# Analysis of the Packet Loss Process for Multimedia Traffic ⋆

György Dán, Viktória Fodor and Gunnar Karlsson

KTH, Royal Institute of Technology,
Department of Microelectronics and Information Technology
{gyuri,viktoria,gk}@imit.kth.se

**Abstract.** In the case of multimedia traffic, like VBR video, the average loss probability is not sufficient to investigate the effects of loss on perceived visual quality, but it is difficult to analytically model the queuing behavior for such traffic. It has been shown that in the case of real-time communications, for which small buffers are used for delay reasons, short range dependence dominates the loss process and so the Markov-modulated Poisson process (MMPP) might be a reasonable source model. In this paper we present an exact mathematical model for the loss process of an MMPP+M/D/1/K queue; we validate it via simulations and compare it to other mathematical models, like the MMPP+M/M/1/K and the Gilbert model, and to simulations with real MPEG-4 video traces. We conclude that the other models give accurate results only in a small set of network scenarios, while our model can capture the loss process of VBR video sufficiently well in most cases. This makes it possible to analyze the effects of forward error correction on transmission quality in various network scenarios.

## 1 Introduction

In the case of flow-type multimedia communications, as opposed to elastic traffic, the average packet loss is not the only measure of interest. The burstiness of the loss process, the number of losses in a block of packets, has a great impact both on the user perceived visual quality and on the possible ways of improving it, for example by forward error correction or receiver-based error concealment.

In this paper we present a model to analyze the packet loss process of a bursty source, for example VBR video, multiplexed with background traffic in a single multiplexer with a finite queue and constant packet sizes. We model the bursty source by an L-state Markov-modulated Poisson process (MMPP) while the background traffic is governed by a Poisson process. We validate our model via simulations and compare the results to simulations made with real video traces.

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It is well known that compressed multimedia, like VBR video, exhibits a self-similar nature [1]. Yoshihara et al. use the superposition of 2-state IPPs to model self-similar traffic in [2] and compare the loss probability of the resulting MMPP/D/1/K queue with simulations. They found that the approximation works well under heavy load conditions and gives an upper bound on the packet loss probabilities. Ryu and Elwalid [3] showed that short term correlations have dominant influence on the network performance under realistic scenarios of buffer sizes for real-time traffic. Thus the MMPP may be a practical model to derive approximate results for the queuing behavior of LRD traffic such as real-time VBR video, especially in the case of small buffer sizes. Recently Cao et al. [4] showed that the traffic generated by a large number of sources tends to Poisson as the load increases due to statistical multiplexing and hence justifying the Poisson model for the background traffic.

The paper is organized as follows. Section 2 gives an overview of the previous work on the modeling of the loss process of a single server queue. In Section 3 we describe our model to calculate the loss probability in a block of packets. In Section 4.1 we validate our model by simulations with MMPP and real VBR traffic sources and compare our results to those obtained for exponential service times. In Section 4.2 we use our model to evaluate the FEC performance for VBR video sources and finally in Section 5 we conclude our work.

## 2 Related Work

In [5], Cidon et al. present an exact analysis of the packet loss process in an M/M/1/K queue, that is the probability of losing $j$ packets in a block of $n$ packets, and show that the distribution of losses may be bursty compared to the assumption of independence. They also consider a discrete time system fed with a Bernoulli arrival process describing the behavior of an ATM multiplexer. In [6], Gurewitz et al. present explicit expressions for the above quantities of interest for the M/M/1/K queue. In [7] the multidimensional generating function of the probability of $j$ losses in a block of $n$ packets is obtained and an easy-to-calculate asymptotic result is given under the condition that $n \leq K + j + 1$.

The above models consider exponentially distributed service times. Most multimedia standards however use constant packet sizes for transmission, and real-time multimedia streams complying to the same standard are likely to share the same service class in the network (like for example the diffserv expedited forwarding). So the packet size distributions will differ significantly from exponential and tend to be rather deterministic. In this case the M/M/1/K queuing model overestimates the loss probability and has a different loss process.

Models with general and deterministic service times have been proposed for calculating various measures of queuing performance. In [8], Ait-Hellal et al. present an asymptotic result for a system where the service times and the interarrival times are stationary ergodic, in particular they show that if the block lengths $k$ and redundancy $j$ is large enough, then the frame loss probabilities can be made arbitrarily small. In [9] the conditional loss probability (CLP) is derived for the N*IPP/D/1/K queue and it is shown that the CLP can be orders of magnitude higher than the loss probability.

The performance of an MMPP/G/1/K queue was evaluated in [10] considering the superposition of multimedia and data traffic at a single server queue, and the corresponding delay distribution was given. The waiting time and queue length distribution of the N/G/1/K queue (N stands for the Neuts process) was derived in [11] including the MMPP/G/1/K queue as a special case. Even though the waiting time and queue length distribution of the MMPP/G/1/K queue has been derived, a more thorough analysis of the packet loss process has not yet been done.

Another approach to calculate the probability of $j$ losses in a block of $n$ packets is to use a channel model, for example the Gilbert model, to describe the correlation of losses and derive the probability of losses in a block. This approach is followed in [12] to evaluate the efficiency of FEC for VBR video transmission. Though it is easy to calculate the block loss probabilities using this method, one has to choose the parameters of the loss model. A direct correlation between the source characteristics and the corresponding loss process can therefore not be investigated.

## 3 Model description

We consider a system with fixed size packets having transmission time $D$. Packets arrive to the system from two sources, a Markov-modulated Poisson process (MMPP) and a Poisson process, representing the tagged source and the background traffic respectively. The packets are stored in a buffer that can host up to K packets, and are served according to a FIFO policy. Every $n$ consecutive packets from the tagged source form a block, and we are interested in the probability distribution of the number of lost packets in a block in the steady state of the system. Throughout the calculations we use notations similar the those in [5].

