Computational Game Theory

Lecture 10

P2/2025

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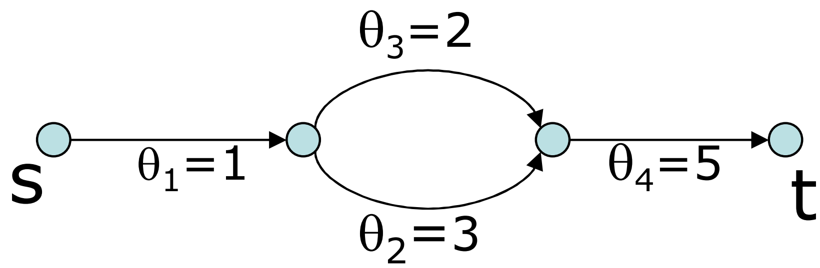
Implementation theory
Mechanism design

- Game theory
  - Set of players *N*
  - Preferences over outcomes
    - Strategic: Action profile *a*
    - Extensive: Terminal histories *z*
  - What is a reasonable solution?
    - Equilibrium concepts

- Implementation theory - Mechanism design
  - Set of players *N*
  - Preference profiles over outcomes
    - Partially unknown
  - Create rules of a game
    - Solution should lead to specific outcome

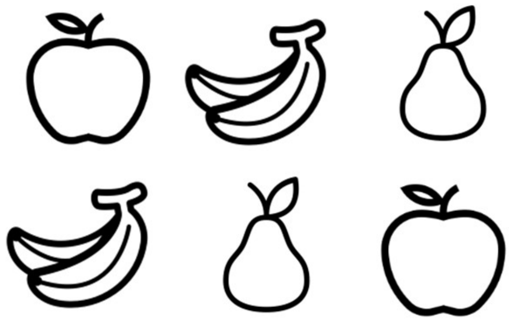
Example: Shortest path



- Communication network digraph
  - Two special nodes: *s* and *t*
- Find least cost path from *s* to *t*
  - Based on costs reported by the players
  - Edges report their cost Θ_i
- How would you find the shortest path?
  - Will players report their real costs?

Detour: Social Choice Theory

- Input
  - Set of individual preferences
- Output
  - Single preference relation
  - Aggregate preference of the "society"
- Is aggregation of individual preferences possible?
  - Formal model
  - Possibility/impossibility results



Example: Voting

- Set of players N={1,2,3}
- Set of candidates A={a,b,c}
- Preference profiles of the players

$$a \succ_1 b \succ_1 c \qquad a \succ_1 b \succ_1 c$$
$$c \succ_2 a \succ_2 b \qquad c \succ_2 a \succ_2 b$$
$$b \succ_3 c \succ_3 a \qquad c \succ_3 b \succ_3 a$$

- We would like to have the following outcomes

$$a, b, c \qquad\qquad c$$

- Is there a mechanism that would lead to this outcome?
  - Majority voting
  - Other examples
    - Borda count voting
      - Assign points to every candidate based on individual preferences
    - Range voting
      - Assign score to every candidate from a range
    - Approval voting
      - Range voting on {0,1}

Example: Condorcet's paradox and Strategic voting

- Set of players N={1,2,3}
- Set of candidates A={a,b,c}
- Majority voting to select winner
- Condorcet paradox: Non-cyclic individual preferences may lead to cyclic societal preferences

$$a \succ_1 b \succ_1 c$$
$$c \succ_2 a \succ_2 b$$
$$b \succ_3 c \succ_3 a$$

$$a \succ b \succ c \succ a$$

- Strategic voting

$$c \succ_3 a \longrightarrow c \succ a \succ b$$

- Can we design a scheme that would avoid strategic voting?

