On Emptying a Wireless Network with Minimum-Energy Under Age Constraints

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***Abstract*—Timely information delivery and low energy consumption are of critical importance for a variety of wireless applications. In this paper, we address the link scheduling problem of emptying a network with minimum energy, subject to a maximum peak age constraint for each information source. We formulate the minimum-energy scheduling with age constraints (MESA) problem in its general form and prove that it is NP-hard. We derive fundamental results, such as lower and upper bounds of the minimum energy consumption, and the conditions when a TDMA schedule is optimal. We propose the deadline-first-with-revision (DFR) algorithm for constructing a scheduling solution, and evaluate its performance under two rate functions. Numerical results show that DFR achieves a significant energy reduction compared to a minimum age scheduling solution.**

I. Introduction

The evolution of wireless communications has enabled a variety of applications that provide timely information for people and for connected devices. In the upcoming 5G, more real-time services, such as high-definition video streaming and two-way gaming, are expected to be deployed or enhanced [1]. Meanwhile, the exponentially increasing data traffic and the massive network infrastructure have resulted in high energy consumption, which has become a major concern for the sustainable development of wireless communications [2].

To reduce the negative environmental effect, and meanwhile meet the expectation of users, delivering information in a timely fashion with low energy consumption is of critical importance. Motivated by this, we study the link scheduling problem with respect to energy-efficiency and freshness of information. The latter is measured by the metric of age of information, or simply, age, which is defined as the time elapsed since the most recently received message was generated [3].

Link scheduling is a key aspect of access coordination in a wireless system with a shared medium. It aims to answer the classic question of which links should transmit together and for how long, so as to deliver the data from the transmitters to the respective destinations. Usually, the selection of a schedule is driven by an optimality goal, such as maximum throughput or minimum transmission time. In this paper, the objective of the scheduling problem is to minimize the total transmission energy, subject to the constraint that the information must be delivered before it becomes stale.

As a fundamental problem of wireless communications, link scheduling has a rich literature (see the surveys [4]–[6]). A number of problem types have been investigated. These scheduling problem types differ from each other in many ways, including the details of the network model, performance criteria, and system constraints. In [7], the scheduling problem with the objective of maximizing the stable throughput region for arbitrary link activation constraints is studied. In [8] and [9] scheduling is addressed together with routing in multi-hop networks, reaching structural results like the back-pressure and the drift-plus-penalty algorithms. In [10], the authors address the minimum-time scheduling problem, where the task is to deliver the backlogged queues with given initial size at the transmitters in minimum time. This scheduling problem is in the category of "empty-the-network", which targets to identify an ultimate capability of a network [4]. This kind of scheduling problem is also of relevance to dynamic systems, where scheduling occurs at discrete time instances, each dealing with optimally emptying the currently backlogged data. In this paper, we adopt this problem setup, that is, we focus on the scheduling problem that empties the currently queued data at the transmitters, and leave the new arrivals to be considered in the next schedule cycle.

Recently, optimizing link scheduling to improve the freshness of information has attracted a growing interest. Link sceduling with the objective of minimizing the total age or the maximum peak age are studied in e.g., [11]–[13]. Peak age refers to a maximal point in age evolution, achieved immediately before receiving an update [14]. In [15], joint assignment and scheduling for minimizing age of correlated information is addressed.

For link scheduling regarding energy-efficiency, [16] addresses the scheduling problem of emptying the network using minimum transmission energy, subject to the constraint that all data must be delivered within a given time. The authors show that time division multiple access (TDMA) is the optimal solution when the time constraint approaches infinity. Our scheduling problem differs significantly from the above, because the link transmissions are constrained by the metric of age, which is in general different from the delay or the transmission time [17].

In this paper, we address the minimum-energy scheduling with age constraints (MESA) problem. Our main contributions are as follows. First, we formulate the MESA in its general form such that the mathematical formulation applies to a wide range of wireless systems. Then we prove its NP-hardness, and derive fundamental results including when a TDMA schedule is optimal and in which order the links should be activated, as well as bounds of energy consumption. Finally, we propose a heuristic algorithm to compute a near-optimal schedule and perform numerical study for the MESA with two commonly used rate functions.

The rest of the paper is organized as follows. In Section II, we define the system model and formulate the optimization problem, followed by the complexity analysis in Section III and the structural results in Section IV. In Section V we develop the optimization algorithm, and we present the numerical results in Section VI. In Section VII, we provide final concluding remarks.