We assume that the sources feeding the system are independent. The MMPP is described by the infinitesimal generator $Q$ with elements $r_{ij}$ and the arrival rate matrix $\Lambda = diag\{\lambda_1,\ldots,\lambda_L\}$, where $\lambda_i$ is the average arrival rate while the underlying Markov chain is in state $i$. The Poisson process modeling the background traffic has average arrival rate $\lambda$. The superposition of the two sources can be described by a single MMPP with arrival rate matrix $\hat{\Lambda} = \Lambda \oplus \lambda = \Lambda + \lambda I = diag\{\hat{\lambda_1},\ldots,\hat{\lambda_L}\}$, and infinitesimal generator $\hat{Q} = Q$, where $\oplus$ is the Kronecker sum. Packets arriving from both sources have the same length, and thus the same transmission time.

Our purpose is to calculate the probability $P(j,n), n \geq 1, 0 \leq j \leq n$ of $j$ losses in a block of $n$ packets. We define the probability $P^a_{x,l}(j,n), 0 \leq x \leq KD, l = 1\ldots L, n \geq$

$1, 0 \leq j \leq n$ as the probability of $j$ losses in a block of $n$ packets, given that the remaining workload in the system is $x$ just before the arrival of the first packet in the block and the first packet of the block is generated in state $l$ of the MMPP. As the first packet in the block is arbitrary,

$$P(j,n) = \sum_{l=1}^{L} \int_{x=0}^{KD} V(x,l) P^a_{x,l}(j,n). \tag{1}$$

An approximation for $V(x,l)$, the workload distribution of the steady state queue as seen by an arriving packet can be given by the steady state distribution of the MMPP/D/1/K queue as outlined in Appendix A.

The probabilities $P^a_{x,l}(j,n)$ can be derived according to the following recursion. The recursion is initiated for n = 1 with the following relations

$$P^a_{x,l}(j,n) = \begin{cases} 1 & j=0 \\ 0 & j \geq 1 \end{cases} \qquad x \leq (K-1)D,$$

$$P^a_{x,l}(j,n) = \begin{cases} 0 & j=0, j \geq 2 \\ 1 & j=1 \end{cases} \qquad (K-1)D < x. \tag{2}$$

Using the notation $p_m = \frac{\lambda_m}{\lambda_m + \lambda}$ and $\overline{p}_m = \frac{\lambda}{\lambda_m + \lambda}$, for $n \geq 2$ the following equations hold.

$$\begin{aligned} P^a_{x,l}(j,n) = \sum_{m=1}^{L} \int_0^{x+D} & f_{lm}(t) \{ p_m P^a_{x+D-t,m}(j,n-1) + \overline{p}_m P^s_{x+D-t,m}(j,n-1) \} dt + \\ & \int_{x+D}^{\infty} f_{lm}(t) \{ p_m P^a_{0,m}(j,n-1) + \overline{p}_m P^s_{0,m}(j,n-1) \} dt \end{aligned}$$

for $0 \leq x \leq (K-1)D$ and

$$\begin{aligned} P^a_{x,l}(j,n) = \sum_{m=1}^{L} \int_0^{x} & f_{lm}(t) \{ p_m P^a_{x-t,m}(j-1,n-1) + \overline{p}_m P^s_{x-t,m}(j-1,n-1) \} dt + \\ & \int_{x}^{\infty} f_{lm}(t) \{ p_m P^a_{0,m}(j-1,n-1) + \overline{p}_m P^s_{0,m}(j-1,n-1) \} dt \end{aligned} \tag{3}$$

for $(K-1)D < x$. $P^s_{x,l}(j,n)$ is given by

$$\begin{aligned} P^s_{x,l}(j,n) = \sum_{m=1}^{L} \int_0^{x+D} & f_{lm}(t) \{ p_m P^a_{x+D-t,m}(j,n) + \overline{p}_m P^s_{x+D-t,m}(j,n) \} dt + \\ & \int_{x+D}^{\infty} f_{lm}(t) \{ p_m P^a_{0,m}(j,n) + \overline{p}_m P^s_{0,m}(j,n) \} dt \quad 0 \leq x \leq (K-1)D \end{aligned}$$

for $0 \leq x \leq (K-1)D$ and

$$\begin{aligned} P^s_{x,l}(j,n) = \sum_{m=1}^{L} \int_0^{x} & f_{lm}(t) \{ p_m P^a_{x-t,m}(j,n) + \overline{p}_m P^s_{x-t,m}(j,n) \} dt + \\ & \int_{x}^{\infty} f_{lm}(t) \{ p_m P^a_{0,m}(j,n) + \overline{p}_m P^s_{0,m}(j,n) \} dt \end{aligned} \tag{4}$$

for $(K-1)D < x$. The probability $P^s_{x,l}(j,n), 0 \leq x \leq KD, l = 1 \ldots L, n \geq 1, 0 \leq j \leq n$ is the probability of $j$ losses in a block of $n$ packets, given that the remaining workload in the system is $x$ just before the arrival of a packet from the background traffic and the MMPP is in state $l$. In (3) and (4) $f_{lm}(t)$ denotes the interarrival-time distribution of the joint arrival process and is given in Appendix B.