Aggregation of Preferences

- Set of players N, $|N|=n$
- Set of consequences C
- Set L of total orderings on C
- Preference relation $\succ_i \in L$ for every player i
  - Set of preference profiles $P=L^n$

- Welfare function $F:P\rightarrow L$
  - Aggregation of preference relations

- Social choice function $f:P\rightarrow C$
  - Aggregation into a single choice
- Social choice rule $f:P\rightarrow 2^C$
  - Aggregation into a set of choices

Example: Borda Count Voting

- Set of players N={1,2,3,4,5}
- Set of alternatives C={A,B,C,D,E}
- Preference relations

$$A \succ_i B \succ_i C \succ_i D \succ_i E \quad i=1,2,3$$
$$C \succ_i D \succ_i E \succ_i B \succ_i A \quad i=4$$
$$E \succ_i C \succ_i D \succ_i B \succ_i A \quad i=5$$

- Borda count voting results
  - A=17, B=16, C=18, D=13, E=10
  - Winner: C

- Social welfare function
$$F(.) = C \succ A \succ B \succ D \succ E$$
- Social choice rule
$$f(.) = C$$

Welfare function properties

- Unanimity

  $$a \succ_i b \; \forall i \Rightarrow a \succ b \quad \text{for} \succ = F(\succ_1, \ldots, \succ_n)$$

  - Case of complete agreement

- Non-imposition (citizen sovereignty)

  $$\neg \exists a, b \; a \succ b \quad \forall \succ_1, \ldots, \succ_n \in L$$

  - Every ordering can be achieved
    - consequence of unanimity

- Dictatorial
  - Player *i* is a dictator in *F* if

    $$F(\succ_1, \ldots, \succ_n) = \succ_i \quad \forall \succ_1, \ldots, \succ_n \in L$$

  - The aggregate always reflects *i*'s preferences
  - Welfare function *F* is dictatorial if $\exists$ *i* dictator

More properties

- Monotonicity
  - If *a* is promoted by at least one player then *a* should not be worse off in the aggregate ordering

- Independence of irrelevant alternatives

$$a \succ_i b \Leftrightarrow a \succ'_i b \quad \Rightarrow \quad a \succ b \Leftrightarrow a \succ' b$$
$$\succ = F(\succ_1, \ldots, \succ_n)$$
$$\succ' = F(\succ'_1, \ldots, \succ'_n)$$
$$\succ_1, \ldots, \succ_n, \succ'_1, \ldots, \succ'_n \in L$$

  - Preference between *a* and *b* should not depend on
    - The preferences w.r.t. third alternatives
    - The existence of third alternatives

Example: Borda Count Voting

- N={1,2,3,4,5}, C={A,B,C,D,E}
- Preference relations

$$A \succ_i B \succ_i C \succ_i D \succ_i E \quad i=1,2,3$$
$$C \succ_i D \succ_i E \succ_i B \succ_i A \quad i=4$$
$$E \succ_i C \succ_i D \succ_i B \succ_i A \quad i=5$$

- Borda count voting results
  - A=17, B=16, C=18, D=13, E=10
  - Winner: C

$$F(.) = C \succ A \succ B \succ D \succ E$$
$$f(.) = C$$

- New preference relations

$$A \succ_i B \succ_i C \succ_i D \succ_i E \quad i=1,2,3$$
$$C \succ_i B \succ_i E \succ_i D \succ_i A \quad i=4$$
$$E \succ_i C \succ_i B \succ_i D \succ_i A \quad i=5$$

- Borda count voting results
  - A=17, B=19, C=18, D=10, E=11
  - Winner: B

$$F(.) = B \succ C \succ A \succ E \succ D$$
$$f(.) = B$$

- Unanimous, non-dictatorial, monotonic, *non*-IIA

Arrow's impossibility theorem

- For a welfare function over a set of more than two outcomes ($|C| \geq 3$) the three conditions
  - unanimity
  - independence of irrelevant alternatives
  - non-dictatorship

  are inconsistent.

  (assuming that all preference relations are allowed)

- Relax some conditions
  - Limit the set of preference relations
    - Single peaked in one dimension – distance from most preferred (Majority rule)
  - Quasi-transitive welfare function
    - Example: 100sek ~ 101sek, 101sek~102sek, etc but 100sek<200sek
    - Majority rule satisfies the rest

Implementation Problem

- Set of players N, $|N|=n$
- Set of consequences C
- Set L of total orderings on C
- Preference relations $\succ_i \in L$ for every player i
  - Set of preference profiles $P=L^n$
- Set Γ of game forms $G=<N,(A_i),g>$ with consequences in C
  - Set of players N
  - Sets of actions A_i
  - Outcome function $g:A \to C$

Environment (N,C,P,Γ)

- Choice function $f:P \to C$
  - Aggregation into a single choice
- Choice rule $f:P \to 2^C$
  - Aggregation into a set of choices

