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THIN INTERPOLATING SEQUENCES AND THREE ALGEBRAS OF BOUNDED FUNCTIONS

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ABSTRACT. We consider the closed subalgebra $\mathbf{A}$ of H^∞ generated by the thin interpolating Blaschke products, the smallest C^* subalgebra $\mathbf{B}$ of L^∞ containing $\mathbf{A}$, and the Douglas algebra $\mathbf{E}$ generated by the complex conjugates of thin interpolating Blaschke products. Our main result is that every $\mathbf{E}$-invertible inner function is a finite product of thin interpolating Blaschke products, making $\mathbf{B} = C_{\mathbf{E}}$. We apply results of Chang and Marshall to prove that $\mathbf{A} = \mathbf{B} \cap H^\infty$, that finite convex combinations of finite products of thin interpolating Blaschke products are dense in the closed unit ball of $\mathbf{A}$, and that the corona theorem holds for $\mathbf{A}$.

0. Introduction. $\mathbf{D}$ will always be the open unit disc $\{z \in \mathbf{C}\colon |z| < 1\}$, and $\mathbf{T} = \partial\mathbf{D}$ will be the unit circle $\{z \in \mathbf{C}\colon |z| = 1\}$. $H^\infty = H^\infty(\mathbf{D})$ is the Banach algebra of bounded analytic functions on $\mathbf{D}$, supplied with the uniform norm. It is well known that we can regard H^∞ as a closed subalgebra of $L^\infty = L^\infty(\mathbf{T})$ via nontangential boundary values.

A sequence $\{z_n\} \subset \mathbf{D}$, finite or infinite, is an (H^∞) interpolating sequence if every interpolation problem

$$f(z_n) = a_n \quad \text{for all } n$$

with $\{a_n\}$ bounded, has solution $f \in H^\infty$. Clearly, every finite sequence that does not contain the same point twice is an interpolating sequence. A famous theorem of Lennart Carleson (see Carleson [**1958**], Garnett [**1981**, Chapter VII]) states that a sequence $\{z_n\}_0^\infty \subset \mathbf{D}$ is interpolating if and only if

$$\prod_{k,k\neq n} \rho(z_k, z_n) \geq \delta, \qquad n = 0, 1, 2, \ldots,$$

for some constant $\delta > 0$. Here ρ denotes the pseudohyperbolic metric

$$\rho(z, \varsigma) = |(z - \varsigma)/(1 - \bar{\varsigma}z)|, \qquad z, \varsigma \in \mathbf{D}.$$

Call an interpolating sequence $\{z_n\}$ *thin* if it is finite or

$$\prod_{k,k\neq n} \rho(z_k, z_n) \to 1 \quad \text{as } n \to \infty.$$

By a recent joint paper by Carl Sundberg and Thomas Wolff [**1983**], a sequence is thin interpolating if and only if it is an interpolating sequence for the uniform algebra $QA = H^\infty \cap \mathrm{VMO}(\mathbf{T})$, where $\mathrm{VMO}(\mathbf{T})$ is the space of functions on $\mathbf{T}$ with vanishing mean oscillation.

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A Blaschke product associated to a thin interpolating sequence is called a *thin interpolating* Blaschke product. It should be pointed out that in our terminology, a Blaschke product multiplied by a unimodular constant is also a Blaschke product.

Let $\mathbf{A}$ be the closed subalgebra of H^∞ generated by the thin interpolating Blaschke products, and let $\mathbf{B} = [\mathbf{A}, \overline{\mathbf{A}}]$ be the smallest (closed) C^* subalgebra of L^∞ containing $\mathbf{A}$. In other words, $\mathbf{B}$ is generated by ratios of thin interpolating Blaschke products. $\mathbf{E} = [H^\infty, \overline{\mathbf{A}}]$ will be the Douglas algebra (for a definition, see Garnett [**1981**]) generated by H^∞ and the complex conjugates of all thin interpolating Blaschke products.