### 3.1 Numerical Evaluation

The above set of an infinite number of integral equations can be solved using numerical integration, so that the infinite number of integral equations is substituted by a finite number of linear equations. More precisely the finite integrals in equations (3) and (4) are calculated numerically while the infinite integrals - as the integrand only depends on $t$ in $f_{lm}(t)$ - can be evaluated analytically as shown in Appendix B (31). We introduce $\Delta$ the step size for the numerical integration such that $D = N\Delta$, and so instead of equations (2-4) we can write

$$P^a_{i,l}(j,n) = \begin{cases} 1 & j = 0 \\ 0 & j \geq 1 \end{cases} \qquad i \leq (K-1)N,$$

$$P^a_{i,l}(j,n) = \begin{cases} 0 & j = 0, j \geq 2 \\ 1 & j = 1 \end{cases} \qquad (K-1)N < x. \tag{5}$$

For $n \geq 2$ the following recursive equations hold.

$$P^a_{i,l}(j,n) = \sum_{m=1}^{L} \sum_{\tau=0}^{i+N} f_{lm}(\tau\Delta) c^{i+N}_{\tau} \{ \frac{\lambda_m}{\lambda_m + \lambda} P^a_{i+N-\tau,m}(j,n-1) + \overline{p}_m P^s_{i+N-\tau,m}(j,n-1) \} + \int_{i\Delta+D}^{\infty} f_{lm}(t) \{ p_m P^a_{0,m}(j,n-1) + \overline{p}_m P^s_{0,m}(j,n-1) \} dt \tag{6}$$

for $0 \leq i \leq (K-1)N$ and

$$P^a_{i,l}(j,n) = \sum_{m=1}^{L} \sum_{\tau=0}^{i} f_{lm}(\tau\Delta) c^{i}_{\tau} \{ \frac{\lambda_m}{\lambda_m + \lambda} P^a_{i-\tau,m}(j-1,n-1) + \overline{p}_m P^s_{i-\tau,m}(j-1,n-1) \} + \int_{i\Delta}^{\infty} f_{lm}(t) \{ p_m P^a_{0,m}(j-1,n-1) + \overline{p}_m P^s_{0,m}(j-1,n-1) \} dt \tag{7}$$

for $(K-1)N < i$. $P^s_{i,l}(j,n)$ is given by

$$P^s_{i,l}(j,n) = \sum_{m=1}^{L} \sum_{\tau=0}^{i+N} f_{lm}(\tau\Delta) c^{i+N}_{\tau} \{ \frac{\lambda_m}{\lambda_m + \lambda} P^a_{i+N-\tau,m}(j,n) + \overline{p}_m P^s_{i+N-\tau,m}(j,n) \} + \int_{i\Delta+D}^{\infty} f_{lm}(t) \{ p_m P^a_{0,m}(j,n) + \overline{p}_m P^s_{0,m}(j,n) \} dt \tag{8}$$

for $0 \leq i \leq (K-1)N$ and

$$P^s_{i,l}(j,n) = \sum_{m=1}^{L}\sum_{\tau=0}^{i} f_{lm}(\tau\Delta)c^i_\tau\{\frac{\lambda_m}{\lambda_m+\lambda}P^a_{i-\tau,m}(j,n)+\overline{p}_m P^s_{i-\tau,m}(j,n)\}+ \int_{i\Delta}^{\infty} f_{lm}(t)\{p_m P^a_{0,m}(j,n)+\overline{p}_m P^s_{0,m}(j,n)\}dt \quad (9)$$

for $(K-1)N < i$, where the coefficient $c^i_\tau$ is the $\tau^{th}$ weighting coefficient in the $i$ degree numerical integration. Through carefully choosing the numerical method by increasing $N$, the error induced by the numerical integration decreases at least proportional to $(\frac{1}{N})^5$.

The procedure of computing $P^a_{i,l}(j,n)$ is as follows. First we calculate $P^a_{i,l}(j,1), i = 0\ldots KN$ from the initial conditions (5). Then in iteration $k$ we first calculate $P^s_{i,l}(j,k), k = 1\ldots n-1$ using equations (8) and (9) and the probabilities $P^a_{i,l}(j,k)$, which have been calculated during iteration $k-1$. Then we calculate $P^a_{i,l}(j,k+1)$ using equations (6) and (7).

## 4 Performance Analysis

In this section we show results obtained with the model described in Section 3 for a 3-state MMPP. First we compare our model with simulation results and other mathematical models of loss processes. Then we use our model to assess the performance of FEC in a scenario where a multimedia stream (modeled by a 3-state MMPP) is multiplexed with background traffic (the superposition of a large number of possibly multimedia streams modeled by Poisson arrivals) in a multiplexer with a finite queue. We compare the results to the widely used Gilbert model and simulations with real MPEG traces. The simulations were performed in ns-2.

### 4.1 Model Evaluation

In the following section we compare our model to analytical models and two sets of simulations. We use the simulations to verify the accuracy of our model. In the first set of simulations we simulate an MMPP+M/D/1/K system. Both in our analytical model and in the simulation the MMPP has 3 states, with arrival intensities $\lambda_1 = 116/s, \lambda_2 = 274/s, \lambda_3 = 931/s$ and transition rates $r_{12} = 0.12594, r_{21} = 0.25, r_{23} = 1.97, r_{32} = 2$. The service time in the considered scenarios is 0.5 ms, 0.25 ms, 0.15 ms, 66.84 $\mu$s, 33.42 $\mu$s, and 9.7 $\mu$s. Considering a packet length of 188 bytes, the average bitrate of the MMPP is 540 kbps and the link speeds are 3 Mbps, 6 Mbps, 10 Mbps, 22.5 Mbps, 45 Mbps and 155 Mbps accordingly. For the sake of simplicity we will refer to the link speeds in the following. In the models as well as in the simulations we use the background process to change the average load $\rho$.

To check the accuracy of estimating the loss process of VBR video with an MMPP we have performed simulations with an MPEG-4 encoded video traffic trace multiplexed with a Poisson arrival process at a multiplexer with a finite queue. The MPEG-4 trace is approximately 2700 seconds, thus 67000 frames long, and was used to set the parameters of the 3-state MMPP. The frames of the MPEG stream are packetized to 188 bytes, as given for the transport stream in the MPEG-2 standard [13]. The link capacity in the different scenarios is 3 Mbps, 6 Mbps, 10 Mbps, 22.5 Mbps, 45 Mbps and 155 Mbps. The simulation time in both simulations was between 20 and 40 thousand seconds. The queueing delay is set to 0.5 ms in all cases, resulting in queue lengths from 2 to 60 packets depending on the link speed.