Example: Divorce

- Set of players N={Husband,Wife}
- Set of outcomes C={Divorce, No divorce}
- Preference relations $\succ_i \in L = \{Divorce \succ No\ divorce, \quad No\ divorce \succ Divorce\}$
- Choice function $f: L^2 \rightarrow C$
- Sets of actions A_i={**G**o to **c**ourt, **N**ot **g**o to **c**ourt}
- Outcome function $g: A \rightarrow C$

	GC	NGC
GC	ND	ND
NGC	ND	ND

Vatican mechanism

	GC	NGC
GC	D	D
NGC	ND	ND

Dictatorial

	GC	NGC
GC	D	ND
NGC	ND	ND

Veto

	GC	NGC
GC	D	ND
NGC	D	ND

Dictatorial

N. Baigent, "Mechanism Design: A quick tour"

Implementation Problem

- Planner is given
  - Environment (N,C,P,Γ)
  - Choice rule $f:P\rightarrow 2^C$
  - Solution concept $S: \Gamma\times P\rightarrow A$

- Choose a game form $G\in\Gamma$ that (fully) **S-implement**s f

$$g(S(G,\succ)) = f(\succ) \quad \forall \succ\in P$$

  - Outcome of G coincides with choice rule for all preference profiles

- Choose a game form $G\in\Gamma$ that **truthfully S-implement**s f
  - $G=<N,(A_i),g>$ with $A_i\subseteq P$
  - and for every $\succ\in P$
    - Reporting the true preference is a solution of the game

$$a^* \in S(G,\succ), \quad where \quad a_i^* = \succ, \forall i \in N$$

    - The outcome corresponding to truthful reporting is in $f(\succ)$

$$g(a^*) \in f(\succ)$$

  - G is called incentive compatible

- Note the difference between the two definitions
  - There might be non-truthful solutions that do not implement f
  - Not every outcome in the choice rule corresponds to a solution

Example: Divorce

- Set of players N={Husband,Wife}
- Set of outcomes C={Divorce, No divorce}
- Preference relations $\succ_i \in L = \{Divorce \succ No\ divorce, \quad No\ divorce \succ Divorce\}$
- Choice function $f{:}L^2 \to C$
- Sets of actions A_i={**G**o to **c**ourt, **N**ot **g**o to **c**ourt}
- Outcome function $g{:}\ A \to C$

	GC	NGC
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Vatican mechanism

	GC	NGC
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Dictatorial

	GC	NGC
GC	D	ND
NGC	ND	ND

Veto

	GC	NGC
GC	D	ND
NGC	D	ND

Dictatorial

N. Baigent, "Mechanism Design: A quick tour"

Implementation in Dominant Strategies

- Consider the strategic game $G=<N,(A_i),(\succ_i)>$.
  The profile $a^*\in A$ is a dominant strategy equilibrium if
  $$(a_{-i},a_i^*)\succ(a_{-i},a_i)\quad\forall a\in A, i\in N$$
  - Best response to every collection of actions of the other players

- Revelation principle for DSE-implementation
  - Let $<N,C,P,\Gamma>$ be an environment in which Γ is the set of strategic game forms.
    If a choice rule $f:P\to 2^C$ is DSE implementable then
    - f is truthfully DSE-implementable
    - there is a strategic game form $G^*=<N,(A_i),g^*>$ in which A_i is the set of all preference relations (instead of profiles) s.t. $\forall\succ\in P$ the action profile $\succ$ is a dominant strategy equilibrium of the strategic game $<G^*,\succ>$ and $g^*(\succ)\in f(\succ)$
  - Truthful DSE implementation is called *strategyproof*
    - Incentive compatible in dominant strategies
    - Not the same as group-strategyproof (collusion)

Example: Divorce

- Set of players: N={Husband,Wife}
- Sets of actions: A_i={**G**o to **c**ourt, **N**ot **g**o to **c**ourt}
- Set of outcomes: C={**D**ivorce, **N**o **d**ivorce}
- Outcome function: $g: A \rightarrow C$

	GC	NGC
GC	D	ND
NGC	ND	ND

Veto

- Choice rule: Divorce if both prefer it

Is this a DSE implementation?
Is this a truthful DSE-implementation?