Our main result is that every $\mathbf{E}$-invertible inner function is in fact a finite product of thin interpolating Blaschke products, and hence $\mathbf{B} = C_{\mathbf{E}}$. Here $C_{\mathbf{E}}$ denotes the C^* algebra generated by the $\mathbf{E}$-invertible inner functions and their complex conjugates. Another consequence is that the finite products of thin interpolating Blaschke products are the only inner functions there are in $\mathbf{B} \cap H^\infty$, and hence by Theorem 4.1 in the joint paper [**1977**] by Sun-Yung Chang and Donald Marshall, $\mathbf{A} = \mathbf{B} \cap H^\infty$, and finite convex combinations of finite products of thin interpolating Blaschke products are dense in the closed unit ball of $\mathbf{A}$. There are some other properties of $\mathbf{A}, \mathbf{B}$, and $\mathbf{E}$ which follow from the results stated in Chang-Marshall [**1977**]; for example, $\mathbf{A}$ has the corona property, that is, $\mathbf{D}$ is dense in the maximal ideal space $\mathcal{M}(\mathbf{A})$ of $\mathbf{A}$.

The author would like to mention the following related open problem, which has been posed by Peter Jones [**1981**, p. 320] and John Garnett [**1981**, p. 430; **1984**]: Do the interpolating Blaschke products generate H^∞?

1. Basic concepts. Let $\mathcal{M}(H^\infty)$ be the maximal ideal space of H^∞, that is, the space of all (nonzero) complex homomorphisms on H^∞, provided with the Gelfand topology. The famous corona theorem states that $\mathbf{D}$ is dense in $\mathcal{M}(H^\infty)$. For $f \in H^\infty$, its Gelfand transform $\hat{f}$ is a continuous function on $\mathcal{M}(H^\infty)$ which extends f. When it cannot cause any confusion, we will usually omit the distinction between a function and its Gelfand transform.

The Gleason parts are the connectivity components of $\mathcal{M}(H^\infty)$ when endowed with the norm topology of the dual Banach space $(H^\infty)^*$ (Hoffman [**1967**, p. 103]). For instance, $\mathbf{D}$ is a Gleason part of $\mathcal{M}(H^\infty)$. Let $\mathcal{P}(m)$ denote the Gleason part containing the point $m \in \mathcal{M}(H^\infty)$.

For $\varsigma \in \mathbf{D}$, let

$$L_\varsigma(z) = (z + \varsigma)/(1 + \bar{\varsigma} z), \qquad z \in \mathbf{D},$$

which maps $\mathbf{D}$ homeomorphically onto itself. According to Hoffman [**1967**] (see also Garnett [**1981**, Chapter X]), we can introduce for every $m \in \mathcal{M}(H^\infty)$ an analytic mapping $L_m \colon \mathbf{D} \to \mathcal{M}(H^\infty)$ which varies continuously with m such that $L_m(0) = m$, $L_m(\mathbf{D}) = \mathcal{P}(m)$, and if $m = \varsigma \in \mathbf{D}$, $L_m = L_\varsigma$. L_m is constant if and only if m is not in the closure of any interpolating sequence. We define $\mathcal{F}$ to be the set of all points in $\mathcal{M}(H^\infty)$ which are in the closure of a *thin* interpolating sequence. Proposition 2.2 tells us that $\mathcal{F}$ is the union of a family of nontrivial Gleason parts, and by Proposition 2.3, L_m is a homeomorphism for $m \in \mathcal{F}$, a fact which is well known (see Hoffman [**1967**, pp. 106–108]).

We should mention that is is customary to identify the Šilov boundary of $\mathcal{M}(H^\infty)$ with $\mathcal{M}(L^\infty)$, the maximal ideal space of L^∞.

2. Results. We shall need the following lemma.

LEMMA 2.1. *Let $\{z_n\}_0^\infty$ be a thin interpolating sequence. If the sequence $\{\varsigma_n\}_0^\infty$ does not contain the same point twice and $\rho(z_n, \varsigma_n) \le r$ for all n for some $r < 1$, the sequence $\{\varsigma_n\}_0^\infty$ is also thin interpolating.*

PROOF. Since $\{z_n\}_0^\infty$ is a thin interpolating sequence, there is a positive number $\delta(N)$ tending to 1 as $N \to \infty$, such that

$$\prod_{n, n\ge N, n\ne k} \rho(z_n, \varsigma_n) \ge \delta(N) \quad \text{for all } k \ge N.$$