Besides the simulations we compare our model with deterministic service times to a finite queue fed by sources with the same characteristics as above but with exponentially distributed service times with mean values given above. The model for the resulting MMPP+M/M/1/K queue is a generalization of the multiple stream model in [5].

A widely used channel model for the evaluation of multimedia transmission schemes in error prone environments is the Gilbert-model [14], which is a two-state time discrete Markov model. State 0 corresponds to the reception of a packet, while state 1 to the loss of a packet. The distribution of the length of the error bursts $B$ is described by the transition rates $p$ and $q$ ($p+q \leq 1$) as

$$P\{B=i\} = (1-q)^{i-1}q. \tag{10}$$

If $p+q=1$ then the model is called Bernoulli. The loss probability in the Gilbert model is given by $P_{loss} = \frac{p}{p+q}$. The parameters of the Gilbert model can be tuned in different ways, [14] uses a method based on the loss burst distribution. We use the following method based on the loss probabilities in a block of packets $P(j,n)$

$$p = q(1-P(0,1))/P(0,1), \qquad q = \sum_{i-1}^{n} P(i,i)/(\sum_{i=1}^{n} iP(i,i)). \tag{11}$$

In order to compare our model and the Gilbert model we derive formulas to calculate the expected burst length and the probabilities $P(j,n)$ of the loss process based on the Gilbert model. The expected loss burst length can be calculated based on the probabilities $P(j,n)$

$$E[B] = \sum_{n=1}^{\infty} P(B \geq n) = \sum_{n=1}^{\infty} P(n,n), \tag{12}$$

and using the Gilbert model

$$E[B_G] = \sum_{i=1}^{\infty} i(1-q)^{i-1}q = \frac{1}{q}. \tag{13}$$

We use the Gilbert model to calculate the probabilities $P(j,n)$ and compare them to the probabilities given by the MMPP+M/D/1/K model in order to verify if the Gilbert model can assess the loss process of the queue. The probabilities $P(j,n)$ for the Gilbert model can be calculated in a similar way to the one shown in Section 3 and are given by the following equation:

$$P(j,n) = \frac{q}{p+q} P_0(j,n) + \frac{p}{p+q} P_1(j,n), \tag{14}$$

where the probabilities $P_i(j,n)$ are the conditional probabilities of loosing $j$ packets in a block of $n$ packets given that the first packet arrives in state $i, (i \in \{0,1\})$, and thus will or will not be lost. $P_i(j,n)$ are given by the following recursive equations:

$$\begin{aligned} P_0(j,n) &= (1-p)P_0(j,n-1) + pP_1(j,n-1) \\ P_1(j,n) &= qP_0(j-1,n-1) + (1-q)P_1(j-1,n-1). \end{aligned} \tag{15}$$

Figures 1, 2 and 3 show the probability of loosing $j$ packets in a block of 22 packets ($n = 22$) at three different load levels on a 3 Mbps and a 22.5 Mbps link. The figures show that the results of the model with exponentially distributed service times differ significantly. Furthermore the Gilbert model can only capture the loss process at very low loss levels and a high level of statistical multiplexing, and for small block lengths but not for small values of $j$. The figures also show that the loss process in the multiplexer fed by real traces is rather similar to the one given by our model using an MMPP.

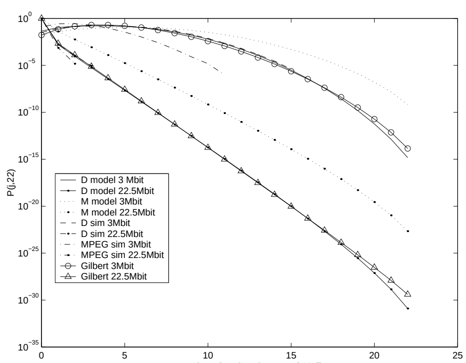


**Fig. 1.** Probability of $j$ packets lost out of 22 on a 3 Mbps and a 22.5 Mpbs link at ρ=0.59

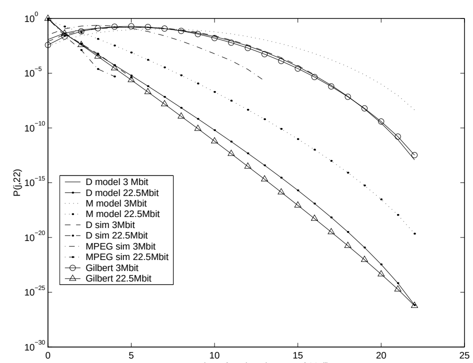


**Fig. 2.** Probability of $j$ packets lost out of 22 on a 3 Mbps and a 22.5 Mpbs link at ρ=0.75

Figure 4 shows the average loss run length as a function of the average load for two different link capacities. The figures shows that the Gilbert model can capture the burst

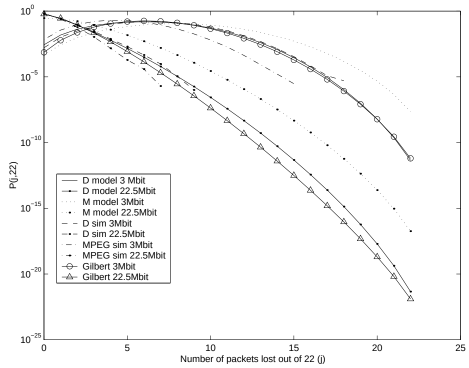


**Fig. 3.** Probability of $j$ packets lost out of 22 on a 3 Mbps and a 22.5 Mpbs link at ρ=0.91

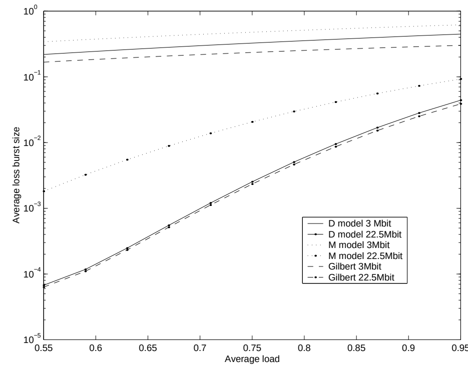


**Fig. 4.** Expected burst length on a 3 Mbps and a 22.5 Mbps link vs the average load.

size distribution if the level of statistical multiplexing is high. The results given by the MMPP+M/M/1/K model differ significantly from the loss process of the simulations.