II. System Model and Problem Formulation

We consider a wireless network that consists of $\mathcal{N} = \{1, 2, \dots, N\}$ transmitter-receiver pairs, or links, sharing a common channel. We denote by TX_n and RX_n the transmitter and the receiver of link n, respectively. Each link is associated with an information source, denoted by S_n, $n \in \mathcal{N}$. The information sources generate update packets from N independent ongoing processes. The packets carry the time stamp of the information sample, and are queued under the discipline of first-come-first-served (FCFS) at the associated transmitter before being delivered to the respective destination.

Time is slotted and is divided into subsequent scheduling cycles. We denote by a_{n0} the the initial age of source S_n at the beginning of a schedule cycle, denoted by t_0. Furthermore, we denote by U_{ni} the i^{th} packet from S_n, and by τ_{ni} its time stamp. We consider that there is a maximum peak age requirement A_n for each source S_n, $\forall n \in \mathcal{N}$, determined by the services in question.

We follow the physical model, a.k.a. the signal-to-interference-and-noise ratio (SINR) model, to determine if a subset of links can transmit simultaneously and at what rate. Let $\mathcal{H}$ denote the set of all subsets of $\mathcal{N}$, excluding the empty set. Clearly, $|\mathcal{H}| = 2^N - 1$. We use the term *group* to refer to a subset of links. The transmission rate of link n in group g is determined by the value of $\text{SINR}(n, g)$, calculated as follows.

$$\text{SINR}(n, g) = \frac{P_n G_{nn}}{\sum_{m \in g, m \neq n} P_m G_{mn} + \sigma_n^2}, \tag{1}$$

where P_n is the transmit power of TX_n, which is assumed to be fixed in this study, G_{mn} is the channel gain between TX_m and RX_n, incorporating the effects of path loss, shadowing and fading. During a schedule interval, the channel gains are constant. The noise variance at RX_n is σ_n^2.

Let us denote by $v(n, g)$ the (integer) number of packets of link n being transmitted in a time slot if group g is active.

$$v(n, g) = f(\text{SINR}(n, g)), \tag{2}$$

where $f(.)$ is a non-decreasing function determined by the network configuration and the modulation scheme.

Let us denote by t_j the end of the jth time slot, counting from the starting point t_0. The age of source S_n at t_j is defined as follows.

$$a_{nj} = \begin{cases} t_j - \tau_{ni} & \text{link } n \text{ is active at } [t_{j-1}, t_j) \text{ and among} \\ & \text{all packets from } S_n \text{ deliverd in this} \\ & \text{time slot, } U_{ni} \text{ is the newest one;} \\ a_{n,j-1} + 1 & \text{link } n \text{ is not active at } [t_{j-1}, t_j). \end{cases} \tag{3}$$

For ease of exposition, let us define the binary variable

$$T_{gj} = \begin{cases} 1 & \text{link group } g \text{ is active at } [t_{j-1}, t_j); \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

If there is any link transmitting at the j^{th} time slot, we set the binary variable $\iota_j = 1$. Otherwise, $\iota_j = 0$. Then we have $\iota_j = \sum_{g \in \mathcal{H}} T_{gj}$.

Before a schedule is determined, its length is unknown. However, since it does not make sense to have a vacant slot during a schedule, at least one packet is delivered in a time slot. The total schedule length is therefore at most $\sum_{n \in \mathcal{N}} d_n$. Defining $\mathcal{J} = \{1, 2, \dots, \sum_{n \in \mathcal{N}} d_n\}$, we formulate the MESA as follows.

$$\underset{\{T_{gj},\ \iota_j \in \{0,1\}\}}{\text{minimize}} \quad \sum_{g \in \mathcal{H},\ j \in \mathcal{J}} P_g T_{gj} \tag{5a}$$

$$\text{subject to} \quad (1),\ (2),\ \text{and } (3),$$

$$P_g = \sum_{n \in g} P_n \qquad \forall g \in \mathcal{H}, \tag{5b}$$

$$\sum_{g \in \mathcal{H},\ j \in \mathcal{J}} v(n, g) T_{gj} \geq d_n \qquad \forall n \in \mathcal{N}, \tag{5c}$$

$$\sum_{g \in \mathcal{H}} T_{gj} = \iota_j \qquad \forall j \in \mathcal{J}, \tag{5d}$$

$$\iota_{j-1} \geq \iota_j \qquad \forall j \in \mathcal{J} \setminus \{1\}, \tag{5e}$$

$$a_{nj} \iota_j \leq A_n \qquad \forall n \in \mathcal{N}, \forall j \in \mathcal{J}. \tag{5f}$$