By a lemma of Garnett [**1981**, Lemma VII.5.3],

$$\prod_{n, n\ge N, n\ne k} \rho(\varsigma_n, \varsigma_k) \ge (\delta(N) - 2r/(1+r^2))(1 - 2r\delta(N)/(1+r^2))^{-1}$$

for all $k \ge N$ if N is sufficiently large. Since $\lim_{k\to\infty} \rho(\varsigma_n, \varsigma_k) = 1$ for every n (use Lemma I.1.4 of Garnett [**1981**]),

$$\begin{aligned}\liminf_{k\to\infty} \prod_{n, n\ne k} \rho(\varsigma_n, \varsigma_k) &= \liminf_{k\to\infty} \prod_{n, n\ge N, n\ne k} \rho(\varsigma_n, \varsigma_k) \\ &\ge (\delta(N) - 2r/(1+r^2))(1 - 2r\delta(N)/(1+r^2))^{-1},\end{aligned}$$

again if N is sufficiently large. Letting $N \to \infty$ and thus $\delta(N) \to 1$, the right-hand side expression tends to 1, and the assertion follows.

As a consequence, we have the following

PROPOSITION 2.2. *$\mathcal{F}$ is the union of a family of nontrivial Gleason parts.*

PROOF. By the definition of $\mathcal{F}$, every point in $\mathcal{F}$ belongs to a nontrivial Gleason part. So, let $m_0 \in \mathcal{F}$ be arbitrary, and suppose $m \in \mathcal{P}(m_0)$. It suffices to show that $m \in \mathcal{F}$. By assumption, there exists a thin interpolating sequence $\{z_n\}_0^\infty$ such that $m_0 \in \overline{\{z_n\}_0^\infty}$, and thus there is a net $\{z_{n(\alpha)}\}_\alpha$ converging to m_0. Since m belongs to $\mathcal{P}(m_0)$, there exists a $\varsigma \in \mathbf{D}$ such that $\lim_\alpha L_{z_{n(\alpha)}}(\varsigma) = L_{m_0}(\varsigma) = m$. But by Lemma 2.1, the sequence

$$\varsigma_n = L_{z_n}(\varsigma) = (\varsigma + z_n)/(1 + \bar{z}_n\varsigma), \qquad n = 0, 1, 2, \ldots,$$

is also thin interpolating since $\rho(\varsigma_n, z_n) = |\varsigma| < 1$, and because $\lim_\alpha \varsigma_{n(\alpha)} = m$, the assertion follows.

PROPOSITION 2.3. *A thin interpolating Blaschke product b has modulus 1 on those Gleason parts of $\mathcal{M}(H^\infty)$ which do not contain a point in the zero set $Z(\hat{b}) = \{m \in \mathcal{M}(H^\infty) : \hat{b}(m) = 0\}$. Since $Z(\hat{b})$ equals the closure of $Z(\hat{b}) \cap \mathbf{D}$ and $\mathcal{F}$ is the union of a family of Gleason parts, $\hat{b}$ has in particular modulus 1 on $\mathcal{M}(H^\infty)\backslash\mathcal{F}$. Also, no two points in $Z(\hat{b})\backslash\mathbf{D}$ are contained in the same Gleason part of $\mathcal{F}$. Indeed, for every $m \in Z(\hat{b})\backslash\mathbf{D}$ there is a $\theta \in \mathbf{R}$ such that*

$$\hat{b} \circ L_m(z) = e^{i\theta} z, \qquad z \in \mathbf{D}.$$

PROOF. The assertion is trivial if b is a finite Blaschke product. So, let $\{z_n\}_0^\infty$ be the zeros of b. That $Z(\hat{b})$ equals the closure of $Z(\hat{b}) \cap \mathbf{D} = \{z_n\}_0^\infty$ follows from Lemma IX.3.3 in Garnett [**1981**]. Since b is thin interpolating,

$$|(b \circ L_{z_n})'(0)| = (1 - |z_n|^2)|b'(z_n)| \to 1$$

as $n \to \infty$, and hence $|(b \circ L_m)'(0)| = 1$ for all $m \in Z(\hat{b}) \backslash \mathbf{D}$. On the other hand, $\|\hat{b} \circ L_m\| \leq 1$, and Schwarz' lemma now tells us that

$$\hat{b} \circ L_m(z) = e^{i\theta} z, \qquad z \in \mathbf{D},$$

for all $m \in Z(\hat{b}) \backslash \mathbf{D}$ with a $\theta \in \mathbf{R}$ depending on m.