### 4.2 FEC performance

Forward error correction (FEC) has been proposed to recover from information losses in real-time applications, where the latency introduced by retransmission schemes is not acceptable. FEC increases the redundancy of the transmitted stream and recovers losses based on the redundant information. There are two main directions of FEC design to recover from packet losses. One solution, proposed by the IETF and implemented in Internet audio tools is to add a redundant copy of the original packet to one of the subsequent packets[15]. The other set of solutions, considered in this paper, use block coding schemes based on algebraic coding, e.g. Reed-Solomon coding [16]. The error correcting capability of RS codes with $k$ data packets and $c$ redundant packets is $c$ if data is lost, which is the case if coding is used to recover packet losses.

The performance of an FEC scheme is largely affected by the distribution of the loss process, e.g. the probability of loosing more than $c$ packets in a block of $k+c$ packets. Given the probabilities $P(j,n)$ the uncorrected loss probability for an RS(k,c+k) scheme can be calculated as

$$P^{*}_{loss} = \frac{1}{c+k} \sum_{j=c+1}^{c+k} jP(j,c+k). \tag{16}$$

Figures 5, 6, 7 and 8 show the average loss probability, and the uncorrected loss probabilities for RS(10,11) and RS(20,22) codes for the MMPP+M/M/1/K, and the MMPP+M/D/1/K models and the simulation of the MMPP+M/D/1/K system for different link speeds. Both RS codes used introduce an overhead of 10%, so that the sources, if they decide to use FEC and want to keep their bitrate unchanged, have to decrease the amount of useful information sent by 10%. As compensation they expect lower probability of incorrigible loss. In the case of VBR compressed video the decrease of the loss probability can compensate for a certain reduction in the source rate, and thus one can achieve better perceived visual quality.

We have chosen two different block lengths to illustrate the effect of the block length on the error correcting capability of the RS codes. The figures show that our model slightly underestimates the probability of uncorrected packet loss, which is due to the approximation of the workload distribution, but it models the packet loss distribution accurately under all load and loss conditions. Comparing the losses of streams using RS(10,11) and RS(20,22) it can be seen that although in general increasing the block size (c+k) results in much more efficient block codes, at high loss rates (e.g. higher than the overhead introduced by the FEC code) the contrary is true, shorter block codes are more efficient.

Figures 9, 10, 11 and 12 show the average loss probability, and the probability of uncorrected loss for RS(10,11) and RS(20,22) codes for different link speeds for the MMPP+M/D/1/K model, simulations with a real MPEG trace and the Gilbert model. Comparing the loss probabilities of the simulations with real MPEG traces and the results of the model we can see that as long as the multimedia source has a significant influence on the queuing behavior, the model overestimates the loss probabilities. However the gain of using different FEC schemes is similar both for the model and the simulation with the real traces. In figure 11 the tagged source achieves the loss probability of $10^{-4}$ at $\rho = 0.77$ by using RS(22,20) according to our model. Without FEC the same loss level is experienced for $\rho = 0.6$. From a different perspective, by using RS(22,20) the source can decrease the probability of uncorrected loss by 1 to 2 orders of magnitude at reasonable loss levels. In this scenario the perceived visual quality improves, if the decrease of two orders of magnitude in the loss probability compensates for 10% decrease of the effective bandwith. Similar behavior can be seen in the case of the MPEG stream. Thus, the model captures the loss process accurately.

Comparing the Gilbert model to the results of our model we can conclude that at low levels of statistical multiplexing and long block sizes ($n = 22$) the results of the Gilbert model are very inaccurate. It can only capture the loss process at very low loss levels and nearly independent losses (Fig. 12). It has one drawback: its parameters have to be set correctly. Our model can be used to set the parameters of the Gilbert model as described before. In other scenarios our model is the only model that can capture the loss process of a bursty source with constant packet sizes.

## 5 Conclusion

In this paper we presented a model to evaluate the probability of loosing $j$ packets in a block of $n$ packets in an MMPP+M/D/1/K queue. Via simulations we have shown that

