\bar{\partial}\chi_\lambda \wedge \bar{\partial}\mathbf{\Phi} \wedge \mathbf{\Phi};$$

the right-hand side is $\bar{\partial}$-closed because it is a $(0,2)$-form, and the singularity of the first factor is swallowed by $\bar{\partial}\chi_\lambda$. The form $\mathbf{g}'' = \mathbf{h} + (1 - (\lambda + 4)\widehat{A^2})\mathbf{y} \in \Lambda^2_{(0,1)}$ is then $\bar{\partial}$-closed, that is, $\bar{\partial}\mathbf{g}'' = 0$. Let $\mathbf{g}''' \in \Lambda^2_{(0,0)}$ solve $\bar{\partial}\mathbf{g}''' = \mathbf{g}''$, and set $\mathbf{g}_0 = \mathbf{g}' - P_\mathbf{f}\mathbf{g}''' \in \Lambda^1_{(0,0)}$. Then $P_\mathbf{f}\mathbf{g}_0 = \chi_\lambda$, and $\bar{\partial}\mathbf{g}_0 = \bar{\partial}\chi_\lambda \wedge \mathbf{\Phi} - (1 - (\lambda + 4)\widehat{A^2})P_\mathbf{f}\mathbf{y}$. Let $\mathbf{x} \in \Lambda^1_{(0,0)}$ solve

$$\bar{\partial}\mathbf{x} = P_\mathbf{f}\mathbf{y} - (1 - (\lambda + 4)\widehat{A^2})^{-1}\bar{\partial}\chi_\lambda \wedge \mathbf{\Phi};$$

again the singularity of the inverted analytic function is absorbed by the factor $\bar{\partial}\chi_\lambda$. Also, the right-hand side is $\bar{\partial}$-closed due to its connection with $\bar{\partial}\mathbf{g}_0$. If we set $\mathbf{g} = \mathbf{g}_0 + (1 - (\lambda + 4)\widehat{A^2})\mathbf{x} \in \Lambda^1_{(0,0)}$, then $\bar{\partial}\mathbf{g} = 0$ and

$$P_\mathbf{f}\mathbf{g} = \chi_\lambda + (1 - (\lambda + 4)\widehat{A^2})P_\mathbf{f}\mathbf{x}.$$

It follows that

$$\frac{1 - P_{\mathbf{f}}\mathbf{g}}{1 - (\lambda + 4)\widehat{A^2}} = \frac{1 - \chi_\lambda}{1 - (\lambda + 4)\widehat{A^2}} - P_{\mathbf{f}}\mathbf{x},$$

the supremum norm of which we can control by tracing back our moves and making sure the solutions to the $\bar{\partial}$ problems are as nice as we need them to be. This is guaranteed by the following lemma, which follows from the existence on the bidisk of bounded continuous solutions to $\bar{\partial}$ problems with data of the same degree of regularity [3, p. 676].

LEMMA. *Let $\mathscr{R}$ be the ring $C((\overline{\Pi} \cup \{\infty\})^2)$. Suppose $\mathbf{v} \in \Lambda^s_{(0,r)}$ is $\bar{\partial}$-closed, and that $(z_1 + 1)^{n+2}(z_2 + 1)^{n+2}\mathbf{v} \in \Lambda^s_{(0,r)}$ for some $n = 0, 1, 2, \ldots$. Then there is a $\mathbf{u} \in \Lambda^s_{(0,r-1)}$ solving $\bar{\partial}\mathbf{u} = \mathbf{v}$, such that $(z_1 + 1)^n(z_2 + 1)^n\mathbf{u} \in \Lambda^s_{(0,r-1)}$, and the following estimate holds:*

$$\|(z_1 + 1)^n(z_2 + 1)^n\mathbf{u}\| \leqslant C\|(z_1 + 1)^{n+2}(z_2 + 1)^{n+2}\mathbf{v}\|.$$

Using the lemma, we obtain in place of (4.17) in [1] that

$$|\mathscr{C}[\phi](\lambda)| \leqslant C(\operatorname{Re}\lambda)^{-2}(1 + |\lambda|)^{12}A(\lambda)^6, \qquad \operatorname{Re}\lambda > 0,$$

which leads to the same conclusions as before [1].

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DEPARTMENT OF MATHEMATICS, UPPSALA UNIVERSITY, BOX 480, S-751 06 UPPSALA, SWEDEN; haakan@math.uu.se