It remains to show that $\hat{b}$ has modulus 1 on those Gleason parts of $\mathcal{M}(H^\infty)$ which do not contain a point in $Z(\hat{b})$. To this end, let $m \in \mathcal{M}(H^\infty) \backslash \bigcup_{m_0 \in Z(\hat{b})} \mathcal{P}(m_0)$ be arbitrary; by the corona theorem, there is a net $\{\varsigma_\alpha\}_\alpha$ in $\mathbf{D}$ which converges to m. Our first step is to show that

$$\lim_\alpha \inf_n \rho(\varsigma_\alpha, z_n) = 1. \tag{2.1}$$

If this were not true, there would be a (cofinal) subnet $\{\beta\}$ of $\{\alpha\}$, an $n(\beta) \in \mathbf{N}$ for every β, and an ε, $0 < \varepsilon < 1$, such that (see Garnett [**1981**, p. 401])

$$\rho(\varsigma_\beta, z_{n(\beta)}) = \sup\{|f(\varsigma_\beta)| \colon f \in H^\infty, \|f\| \leq 1, f(z_{n(\beta)}) = 0\} \leq \varepsilon$$

for all β, and, consequently,

$$\sup\{|f(\varsigma_\beta) - f(z_{n(\beta)})| \colon f \in H^\infty, \|f\| \leq 1, f(z_{n(\beta)}) = 0\} \leq 2\varepsilon < 2$$

for all β. Let $m_0 \in \overline{\{z_{n(\beta)}\}_\beta} \backslash \{z_{n(\beta)}\}_\beta$ be arbitrary. Then

$$|\hat{f}(m) - \hat{f}(m_0)| \leq \sup_\beta |f(\varsigma_\beta) - f(z_{n(\beta)})| \leq 2\varepsilon < 2$$

for all $f \in H^\infty$ with $\|f\| \leq 1$. Hence m and m_0 lie in the same Gleason part, which is clearly impossible. Thus (2.1) follows.

Now the time has come to show that

$$|\hat{b}(m)| = \lim_\alpha |b(\varsigma_\alpha)| = 1 \tag{2.2}$$

Observe that in order to show that (2.2) holds, we may assume without loss of generality that

$$\inf_n (1 - |z_n|^2)|b'(z_n)| = \inf_n \prod_{k, k \neq n} \rho(z_k, z_n) = \delta,$$

where $\delta \in (0,1)$ is arbitrarily close to 1, by dividing b by finitely many of its Blaschke factors, since all finite Blaschke products satisfy (2.2). Choose δ so that $r(\delta)$ of Lemma X.1.4 in Garnett [**1981**] is close to 1, and let $\lambda(\delta)$ be as prescribed in that lemma. Since

$$\lim_\alpha \inf_n \rho(\varsigma_\alpha, z_n) = 1,$$

there is an index $\alpha_0(\delta)$ such that

$$\inf_n \rho(\varsigma_\alpha, z_n) \geq \lambda(\delta) \quad \text{for all } \alpha \geq \alpha_0(\delta),$$

that is,

$$\varsigma_\alpha \in \mathbf{D} \backslash \bigcup_n U_n \quad \text{for } \alpha \geq \alpha_0(\delta),$$

where $U_n = \{z \in \mathbf{D} \colon \rho(z, z_n) < \lambda(\delta)\}$. Hence by Lemma X.1.4 in Garnett [**1981**],

$$|b(\varsigma_\alpha)| \geq r(\delta) \quad \text{for all } \alpha \geq \alpha_0(\delta).$$

Since $r(\delta)$ was close to 1, we conclude that $\lim_\alpha |b(\varsigma_\alpha)| = 1$. The proof of the proposition is complete.

It is a consequence of Proposition 2.3 that $\mathcal{F}$ is an open subset of $\mathcal{M}(H^\infty)$. If $\mathcal{M}(\mathbf{E})$ denotes the maximal ideal space of the Douglas algebra $\mathbf{E}$, Proposition 2.3 has the following corollary when combined with Theorem IX.1.3 in Garnett [**1981**].