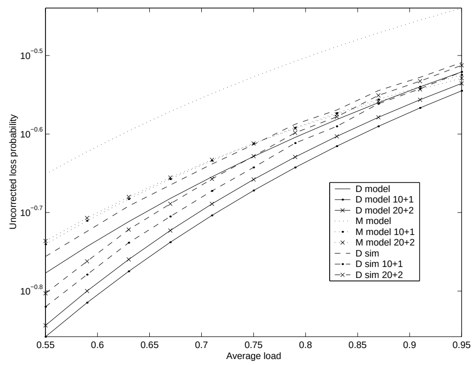


**Fig. 5.** Probability of uncorrected packet loss vs. average load on a 3 Mbps link,K=2

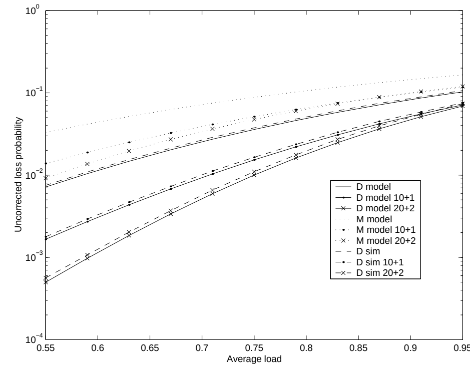


**Fig. 6.** Probability of uncorrected packet loss vs. average load on a 10 Mbps link, K=5

the model is accurate, and in the case of a high level of statistical multiplexing it can be used to model the behavior of VBR coded video streams. We have compared our model with an MMPP+M/M/1/K model and showed that the results differ significantly, both in the average packet loss, as well as in the packet loss process. Thus our model helps in getting further insight into the packet loss process of queues with deterministic service time and bursty input traffic. Using the model and simulations we have shown that the use of FEC can significantly decrease the probability of uncorrected packet loss at reasonable loss levels, and can increase perceived quality. We have compared our model with the Gilbert model, and came to the conclusion that the Gilbert model can only capture the loss process if the level of statistical multiplexing is very high, and the loss probability low. In this case the Gilbert model is easy to use to evaluate for example FEC performance. Our model is still needed to determine the parameters of the Gilbert model so that it can emulate a realistic loss process.

## 6 Appendices

## A Workload distribution

The Laplace transform of the virtual waiting time distribution of the MMPP/G/1/K queue is given in [11]. Following the arguments presented there one can derive the Laplace transform of the workload distribution

$$V(s) = \frac{1}{[\mu - \overline{\pi_0}(\hat{Q}-\hat{\Lambda})^{-1}\overline{e}]}\{\overline{\pi_0}[-s(sI-\hat{\Lambda}+\hat{Q})^{-1}(\hat{Q}-\hat{\Lambda})^{-1}$$

$$+\sum_{k=1}^{N-1} T[\hat{\Lambda}S]^{k-1}(sI+\hat{Q})S[G^*(s)]^k - [G^*(s)]^{N-1}\sum_{k=0}^{N-1}\overline{\pi_k}T[\hat{\Lambda}S]^{N-k-1}]]$$

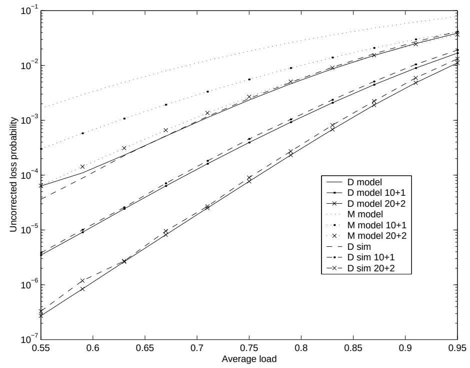


**Fig. 7.** Probability of uncorrected packet loss vs. average load on a 22.5 Mbps link,K=10

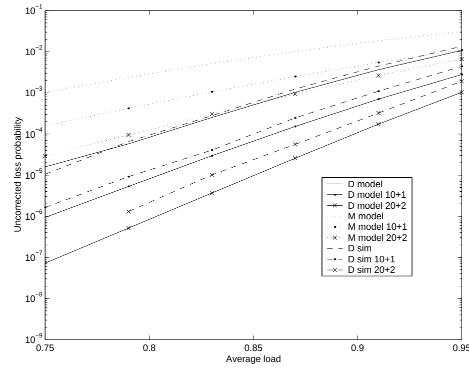


**Fig. 8.** Probability of uncorrected packet loss vs. average load on a 45 Mbps link, K=20

$$+[G^*(s)]^{N-1}\sum_{k=1}^{N-1}\overline{\pi_k}\sum_{j=N-k}^{\infty}\left[\sum_{k=0}^{j}A_kT[\hat{\Lambda}S]^{n-k}-G^*(s)T[\hat{\Lambda}S]^{n}\right]\},$$

where $S=(\hat{\Lambda}-sI-\hat{Q})^{-1}$, $T=(sI-\hat{\Lambda}+\hat{Q})^{-1}$, $G^*(s)$ is the Laplace transform of the service time distribution. $A_k$ is an $L\times L$ matrix whose $(l,m)$th element denotes the conditional probability of the MMPP reaching phase $m$ and having $k$ arrivals during a service time, starting from phase $l$. Instead of calculating the inverse Laplace transform of the above expression we use an approximation based on the steady state distribution of the queue length to calculate the workload distribution of the steady state system. A way to calculate the steady state queue length distribution, $\pi(i,l)$ $(0\le i\le K, 1\le l\le L)$, of an MMPP/G/1/K queue is described in [11]. The queue length distribution as seen by an arriving packet, $\Pi(i,l)$ $(0\le i\le K, 1\le l\le L)$, can be calculated from the steady state queue length distribution as

$$\Pi(i,l)=\frac{\pi(i,l)\lambda_l}{\sum_{l=1}^{L}\lambda_l\sum_{i=0}^{K}\pi(i,l)}. \tag{17}$$

Given the queue length distribution as seen by an arriving packet $\Pi(i,l)$, $0\le i\le K, 1\le l\le L$ the workload distribution $V(i,l)$ is approximated by

$$V(i,l)=\begin{cases}\Pi(0,l) & i=0\\ \Pi(\lceil i/N\rceil,l)/N & 0<i<=NK\end{cases}, \tag{18}$$

where $N$ is defined in Section 3.1.