The objective function (5a) is to compute the total transmission energy in a scheduling solution, and P_g is the transmit power of group g, calculated by (5b). The constraints in (5c) ensure that all packets are delivered by the end of a schedule. The equalities in (5d) are derived by the definition of ι_j, and they also guarantee that at most one group is active in a time slot. The constraint set (5e) is introduced to exclude the case where no packet is delivered in a time slot during a schedule. Putting together (5d) and (5e), one can verify that $\iota_j = 0$ indicates that all queued packets are delivered and the calculated schedule ends before t_j. By definition, the maximum peak age is also the maximum age during the schedule. Hence we have the maximum age constraints defined in (5f). Note that, after all packets are emptied, (5f) does not take effect because $\iota_j = 0$. Therefore, in (5) the schedule length $|\mathcal{J}|$ can be overestimated without loss of the optimality. We summarize the key notations in Table I.

III. Complexity Consideration

Theorem 1. *The MESA, as defined in* (5)*, is NP-hard.*

TABLE I

Notation	Description
$\mathcal{N}$	The set of links
S_n	The source of link n
TX_n	The transmitter of link n
RX_n	The receiver of link n
d_n	The number of packets queued at TX_n
g	A subset of links
$\mathcal{H}$	The set of all groups
P_n	The transmit power of link n
P_g	The transmit power of group g
$v(n,g)$	The number of packets of link n being transmitted in a time slot if group g is active
t_0	The initial time
t_j	The time corresponds to the end of the j^{th} time slot
U_{ni}	The i^{th} packet of link n
τ_{ni}	The time stamp carried by U_{ni}
a_{n0}	The initial age of S_n at t_0
a_{nj}	The age of S_n at t_j
A_n	The maximum peak age requirement of S_n

Proof: We consider a special case of the MESA where the transmission rate is binary. That is,

$$v(n,g) = \begin{cases} 1 & \text{if } \mathrm{SINR}(n,g) \geq \gamma, \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Here γ is a threshold given by the network in question. Then for any feasible scheduling solution of (5), the energy consumption is constant. The proof is as follows. For any link $n \in \mathcal{N}$, to empty its demand d_n, the energy to be consumed can be written as $\frac{P_n d_n}{\bar{v}_n}$, where $\bar{v}_n$ is the average rate when link n is active. For this case, since link n always transmits at rate one (if active), we have $\bar{v}_n = 1$, $\forall n \in \mathcal{N}$. Therefore, for any feasible schedule, the total energy consumption for all links, i.e., the objective cost of (5), equals $\sum_{n\in\mathcal{N}} P_n d_n$. Consequently, the optimization problem of (5) is in fact to determine whether or not there is a feasible solution such that the age constraints in (5f) are satisfied. Letting $A_n = A$, $\forall n \in \mathcal{N}$, then (5) is the decision problem of the so-called min-max peak age scheduling problem (MPASP), which aims to find the optimal scheduling solution that minimizes the maximum peak age of all information sources [13]. The MPASP has been proved to be NP-hard by a polynomial reduction from the 3-SAT problem, which in NP-complete [18]. Hence we conclude the hardness of the MESA. ■

Furthermore, it has been proved that the MPASP is NP-hard even if the candidate groups are given as problem input (see [13] for details). Therefore, following arguments similar to the proof of Theorem 1, we can reach the following corollary.

Corollary 2. *The MESA with given candidate transmission groups is NP-hard.*

Corollary 2 shows that the hardness of the MESA is not merely resident in constructing the link groups under a given interference model. Theorem 1 and Corollary 2, together with their proofs, imply that one could hardly expect that the use of simple rate functions, including binary ones, would reduce complexity.

IV. Structural Results

It is known that for minimum-energy scheduling without age constraints, TDMA is the optimal solution [16]. It is thus fundamental to understand under what conditions TDMA is optimal when imposing age constraints. Moreover, for minimum-energy scheduling or minimum-energy scheduling with total transmission time constraint, the order of activating the selected link groups can be arbitrary, which is not the case for the MESA. Hence in the following theorem, we present a sufficient and necessary condition for when TDMA is optimal for the MESA, together with the optimal link activation order.