COROLLARY 2.4. $\mathcal{M}(\mathbf{E}) = \mathcal{M}(H^\infty)\backslash\mathcal{F}$.

By the Douglas-Rudin theorem,

$$[H^\infty, \overline{H^\infty}] = L^\infty,$$

and since the set $\mathcal{M}(H^\infty)\backslash(\mathcal{F} \cup \mathcal{M}(L^\infty))$ is nonempty (there are, for example, one-point Gleason parts not contained in the Šilov boundary $\mathcal{M}(L^\infty)$), Corollary 2.4 has the following consequence.

COROLLARY 2.5. $\mathbf{A}$ *is a proper subalgebra of* H^∞.

We are now ready to present our main result.

THEOREM 2.6. *Every* $\mathbf{E}$*-invertible inner function is a finite product of thin interpolating Blaschke products.*

PROOF. Let u be an arbitrary $\mathbf{E}$-invertible inner function, and denote by $Z(\hat{u})$ the zero set $\{m \in \mathcal{M}(H^\infty)\colon \hat{u}(m) = 0\}$. Our first step will be to show that the zeros of u, when counted with respect to multiplicity, form a finite union of thin interpolating sequences.

By Theorem IX.1.3 in Garnett [**1981**], $|\hat{u}(m)| = 1$ on $\mathcal{M}(\mathbf{E}) = \mathcal{M}(H^\infty)\backslash\mathcal{F}$, and consequently, $Z(\hat{u}) \subset \mathcal{F}$.

Introduce ε, $0 < \varepsilon < 1$, to be determined later. For every thin interpolating Blaschke product b, let

$$\Omega_\varepsilon(b) = \{m \in \mathcal{M}(H^\infty)\colon |\hat{b}(m)| < \varepsilon\}.$$

Since $Z(\hat{u}) \subset \mathcal{F}$, a compactness argument provides us with finitely many thin interpolating Blaschke products $b_1, \ldots, b_{n(\varepsilon)}$ such that

$$Z(\hat{u}) \subset \bigcup_{k=0}^{n(\varepsilon)} \Omega_\varepsilon(b_k).$$

If $\{z_j(b_k)\}_{j=0}^\infty$ are the zeros of b_k, let δ, $0 < \delta < 1$, equal

$$\delta = \min_{1 \le k \le n(\varepsilon)} \inf_j (1 - |z_j(b_k)|^2)|b_k'(z_j(b_k))|,$$

and put $\varepsilon = r(\delta)$, where $r(\delta)$ is as in Lemma X.1.4 in Garnett [**1981**]. According to that lemma, there exists a $\lambda(\delta)$, $0 < \lambda(\delta) < 1$, such that

$$\Omega_\varepsilon(b_k) \cap \mathbf{D} \subset \bigcup_{j=0}^\infty \Delta_{j,k}, \qquad k = 1, \ldots, n(\varepsilon),$$

where $\Delta_{j,k}$ is the disc $\{z \in \mathbf{D}\colon \rho(z, z_j(b_k)) < \lambda(\delta)\}$.

We now claim that the number of zeros of u in $\Delta_{j,k}$ (counted with respect to multiplicity, as always) has a bound that is independent of j and k. Once this is established, Lemma 2.1 will tell us that the zeros of u, when counted with respect to multiplicity, form a finite union of thin interpolating sequences.

We argue by contradiction. So, assume that there is a k, $1 \leq k \leq n(\varepsilon)$, and an increasing sequence $\{j_n\}_{n=0}^{\infty}$ of positive integers such that the number of zeros of u in $\Delta_{j_n,k}$ tends to infinity as $n \to \infty$. Then $\hat{u}(m) = 0$ on $\mathcal{P}(m_0)$ for an arbitrary cluster point $m_0 \in \mathcal{M}(H^\infty)\backslash\mathbf{D}$ of the sequence $\{z_{j_n}(b_k)\}_{j=0}^{\infty}$. But then $\bigcup_{k=0}^{n(\varepsilon)} \Omega_\varepsilon(b_k)$ would cover the closure of $\mathcal{P}(m_0)$, which is impossible in view of Proposition 2.3.

From what we have done so far, it follows that $u = bv$, where b is a finite product of thin interpolating Blaschke products, and v is a singular inner function. Our final step is to show that v equals a unimodular constant.