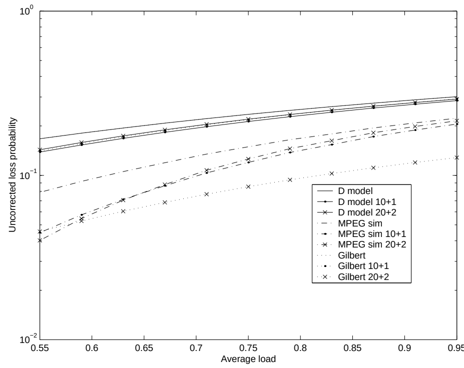


**Fig. 9.** Probability of uncorrected packet loss vs. average load on a 3 Mbps link,K=2

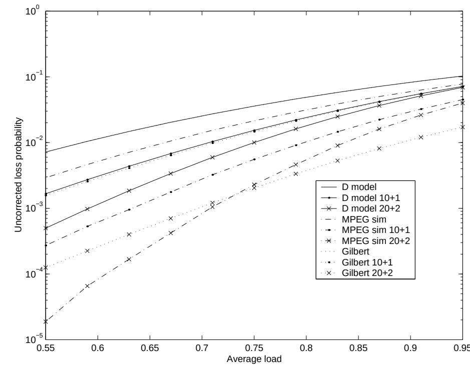


**Fig. 10.** Probability of uncorrected packet loss vs. average load on a 10 Mbps link, K=5

## B Interarrival-time distribution

The probability $f_{lm}(t)$ denotes the joint conditional probability that the time between two arrivals from the joint arrival stream is $X_k = t$ and the state of the MMPP at the moment of the arrival is $J_{k+1} = m$ given that at the time of the last arrival the MMPP was in state $J_k = l$. A straightforward way to calculate $f_{lm}(t)$ is using the equality

$$f_{lm}(t) = [e^{(\hat{Q}-\hat{\Lambda})t}\hat{\Lambda}]_{l,m}. \tag{19}$$

Instead we may calculate $f_{lm}(t)$ using the Laplace transform

$$f^*(s) = \mathcal{L}\left\{e^{(\hat{Q}-\hat{\Lambda})t}\hat{\Lambda}\right\} = [sI - \hat{Q} + \hat{\Lambda}]^{-1}\hat{\Lambda}. \tag{20}$$

By performing the matrix inversion and multiplication followed by the inverse Laplace transform we get that the interrarival time distribution is the weighted sum of exponentially distributed random variables

$$f_{lm}(t) = \sum_{i=1}^{L} A_{i,lm} e^{\alpha_i t}, \tag{21}$$

where $\alpha_i$ is the $i^{th}$ root of $det[sI - \hat{Q} + \hat{\Lambda}]$ and can be calculated analytically for $L \leq 4$.

In the following we show how the calculation proceeds for $L = 3$. To calculate the roots $\alpha_i$ we rewrite $t(s) = det[sI - \hat{Q} + \hat{\Lambda}]$ in (20) to the form

$$t(s) = a_3 s^3 + a_2 s^2 + a_1 s + a_0, \tag{22}$$

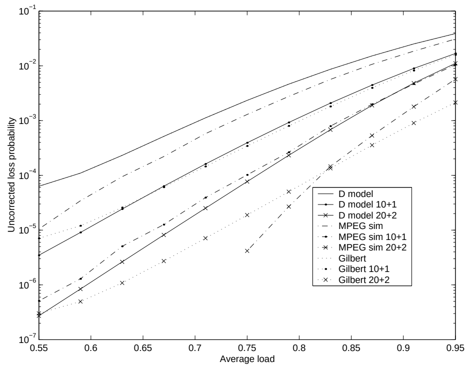


**Fig. 11.** Probability of uncorrected packet loss vs. average load on a 22.5 Mbps link,K=10

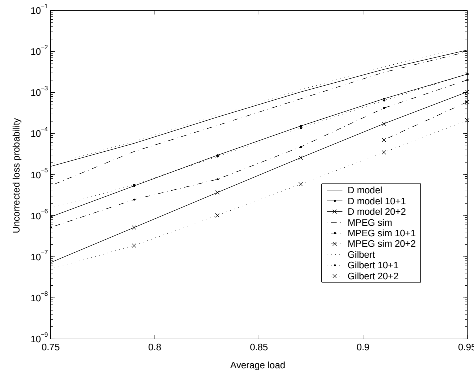


**Fig. 12.** Probability of uncorrected packet loss vs. average load on a 45 Mbps link, K=20

where

$$\begin{aligned}
a_3 &= 1\\
a_2 &= r_{12}+r_{13}+\lambda_1+r_{31}+r_{32}+\lambda_3+r_{21}+r_{23}+\lambda_2\\
a_1 &= \lambda_2*r_{31}+r_{13}*\lambda_3-r_{21}^2+r_{13}*r_{23}+r_{13}*r_{32}+r_{13}*r_{21}+r_{21}*r_{31}+\\
&\quad r_{21}*r_{32}+r_{21}*\lambda_3+r_{23}*r_{31}+r_{23}*\lambda_3+\lambda_2*r_{32}+\lambda_2*\lambda_3+r_{12}*r_{31}+\\
&\quad r_{12}*r_{32}+r_{12}*\lambda_3+r_{12}*r_{21}+r_{12}*r_{23}+r_{12}*\lambda_2+r_{12}*\lambda_2+r_{13}*\lambda_2+\\
&\quad \lambda_1*r_{31}+\lambda_1*r_{32}+\lambda_1*\lambda_3+\lambda_1*r_{21}+\lambda_1*r_{23}+\lambda_1*\lambda_2\\
a_0 &= r_{12}*\lambda_2*r_{32}+r_{12}*\lambda_2*\lambda_3+r_{13}*r_{21}*\lambda_3-r_{21}^2*r_{31}-r_{21}^2*r_{32}-r_{21}^2*\lambda_3+\\
&\quad r_{12}*r_{21}*r_{31}+r_{12}*r_{21}*r_{32}+r_{12}*r_{21}*\lambda_3+r_{12}*r_{23}*r_{31}+r_{12}*r_{23}*\lambda_3+\\
&\quad r_{13}*r_{23}*\lambda_3+r_{13}*\lambda_2*r_{32}+r_{13}*\lambda_2*\lambda_3+\lambda_1*r_{21}*r_{31}+\lambda_1*r_{21}*r_{32}+\\
&\quad \lambda_1*r_{21}*\lambda_3+\lambda_1*r_{23}*r_{31}+\lambda_1*r_{23}*\lambda_3+\lambda_1*\lambda_2*r_{31}+\\
&\quad \lambda_1*\lambda_2*r_{32}+\lambda_1*\lambda_2*\lambda_3-r_{31}*r_{21}*r_{23}. \qquad (23)
\end{aligned}$$