Theorem 3. *Define $\mathcal{T}^1$ as the ordered TDMA schedule where the links are activated in the following order: starting from $j=1$, in the j^{th} time slot, link $n = argmin_{n\in\mathcal{N}_j}(A_n - a_{n,j-1})$ is active, here $\mathcal{N}_j$ is the set of links with non-empty demand at t_{j-1}. Ties, if any, can be broken arbitrarily. For any MESA instance, the optimal solution is TDMA if and only if $\mathcal{T}^1$ is feasible.*

Proof: It is easy to see that $\mathcal{T}^1$ is optimal for minimum-energy scheduling since $\mathcal{T}^1$ is TDMA. After imposing the age constraints in (5f), the solution space of the MESA is a subset of the corresponding minimum-energy scheduling problem. Hence $\mathcal{T}^1$ must be optimal for the MESA if it is a feasible solution, and the sufficiency follows directly.

For the necessity, we show in the following that if $\mathcal{T}^1$ is infeasible, then there is no feasible TDMA solution for the MESA. Suppose the opposite, and let us denote by Ω_1 a feasible TDMA solution where the links are not ordered as in $\mathcal{T}^1$. Then in Ω_1, there exists a time slot, say slot j_1, in which the active link $n_1 \neq n_2 = \mathrm{argmin}_{n\in\mathcal{N}_{j_1}}(A_n - a_{n,j_1-1})$. Denote by j_2 the nearest future time slot in which link n_2 is active in Ω_1. Since Ω_1 is feasible, $a_{n_2,j_2-1} \leq A_{n_2}$ holds for S_{n_2}. Let us denote by τ_1 and τ_2 the time stamps of the most updated packets delivered by n_1 and n_2 before slot j_1, respectively. Then $a_{n_1,j_1-1} = t_{j_1} - 1 - \tau_1$, $a_{n_2,j_1-1} = t_{j_1} - 1 - \tau_2$, and $a_{n_2,j_2-1} = t_{j_2} - 1 - \tau_2$. Hence the above feasibility condition for S_{n_2} can be rewritten as

$$t_{j_2} - 1 - \tau_2 \leq A_{n_2}. \tag{7}$$

Next, we swap the active links of the two time slots and obtain a new TDMA solution Ω_2. The swapping has no impact on the age evolution of S_n, $n \neq n_1, n_2$, $n \in \mathcal{N}$. In addition, the age values of S_{n_1} and S_{n_2} before slot j_1 and after slot j_2 remain the same as in the original solution. For S_{n_2}, it is easy to see that its corresponding age constraint remains satisfied in Ω_2, because some of its packets are scheduled earlier than in Ω_1. For S_{n_1}, in Ω_2 the maximal age between slots j_1 and j_2 is at most $t_{j_2} - 1 - \tau_1$, and the value is achieved when link n_1 is not active in $(t_{j_1}, t_{j_2} - 1)$. By the assumption, we have $A_{n_2} - a_{n_2,j_1-1} < A_{n_1} - a_{n_1,j_1-1}$ in Ω_1, implying that

$$A_{n_2} - (t_{j_1} - 1 - \tau_2) < A_{n_1} - (t_{j_1} - 1 - \tau_1). \tag{8}$$

Combining (7) and (8) we obtain $t_{j_2} - 1 - \tau_1 < A_{n_1}$, which confirms the feasibility of Ω_2. Repeating the swapping for any links that are not activated following the defined order, after a finite number of steps, we reach $\mathcal{T}^1$, which is also feasible. Hence the necessity follows, and we conclude the proof. ■

By Theorem 3 and its proof, we remark that both the problem recognition (i.e., checking the sufficient and necessary condition) and the optimal solution construction can be done in polynomial time. Hence the MESA becomes tractable if TDMA is allowed. Next, we derive the lower and upper bounds for the general MESA.

Theorem 4. *For any feasible MESA instances, the objective value E satisfies $\sum_{n\in\mathcal{N}} \frac{P_n d_n}{v(n,\{n\})} \leq E \leq \sum_{n\in\mathcal{N}} \frac{P_n d_n}{v_{n,min}}$, where $v_{n,min} = v(n,\mathcal{N})$ if $v(n,\mathcal{N}) > 0$, otherwise $v_{n,min} = f(\gamma_n)$ where γ_n is the SINR threshold for link n to be able to transmit.*