Gibbard-Satterthwaite theorem

- Let <N,C,P,Γ> be an environment with
  - At least three alternatives $|C| \geq 3$
  - P is the set of all possible preference profiles $P=L^n$
  - Γ is the set of strategic game forms.

  Let $f:P \rightarrow C$ be a choice function that is DSE implementable and

  $$\forall a \in C \quad \exists \succ \in P \quad s.t. \quad f(\succ) = a$$

  then f is dictatorial.

- Proof based on
  - Arrow's impossibility theorem and
  - Revelation principle for DSE implementation

- Get around it
  - Limit the set of preference relations

M.A. Satterthwite, "Strategy-proofness and Arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions", Journal of Economic Theory 10(2), pp. 187-217, 1975

Example: Solomon's dilemma

- Two players $N=\{A,B\}$ (and a baby to be allocated)
- Set of consequences: $C=\{a,b,c\}$ (Give to *A*,Give to *B*,*Cut*)
- Preference relations

$$a \succ_A^\alpha b \succ_A^\alpha c \quad b \succ_B^\alpha c \succ_B^\alpha a$$

$$a \succ_A^\beta c \succ_A^\beta b \quad b \succ_B^\beta a \succ_B^\beta c$$

- Choice function

$$f(\succ^\alpha) = a \quad f(\succ^\beta) = b$$

- Question: Is the true mother A or B?
- Original mechanism
  - $A_i=\{t_\alpha,t_\beta\}$

$$g(t_\alpha,t_\alpha) = a \quad g(t_\beta,t_\beta) = b \quad g(t_\alpha,t_\beta) = g(t_\beta,t_\alpha) = c$$

- Modified mechanism

$$g(t_\beta,t_\beta) = a$$

- Not DSE implementable

Implementation with Money

- Every player has a type

  $$\theta_i \in \Theta_i$$

  - Could correspond to its preference relation
- Player *i*'s preferences described by a scalar

  $$v_i(\theta,c) \quad \theta \in \Theta, c \in C$$

- Planner is allowed to make transfers
  - Levy a fine m_i on player *i*
  - Subsidize player *i* by $-m_i$

- Utility of player *i* is quasi-linear

  $$u_i(\theta,c) = v_i(\theta,c) - m_i$$

Example: Public project

- N players interested in a public project
  - Valuation of player *i* is θ_i
- Set of outcomes $C=\{0,1\}$
- Utility of player *i* is quasi-linear

$$u_i(\theta_i, c) = v_i(\theta_i, c) - m_i$$

- Project should be implemented if $\sum_{i \in N} \theta_i \geq \gamma$

$$f(\theta) = \begin{cases} 0 & \sum_{i \in N} \theta_i < \gamma \\ 1 & \sum_{i \in N} \theta_i \geq \gamma \end{cases}$$

- Is there a mechanism that would truthfully DSE-implement $f(\theta)$?

Desiderata: Budget balance

- Planner should not subsidize the players
  - r(Θ)=cost of implementing c, given Θ (e.g., r(Θ)=0)
- Ex-ante budget balance
  - Expected payments cover costs

$$E_{\theta\in\Theta}\left[\sum_{i\in N} m_i(\theta)\right] = E_{\theta\in\Theta}\left[r(\theta)\right]$$

- Ex-post budget balance
  - Actual payments cover costs

$$\sum_{i\in N} m_i(\theta) = r(\theta)$$

- Weak budget-balance
  - No net payments from the planner to the players

Desiderata: Individual rationality

- Participants are allowed not to participate
  - Obtain expected utility $\tilde{u}_i(\theta_i)$ when not participating

- Ex-ante individual rationality

$$E_{\theta\in\Theta}[u_i(f(\theta),\theta_i)] \geq E_{\theta_i\in\Theta_i}\tilde{u}_i(\theta_i)$$

  - Expected externality mechanism

- Interim individual rationality

$$E_{\theta_{-i}\in\Theta_{-i}}[u_i(f(\theta_{-i},\theta_i),\theta_i)] \geq \tilde{u}_i(\theta_i)$$

  - Groves mechanism

- Ex-post individual rationality

$$u_i(f(\theta),\theta_i) \geq \tilde{u}_i(\theta_i)$$

Example: Public project

- N players interested in a public project
  - Valuation of player *i* is θ_i
- Set of outcomes $C=\{0,1\}$
- Utility of player *i* is quasi-linear

$$u_i(\theta_i, c) = v_i(\theta_i, c) - m_i$$

- Project should be implemented if $\sum_{i \in N} \theta_i \geq \gamma$

$$f(\theta) = \begin{cases} 0 & \sum_{i \in N} \theta_i < \gamma \\ 1 & \sum_{i \in N} \theta_i \geq \gamma \end{cases}$$

- Is there a mechanism that would truthfully DSE-implement $f(\theta)$?