We denote the roots of (22) with $\alpha_i, i=1,2,3$. Knowing $\alpha_i$ we can perform the partial fraction decomposition of (20) with respect to $s$

$$f^{l,m*}(s)=\sum_{i=1}^{L}\frac{A_i^{lm}}{s-\alpha_i}, \qquad (24)$$

where $A_i^{lm}$ can be calculated as

$$\begin{aligned}
A_1^{lm} &= (c_2^{lm}*\alpha_1^2-c_1^{lm}*\alpha_1+c_0^{lm})/(\alpha_2-\alpha_1)/(\alpha_3-\alpha_1)\\
A_2^{lm} &= (c_2^{lm}*\alpha_2^2-c_1^{lm}*\alpha_2+c_0^{lm})/(\alpha_1-\alpha_2)/(\alpha_3-\alpha_2)
\end{aligned}$$

$$A_3^{lm} = (c_2^{lm} * \alpha_3^2 - c_1^{lm} * \alpha_3 + c_0^{lm})/(\alpha_2 - \alpha_3)/(\alpha_1 - \alpha_3). \tag{25}$$

The coefficients $c_2^{lm}, c_1^{lm}, c_0^{lm}$ are the following:

$$\begin{aligned}
c_2^{11} &= \lambda_1 \\
c_1^{11} &= \lambda_1(r_{31} + r_{32} + \lambda_3 + r_{21} + r_{23} + \lambda_2) \\
c_0^{11} &= \lambda_1(r_{21}r_{31} + r_{21}r_{32} + r_{23}r_{31} + r_{21}\lambda_3 + r_{23}\lambda_3 + \lambda_2 r_{31} + \lambda_2 r_{32} r_{31} + r3 + \lambda_2\lambda_3) \\
c_2^{12} &= 0 \\
c_1^{12} &= \lambda_2 r_{21} \\
c_0^{12} &= \lambda_2(r_{21}r_{31} + r_{21}r_{32} + r1\lambda_3 + r_{13}r_{32}) \\
c_2^{13} &= 0 \\
c_1^{13} &= \lambda_3 r_{13} \\
c_0^{13} &= \lambda_3(r_{21}r_{23} + r_{13}r_{21} + r_{13}r_{23} + r_{13}\lambda_2) \\
c_2^{21} &= 0 \\
c_1^{21} &= \lambda_1 r_{21} \\
c_0^{21} &= \lambda_1(r_{21}r_{31} + r_{21}r_{32} + r_{21}\lambda_3 + r_{23}r_{31}) \\
c_2^{22} &= \lambda_2 \\
c_1^{22} &= \lambda_2(r_{31} + r_{12} + r_{13} + \lambda_1 + r_{32} + \lambda_3) \\
c_0^{22} &= \lambda_2(r_{12}r_{31} + r_{12}r_{32} + r_{12}\lambda_3 + r_{13}r_{32} + r_{13}\lambda_3 + \lambda_1 r_{31} + \lambda_1 r3 + \lambda_1\lambda_3) \\
c_2^{23} &= 0 \\
c_1^{23} &= \lambda_3 r_{23} \\
c_0^{23} &= \lambda_3(r_{23}r_{12} + r_{13}r_{23} + r_{23}\lambda_1 + r_{13}r_{21}) \\
c_2^{31} &= 0 \\
c_1^{31} &= \lambda_1 r_{31} \\
c_0^{31} &= \lambda_1(r_{21}r_{32} + r_{21}r_{31} + r_{23}r_{31} + \lambda_2 r_{31}) \\
c_2^{32} &= 0 \\
c_1^{32} &= \lambda_2 r_{32} \\
c_0^{32} &= \lambda_2(r_{32}r_{12} + r_{13}r_{32} + r_{32}\lambda_1 + r_{21}r_{31}) \\
c_2^{33} &= \lambda_3 \\
c_1^{33} &= \lambda_3(r_{13} + r_{12} + \lambda_1 + r_{21} + r_{23} + \lambda_2) \\
c_0^{33} &= \lambda_3(r_{13}r_{23} + r_{13}r_{21} - r_{21}^2 + r_{12}r_{21} + r_{12}r_{23} + r_{13}\lambda_2 + r_{12}\lambda_2 + \lambda_1 r_{21} + \lambda_1 r_{23} + \lambda_1\lambda_2).
\end{aligned} \tag{26}$$

Thus the Laplace transform of the conditional probability $f^{l,m}(t)$ has the form

$$f^{l,m}(s) = \sum_{i=1}^{L} A_i^{lm} \frac{1}{s - \alpha_i} \tag{27}$$

and $f^{l,m}(t)$ is

$$f^{l,m}(t) = \sum_{i=1}^{L} A_i^{lm} e^{\alpha_i t}. \tag{28}$$

By the definition of $f_{lm}(t)$

$$\sum_{m=1}^{L} f_{lm}(t) = P(X_{k+1} = t | J_k = l), \tag{29}$$

and

$$\int_0^\infty f_{lm}(t) = [(\hat{\Lambda} - \hat{Q})^{-1} \hat{\Lambda}]_{l,m}, \tag{30}$$

which accounts for the evolution of the underlying Markov chain. Based on (21) the infinite integrals in equations (6),(7),(8) and (9) can be calculated as

$$\int_x^\infty f_{lm}(t) dt = -\sum_{i=1}^{L} \frac{A_{i,lm}}{\alpha_i} e^{\alpha_i x}. \tag{31}$$

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