Proof: As we explained in the proof of Theorem 1, for any scheduling solution of (5), to empty the demand of link n, the energy consumption equals $\frac{P_n d_n}{\bar{v}_n}$. Herein $\bar{v}_n$ is the average rate of link n during its activation time. Since the links share the same channel and hence are mutually interfering, the highest rate of link n is achieved when it transmits alone. Clearly, for any feasible MESA instance, $v(n,\{n\}) > 0$. If all links can transmit simultaneously, then we obtain the lowest rate of link n (among the possible $2^N - 1$ groups) equalling $v(n,\mathcal{N})$. Otherwise, the lowest positive rate, i.e., $v_{n,min}$, is obtained at the SINR threshold of RX_n. Therefore, for the average activation rate of link n, we have $v(n,\{n\}) \geq \bar{v}_n \geq v_{n,min}$. Hence the upper and the low bounds of the objective value E are $\sum_{n\in\mathcal{N}} \frac{P_n d_n}{v_{n,min}}$ and $\sum_{n\in\mathcal{N}} \frac{P_n d_n}{v(n,\{n\})}$, respectively. ■

V. Deadline First with Revision (DFR) Algorithm

Since the MESA is hard in general, in what follows we propose an efficient heuristic algorithm. Inspired by the structural results, the principle of the *deadline-first-with-revision (DFR)* algorithm is to construct a feasible schedule with as few as possible links being active in a time slot. That is, by DFR, we intend to compute a scheduling solution approaching TDMA in the feasibility region of the MESA.

A. Algorithm Flow

The DFR algorithm starts from t_0 and constructs a link group for each time slot. For the j^{th} time slot, the algorithm sorts the non-empty links in ascending order of $A_n - a_{n,j-1}$. Then the links are checked one-by-one from the top of the list to set their priorities. In this process, the algorithm will first identify the "must-do" link(s). That is, if any of those links is not included in the active group of this time slot, then information from at least one source would expire, and the schedule task would fail at this step. The "must-do" links could stem from the following two scenarios:

- Link n has packets to be delivered, and $A_n - a_{n,j-1} = 0$. Then link n must be active at slot j.
- There exists at least one source, e.g., S_m, for which the gap between its current age and the corresponding age requirement A_m has decreased to one, and all packets from that source have been delivered. In this case the current schedule needs to be ended in one time slot, so that the age constraint for source S_m can be met. Therefore, in the coming time slot all other links with queued packets need to be emptied.

If there is no link that must be active immediately, then the algorithm will select the most urgent link, defined as $n = argmin_{n\in\mathcal{N}_j}(A_n - a_{n,j-1})$, where $\mathcal{N}_j$ is the set of links with non-empty packets at t_{j-1}. The singleton-link group $\{n\}$ will be active in the j^{th} time slot. If any "must-do" link exists, the algorithm will proceed based on the number of those links.

- If there is only one "must-do" link in this time slot, then evidently the singleton-link group consisting of that link will be the choice.
- If multiple links have to be active in the j^{th} time slot, putting them together into one group is probably suboptimal, since the $v(n,g)$ values will be low, or even zero due to the interference. In view of this, we propose a backward construction strategy. The algorithm first sorts the "must-do" links in the descending order of their signal-to-noise ratio (SNR). Then it puts the first one, say link n_1, which has the largest SNR and hence the maximum rate (if transmitting alone), into the group for the j^{th} time slot. Let us denote by g_k the constructed group for the k^{th} slot. At the current step, we have g_k, $k = 1, 2, \ldots, j$, available, and $g_j = \{n_1\}$. Next, the algorithm will revise these groups so as to activate the remaining "must-do" links before t_j. Specifically, the algorithm will attempt to add the "must-do" links one-by-one into some of the groups. To identify which group should be revised, we define a metric called *average energy per packet*. For a link group g, this metric is calculated as $\bar{e}_g = \frac{P_g}{d_g}$, where $d_g = \sum_{n\in g} v(n,g)$. In order to achieve minimum energy, it is easy to see that a group with lower $\bar{e}_g$ is preferable. Therefore, the algorithm will select the group with the lowest $\bar{e}_g$. We take the second link in the "must-do" list, say link n_2, as an example. Following the above processing, the link will be added to group $g_{k'}$, where $k' = argmin_{k\in\{1,2,\ldots,j\}} \bar{e}_{g_k \cup \{n_2\}}$. The algorithm will continue in this fashion until all "must-do" links have been added into the groups being activated in the first j time slots. Let us denote by j' the earliest time slot in which the original active group is revised. Then the algorithm will re-calculate the age evolution in $[t_{j'}, t_j]$ with the new scheduling solution.

After that, the algorithm will proceed to the $(j+1)^{th}$ slot and repeat the above process, until all packets are emptied.