Groves Mechanism

- Set of players: *N*
  - Player *i* has type θ_i
- Set of outcomes: $\{(c,m): c \in C, m \in R^n\}$
- Players' utilities: $u_i(\theta,c) = v_i(\theta,c) - m_i$
- Choice rule (maximizes social welfare):

$$f(\Theta_1,\ldots,\Theta_n) \in \arg\max_{c\in C} \sum_{i\in N} v_i(\Theta_i,c)$$

- Groves mechanism
  - Set of actions $a_i \in R$
  - Choose optimal consequence based on players' actions

$$c^* = \arg\max_{c\in C} \sum_{i\in N} v_i(a_i,c)$$

  - Require payment from player *i*

$$m_i(a) = h_i(a_{-i}) - \sum_{j\neq i} v_j(a_j,c^*)$$

Groves Mechanism

- The Groves mechanism is truthful
  - Player *i* tries to maximize
    $$u_i(a_i) = v_i(a_i, c^*) + \sum_{j \neq i} v_j(a_j, c^*) - h_i(a_{-i})$$
  - Last term is independent of a_i, so equivalently
    $$u_i(a_i) = v_i(a_i, c^*) + \sum_{j \neq i} v_j(a_j, c^*) = \sum_{j \in N} v_j(a_j, c^*)$$
  - But c^* is a maximizer only if $a_i = \Theta_i$
  - Truthfulness is independent of $h_i(a_{-i})$
    - but $h_i(a_{-i})$ influences the amount of payments
- Gibbard-Satterthwaite theorem?
  - Utility functions are quasi-linear

Clarke pivot rule

- Clarke pivot rule
  $$h_i(a_{-i}) = \max_{c \in C} \sum_{j \neq i} v_j(a_j, c)$$
  - as if player *i* did not exist

- The Groves mechanism with Clark pivot payments is weakly budget balanced (makes no positive transfers)

$$m_i(\theta) = h_i(a_{-i}) - \sum_{j \neq i} v_j(\theta_j, c^*) = \max_{b \in C} \sum_{j \neq i} v_j(a_j, b) - \sum_{j \neq i} v_j(\Theta_j, c^*) \geq 0$$

- The Groves mechanism with Clark pivot payments is interim individually rational if $v_i(c) \geq 0 \quad \forall c \in C, i \in N$

$$v_i(c) - m_i(a) = v_i(c) + \sum_{j \neq i} v_j(c) - \sum_{j \neq i} v_j(b) = \sum_{j \in N} v_j(c) - \sum_{j \neq i} v_j(b) \geq \sum_{j \in N} v_j(c) - \sum_{j \in N} v_j(b) \geq 0$$

Example: Public project

Vickrey-Clarke-Groves mechanism

- Introduce player *n*+1 "government" with
  - Cost γ if the project is undertaken
- Each player reports its valuation a_i
- The project is undertaken iff $\sum_{i \in N} a_i \geq \gamma \Rightarrow x(a) = 1$
- Payments made by the players

$$m_i(a) = h_i(a_{-i}) + x(a)\left(\gamma - \sum_{j \in N \setminus \{i\}} a_j\right)$$

$$h_i(a_{-i}) = \max_{c \in C}\left[x(a_{-i})\left(\sum_{j \in N \setminus \{i\}} v_j(a_j, c) - \gamma\right)\right]$$