Since u is $\mathbf{E}$-invertible, v is, too. Unless v is a unimodular constant, v is not invertible in H^∞. So, assuming that v is not invertible in H^∞, we find an $m \in \mathcal{F}$ such that $\hat{v}(m) = 0$. But since every positive power v^α, $\alpha > 0$, of v belongs to H^∞, $\hat{v}$ must vanish identically on the closure of the analytic disc $\mathcal{P}(m)$. But we already encountered such a situation in this proof—with u instead of v, though—and in exactly the same fashion, we obtain a contradiction. The proof of the theorem is complete.

If $C_{\mathbf{E}}$ denotes the C^* algebra generated by the $\mathbf{E}$-invertible inner functions and their complex conjugates, Theorem 2.6 has the following corollary.

COROLLARY 2.7. $C_{\mathbf{E}} = \mathbf{B}$.

COROLLARY 2.8. *Let b be a finite product of thin interpolating Blaschke products. If $f \in H^\infty$, $\|f\| < 1$, is such that $\overline{f}b$ equals an H^∞ function g almost everywhere on $\mathbf{T}$, then the function*

$$b_f(z) = (b(z) - f(z))/(1 - g(z)), \qquad z \in \mathbf{D},$$

is a finite product of thin interpolating Blaschke products.

PROOF. The assertion is immediate from Theorem 2.6, since b_f is an $\mathbf{E}$-invertible inner function.

REMARK 2.9. One can show that if b is an n-fold product of thin interpolating Blaschke products times a finite Blaschke product, then b_f is also an n-fold product of thin interpolating Blaschke products times a finite Blaschke product.

Chang and Marshall showed in [**1977**] that for an arbitrary Douglas algebra B, the closed unit ball of $H^\infty \cap C_B$ is the norm-closed convex hull of the Blaschke products in $H^\infty \cap C_B$. Here C_B denotes the C^* algebra generated by the B-invertible inner functions and their complex conjugates. They also showed that $B = H^\infty + C_B$, and that $\mathbf{D}$ is dense in the maximal ideal space of $H^\infty \cap C_B$. In our case $B = \mathbf{E}$, their results specialize to prove the following. Observe that an inner function in $\mathbf{B} \cap H^\infty$ is $\mathbf{E}$-invertible.

COROLLARY 2.10. (a) $\mathbf{A} = \mathbf{B} \cap H^\infty$, *and finite convex combinations of finite products of thin interpolating Blaschke products are dense in the closed unit ball of* $\mathbf{A}$,

(b) $\mathbf{E} = H^\infty + \mathbf{B}$,

(c) $\mathbf{D}$ *is dense in the maximal ideal space* $\mathcal{M}(\mathbf{A})$ *of* $\mathbf{A}$.

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ADDED IN PROOF. After this paper was accepted for publication, I received an unpublished note from Keiji Izuchi entitled *Interpolating and sparse Blaschke products*, in which he also obtains Theorem 2.6.

REFERENCES

L. Carleson, [**1958**] *An interpolation problem for bounded analytic functions*, Amer. J. Math. **80**, 921–930.

S.-Y. Chang and D. E. Marshall, [**1977**] *Some algebras of bounded analytic functions containing the disk algebra*, Banach Spaces of Analytic Functions, Lecture Notes in Math., vol. 604, Springer-Verlag, Berlin, Heidelberg, and New York, pp. 12–20.

J. B. Garnett, [**1981**] *Bounded analytic functions*, Academic Press, New York, 1981.

——, [**1984**] *Some open problems concerning H^∞ and BMO*, Linear and Complex Analysis Problem Book, Lecture Notes in Math., vol. 1043, Springer-Verlag, Berlin, Heidelberg, New York, and Tokyo, pp. 330–332.

K. Hoffman, [**1967**] *Bounded analytic functions and Gleason parts*, Ann. of Math. (2) **86**, 74–111.

P. W. Jones, [**1981**] *Ratios of interpolating Blaschke products*, Pacific J. Math. **95**, 311–321.

C. Sundberg and T. H. Wolff, [**1983**] *Interpolating sequences for QA_B*, Trans. Amer. Math. Soc. **276**, 551–581.

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