Remark 1. *In each time slot, despite that the DFR may go backward for revision, by construction the resulting schedule always processes one slot forward in the time line. Since there are a finite number of packets to be delivered under age constraints, the length of any feasible schedule is bounded. Therefore, the algorithm will end in a finite number of steps, with an output of either a feasible solution or a result of "infeasible". For any MESA defined in* (5)*, the number of backward revision rounds is strictly less than $\sum_{n\in\mathcal{N}} d_n$.* □

B. Optimality Property

We verify the rationale of the DFR algorithm by applying it to the tractable case for which the ordered TDMA is optimal.

Theorem 5. *The DFR algorithm achieves the global optimum for any MESA instance for which TDMA is feasible.*

Proof: According to Theorem 3, the ordered TDMA is the optimal solution of such a MESA instance. We compare this optimal schedule with the output of the DFR. Starting from the first time slot, we assume that link n is the one with minimum $A_n - a_{n0}$. If $A_n - a_{n0} = 1$, then n is the "must-do" link as at t_0 none of the links are empty. Note that, there could not be any other "must-do" link, because otherwise TDMA would be infeasible. If $A_n - a_{n0} > 1$, then there is no "must-do" link since it is the minimum age gap. By construction, the DFR algorithm will select n as the most urgent link. Therefore, in any case, the algorithm will construct the singleton-link group $\{n\}$ for the first time slot. The algorithm then continues to the following time slots, until all packets are emptied. Following the similar argument, one can verify that in each slot, the algorithm always makes the same choice as the ordered TDMA does, and in each slot, with the constructed schedule, there is at most one "must-do" link since the ordered TDMA is feasible, thus no backward construction is involved. Therefore, the scheduling solution constructed by the DFR is exactly the ordered TDMA. Hence the conclusion. ■

VI. Numerical Study

We perform numerical study for the MESA with two rate functions, and show how the energy consumption is impacted by the age constraints. Moreover, in order to assess the improvement in energy-efficiency, we compare the optimized schedule with that of minimum peak age scheduling (MPAS) [13]. We derive the MPAS solution with a greedy algorithm, i.e., in each time slot, we sort the sources in descending order of their ages, and add the link associating with the oldest one to the group. Then we iterate through the ordered list and add as more as possible links that could be capable of transmitting together.

A. MESA with Cardinality-based Rates

We first consider the MESA case where the transmission rate is solely determined by the number of concurrently active links. We refer to it as the cardinality-based rates, since the rate values depend on the cardinality of the link group instead of its individual elements. The cardinality-based rates could be used to model or approximate the scenarios where the transmitters have similar distances (and hence similar channel gains) to their receivers, which are located close to each other. An example is a cellular network consisting of a number of mobiles having similar distances (on a circle) to their common serving base station; or a sensor network with a set of sensors surrounding their server.

We run simulations in wireless networks with $N = 30$ links, each associating with an information source. The starting time of a schedule is $t_0 = 500$. The initial ages of the 30 sources, i.e., $a_{n0}, \forall n \in \mathcal{N}$, are uniformly distributed in [50, 300]. Each transmitter has 20 packets to be delivered. The time stamps of the packets are integers uniformly distributed in $(t_0 - a_{n0}, t_0)$. The transmission rate is defined as

$$v(n,g) = \begin{cases} 12 - 2|g| & \text{if } |g| < 6, \\ 1 & \text{if } |g| = 6, \\ 0 & \text{if } |g| > 6. \end{cases} \quad (9)$$

Note that, if the number of packets queued at TX_n is less than $v(n,g)$, then $v(n,g)$ will automatically decrease to the number of the remaining packets of link n.