- Example:
  - Two players: $\theta_i=1$, $\gamma=2$

$$m_1((1,1)) = 0 + 1 * 1 = 1$$

$$m_2((1,1)) = 0 + 1 * 1 = 1$$

  - Three players: $\theta_1, \theta_2=0.9, \theta_3=0.5, \gamma=1.5$

$$m_1(0.9, 0.9, 0.5) = 0 + 1 * 0.1 = 0.1$$

$$m_3(0.9, 0.9, 0.5) = 0.3 + 1 * (-0.3) = 0$$

Pivotal players pay, not budget balanced

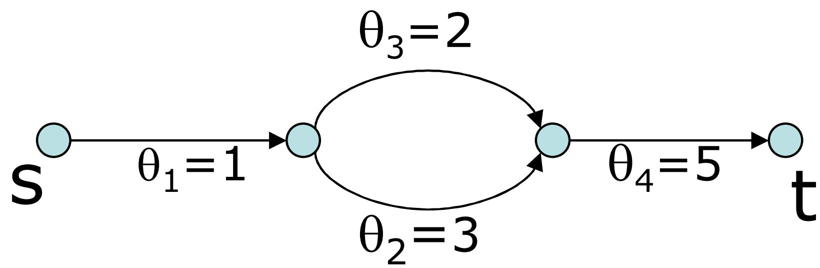
Example: Shortest path

- Communication network digraph
  - Edges are players with cost Θ_i
- Two special nodes: *s* and *t*
- Find cheapest path from *s* to *t* based on costs reported by the players

- Set of players: *N* (edges of the graph)
- Set of outcomes: *C* (all (s,t) paths in the graph)
  - value of player *i* is 0 if not on path, $-\Theta_i$ if on path

- Design a game to find the shortest path
  - Will players report their real costs?
  - Payments are allowed

Example: Shortest path



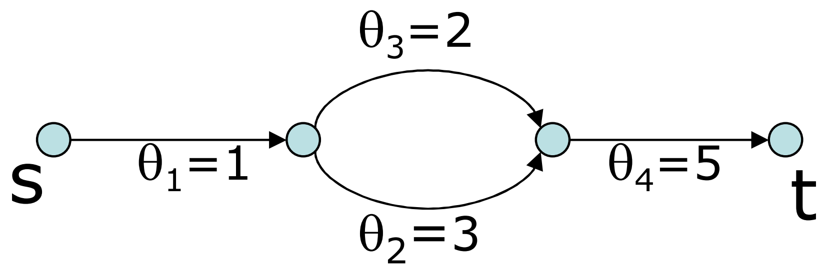
- Shortest path (1,3,4) and d(s,t)=8

- Clarke-Groves mechanism
  - Each edge reports cost: a_i
  - Calculate cheapest path for reported costs: a
  - Payment given to player i
    - $m_i=0$ if i is not on the shortest path
    - $m_i = d(s,t)|_{a_i=\infty} - d(s,t)|_{a_i=0}$ if i is on the shortest path
  - Utility of player i
    - $u_i=0$ if i is not on the shortest path
    - $u_i=m_i-\theta_i$ if i is on the shortest path
- Transfers made by planner

$$m_3 = \theta_2 = 3 \quad \Rightarrow \quad u_3 = \theta_2 - \theta_3 = 1$$
$$u_2 = 0$$

Not budget balanced

Example: Shortest path



- Assume link *i (i=2,3)* reports a'_i instead of θ_i
  - If shortest path is unchanged then irrelevant
  - If link *i* was not on shortest path, but now it is $(a'_i < \theta_i)$

$$u_i(a_{-i}, \theta_i) = 0 \qquad u_i(a_{-i}, a'_i) = a_{-i} - \theta_i < 0$$

  - If link *i* was on shortest path, but now it is not $(a'_i > \theta_i)$

$$u_i(a_{-i,} \theta_i) = a_{-i} - \theta_i > 0 \qquad u_i(a_{-i,} a'_i) = 0$$

Strategyproof

Nash Implementation

- Consider Nash equilibrium solutions of the game
- Revelation principle for Nash implementation
  - Let <N,C,P,Γ> be an environment in which Γ is the set of strategic game forms. If a choice rule $f:P\rightarrow 2^C$ is Nash-implementable then it is truthfully Nash-implementable.
- Note:
  - Players' actions are preference profiles

Example: Divorce

- Set of players: N={Husband,Wife}
- Sets of actions: A_i={**G**o to **c**ourt, **N**ot **g**o to **c**ourt}
- Set of outcomes: C={**D**ivorce, **N**o **d**ivorce}
- Outcome function: $g: A \rightarrow C$

	GC	NGC
GC	D	ND
NGC	ND	ND

Veto

- Choice rule: Divorce if both prefer it

Is this a Nash-implementation?
Is this a truthful Nash-implementation?