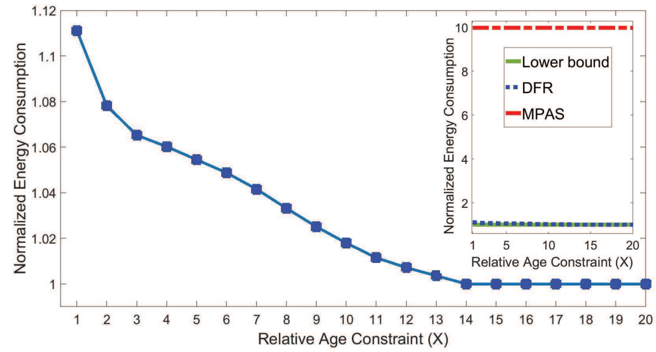

Fig. 1. Energy consumption with age constraints and cardinality-based rates

We set the age constraints $A_n = a_{n0} + x, \forall n \in \mathcal{N}$, where x is a random integer uniformly distributed in $[X, 10 + X]$. For each value of X, we compute the scheduling solutions for 100 MESA instances with the DFR algorithm. Note that, it may happen that some instances with low age constraints are infeasible. Hence we collect the results from $X = 1$, that is, $A_n = a_{n0} + x$, $x \in [1, 11]$, $\forall n \in \mathcal{N}$, for which the DFR obtains feasible solutions for 93 instances of the 100. The energy consumption with the optimized schedule is normalized by its lower bound (see Theorem 4). We present the results in Figure 1, where the *x-axis* represents the the relative age constraint X, and the *y-axis* represents the mean of the normalized energy consumption of the 93 instances. In the inset figure, we compare the resulting energy consumption of the DFR with the greedy algorithm of MPAS, both normalized by the lower bound.

The results show that the energy consumption decreases with the relative age constraint with a decreasing marginal gain. After $X = 14$, the energy consumption becomes a constant and reaches its lower bound. Hence one could roughly estimate that for this dataset TDMA is allowed when $A_n \in [a_{n0} + 14, a_{n0} + 24]$. The inset figure shows that the DFR achieves a significant energy reduction ($\sim 90\%$) in comparison to the MPAS algorithm, which aims to minimize the peak ages.

B. MESA with Shannon Rate Function

Next we consider the MESA case when the bit-rate is given by the Shannon formula. We consider wireless networks with $N = 30$ links that are randomly distributed in an area of 1000×1000 meters. To obtain links with practically

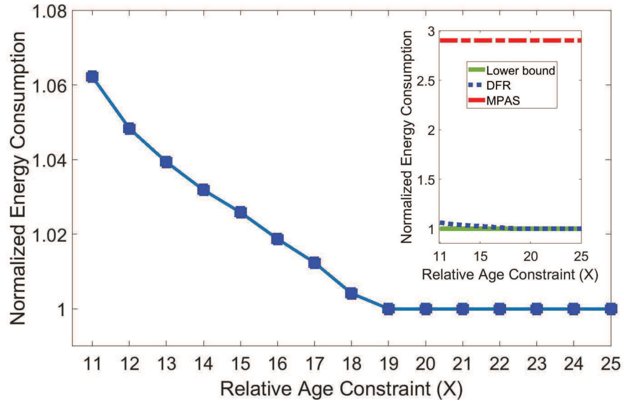

Fig. 2. Energy consumption with age constraints and Shannon rate function

meaningful SNR values, the distance between any transmitter and its associated receiver is restricted to be between 3 and 200 meters. For all links, the transmit power P_n and the noise variance σ_n are uniformly set to 30 dBm and -100 dBm, respectively. The channel gain follows a distance-based propagation model with a path loss exponent of 4. We assume that the bandwidth $B = 2$ KHz, and each packet consists of $\rho = 8$ bits. Following the Shannon formula, the number of packets delivered by link n in a time slot is given as the minimum of the number of available packets at TX_n and $v(n, g)$, which is calculated as

$$v(n, g) = \lfloor \frac{B}{\rho} \log_2(1 + \text{SINR}(n, g)) \rfloor. \tag{10}$$

For each link, 20 packets are queued at its transmitter at t_0. The parameters t_0, a_{n0}, τ_{ni}, and A_n are following the same setting as in the cardinality-based rates case, described in Section VI-A. For each set of the age constraints, i.e., each value of X, we employ the DFR to compute the scheduling solutions for 100 instances, and record the results from the minimum vale of X for which 90% of the instances have a feasible solution. The energy consumption by the optimized schedule is normalized by its corresponding lower bound. We present the mean of the 90 instances in Figure 2, and compare the DFR with the MPAS algorithm in its inset figure.

The results show similar behavior as those in Figure 1. When the age constraints A_n, $n \in \mathcal{N}$, increase, the energy consumption decreases and reaches the lower bound when TDMA becomes feasible. In addition, one could observe that the starting point of X is much larger than the cardinality-based rates case, implying that the Shannon function case is more sensitive to age constraints, especially in the regime of low X. The reason is that under this network setting, some links may generate significant interference to each other, and hence hardly to be active simultaneously. Then the scheduling task is easy to fail under strict age requirements. The observation also indicates that it would be challenging to deploy real-time services in a network with a severely interference-limited channel. Clearly, comparing to the MPAS algorithm, as shown in the inset figure, the DFR is more energy-efficient as it only consumes about one-third energy of the former.