Properties of choice rules

- A choice rule *f:P→C* is monotonic if whenever

  $$c \in f(\succ) \text{ and } c \notin f(\succ') \Rightarrow \exists i \in N, b \in C \quad c \succsim_i b \text{ and } b \succ'_i c$$

  - Outcome degrades if it degrades for at least one player
  - Examples
    - Weakly Pareto efficient outcomes
    - Outcomes top ranked by at least one player

- A choice rule *f:P→C* has no veto power if $c \in f(\succ)$ whenever for at least |N|-1 players $c \succ_i y \quad \forall y \in C$

Nash-implementability

- Let $<N,C,P,\Gamma>$ be an environment in which Γ is the set of strategic game forms
  - If a choice rule is Nash-implementable then it is monotonic
  - If $|N|\geq 3$ then any choice rule that is monotonic and has no veto power is Nash-implementable
- Gibbard-Satterthwaite still applies
  - Choice rule *(instead of function)*
  - Limited domain (preference profiles)

E. Maskin, ”The theory of implementation in Nash equilibrium: a survey,” in Social Goals and Social Organizations, Cambridge Univ. Press, pp. 173–204.,1985

E. Muller, M.A. Satterthwite, ” The equivalence of strong positive association and strategy-proofness”, Journal of Economic Theory 14(2), pp. 412-418, 1977

Example: Solomon's dilemma

- Two players *N={A,B}* (and a baby to be allocated)
- Set of consequences: *C={a,b,c}* (Give to *A*,Give to *B*,*Cut*)
- Preference relations

$$a \succ_A^\alpha b \succ_A^\alpha c \quad b \succ_B^\alpha c \succ_B^\alpha a$$

$$a \succ_A^\beta c \succ_A^\beta b \quad b \succ_B^\beta a \succ_B^\beta c$$

- Choice function

$$f(\succ^\alpha) = a \quad f(\succ^\beta) = b$$

- Question: Is the true mother A or B?
- Original mechanism
  - $A_i=\{t_\alpha, t_\beta\}$

$$g(t_\alpha, t_\alpha) = a \quad g(t_\beta, t_\beta) = b \quad g(t_\alpha, t_\beta) = g(t_\beta, t_\alpha) = c$$

- Is it Nash-implementable?
  - Truthfully-Nash implementable?

Not monotonic for "b"…

Example: Solomon's dilemma v2

- Two players $N=\{1,2\}$ (and an object to be allocated)
- Set of consequences: $C=\{(x,m_1,m_2):x\in\{0,1,2\},m_i\in R\}$
  - $x=0$ nobody gets it
  - m_i fine paid by player i
- Quasi-linear preferences (H: true owner, L: false owner)

$$u_i(H)=v_H-m_i \quad u_i(L)=v_L-m_i \quad v_H>v_L$$

- Choice function (superscript: legitimate owner)

$$f(\succ^1)=(1,0,0) \quad f(\succ^2)=(2,0,0)$$

- Nash-implementation
  - $M=(v_H+v_L)/2$
  - $\varepsilon>0$

Assume player 1 is true owner!
What are the NE?

	Mine	Hers	Mine+
Mine	$(0,\varepsilon,\varepsilon)$	(1,0,0)	$(2,\varepsilon,M)$
His	(2,0,0)	$(0,\varepsilon,\varepsilon)$	(0,0,0)
Mine+	$(1,M,\varepsilon)$	(0,0,0)	$(0,2\varepsilon,2\varepsilon)$

Randomized mechanisms

- Randomized mechanism is a distribution over deterministic mechanisms
  - It is the planner that randomizes
- Incentive compatible randomized mechanism
  - Universal sense
    - Each mechanism is incentive compatible
  - Expectation
    - Truth is a dominant strategy in expectation

N. Nisan, A. Ronen, "Algorithmic Mechanism Design", Games and Economic Behavior vol. 35, pp. 166-196, 2001

Topics not covered

- Bayesian-Nash Implementation
  - Revelation principle
  - Expected externality mechanism (dAGVA)
- Subgame Perfect Implementation
  - Extensive games
- Practical implementability of mechanisms
  - Algorithmic complexity
- Distributed mechanisms

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