VII. Conclusions

The paper addresses the minimum-energy scheduling problem of emptying a network under age constraints. We have formulated the problem in its general form such that the study could apply to a wide range of wireless systems. We have derived theoretical results including the problem's complexity and its upper and lower bounds, and shown that under which condition and scheduling order a TDMA schedule is optimal. An optimization algorithm has been proposed. Numerical results with two rate functions have been presented. The current work has a number of interesting extensions, such as the joint optimization of link scheduling and power control, involving circuit energy, and considering the problem in multi-hop networks.

References

[1] J. G. Andrews, S. Buzzi, W. Choi, S. V. Hanly, A. Lozano, A. C. K. Soong, and J. C. Zhang, "What will 5G be?" *IEEE J. Sel. Areas Commun.*, vol. 32, no. 6, pp. 1065–1082, 2014.

[2] P. Gandotra and R. K. Jha, "A survey on green communication and security challenges in 5G wireless communication networks," *J. Netw. Comput. Appl.* , vol. 96, pp. 39–61, 2017.

[3] S. Kaul, R. Yates, and M. Gruteser, "Real-time status: How often should one update?" in *Proc. of IEEE INFOCOM*, 2012, pp. 2731–2735.

[4] A. Pantelidou and A. Ephremides, "Scheduling in wireless networks," *Found. Trends. Network.*, vol. 4, no. 4, pp. 421–511, 2009.

[5] L. Georgiadis, M. J. Neely, and L. Tassiulas, "Resource allocation and cross-layer control in wireless networks," *Found. Trends. Network.*, vol. 1, no. 1, pp. 1–144, 2006.

[6] A. Capone, I. Filippini, S. Gualandi, and D. Yuan, "Resource optimization in multi-radio multi-channel wireless mesh networks," in *Mobile Ad Hoc Networking: Cutting Edge Directions*, 2nd ed., S. Basagni, M. Conti, S. Giordano, and I. Stojmenovic, Eds. Wiley and IEEE Press, 2013, pp. 241–274.

[7] L. Tassiulas and A. Ephremides, "Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks," *IEEE Trans. Automat. Contr.*, vol. 37, no. 12, pp. 1936–1948, 1992.

[8] S. A. Borbash and A. Ephremides, "Wireless link scheduling with power control and SINR constraints," *IEEE Trans. Inf. Theory*, vol. 52, no. 11, pp. 5106–5111, 2006.

[9] M. J. Neely, "Energy optimal control for time varying wireless networks," *IEEE Trans. Inf. Theory*, vol. 52, no. 7, pp. 2915–2934, 2006.

[10] V. Angelakis, A. Ephremides, Q. He, and D. Yuan, "Minimum-time link scheduling for emptying backlogged traffic in wireless systems: solution characterization and algorithmic framework," *IEEE Trans. Inf. Theory*, vol. 60, no. 2, pp. 1083–1100, 2014.

[11] Q. He, D. Yuan, and A. Ephremides, "Optimal link scheduling that minimizes the age of information in wireless systems," *IEEE Trans. Inf. Theory*, vol. 64, no. 7, pp. 5381–5394, 2018.

[12] I. Kadota, E. Uysal-Biyikoglu, R. Singh, and E. Modiano, "Minimizing the age of information in broadcast wireless networks," in *Proc. of IEEE Allerton*, 2016.

[13] Q. He, D. Yuan, and A. Ephremides, "On optimal link scheduling with min-max peak age of information in wireless systems," in *Proc. of IEEE ICC*, 2016.

[14] M. Costay, M. Codreanu, and A. Ephremides, "Age of information with packet management," in *Proc. of IEEE ISIT*, 2014, pp. 1583–1587.

[15] Q. He, G. Dán, and V. Fodor, "Joint assignment and scheduling for minimizing age of correlated information," *IEEE/ACM Trans. Netw.*, 2018, under revision.

[16] G. D. Nguyen, S. Kompella, C. Kam, J. E. Wieselthier, and A. Ephremides, "Minimum-energy link scheduling for emptying wireless networks," in *Proc. of IEEE WIOPT*, 2015, pp. 207–212.

[17] A. Kosta, N. Pappas, and V. Angelakis, "Age of information: A new concept, metric, and tool," in *Foundations and Trends® in Networking*. Now Publishers, 2017.

[18] M. R. Garey and D. S. Johnson, *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W. H. Freeman, 1979.