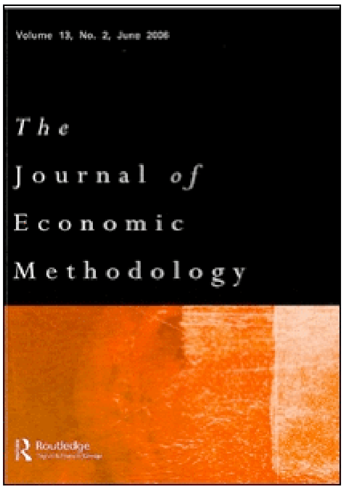

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The roles of stories in applying game theory

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Game-theoretic models consist of a formal game structure and an informal model narrative or story. When game theory is employed to model economic situations, the stories play a central role in interpreting, constructing and solving game structures. We analyse the architecture of game theory and distinguish between game models and the theory proper. We present the different functions of the model narrative in the application of game models to economic situations. In particular, we show how model narratives support the choice of solution concepts defined and provided by the theory proper. We further argue that the narrative's role in interpretation, construction and solution makes it a necessary part of a game model that is intended to be a model of an economic situation. We conclude that game theory is not a universal theory of rationality, but only offers tools to model specific situations at varying degrees and kinds of rationality.

Keywords: game theory; modelling; solution concepts; rationality; stories

JEL codes: B41; C70; D01

1 Introduction

Over the past three decades, game theory has become one of economists' major theoretical tools.[1] An attempt to elucidate the practice of economic theorizing must therefore investigate the specifics of game-theoretic theorizing. This investigation is particularly important, because available accounts of economic model building and model use do not capture some important features of the application of game theory in economics.

Game theory, like other modes of economic theorizing, is model-based. The practice of economic theorizing proceeds by constructing, analysing and manipulating models, and then transferring the results of this analysis to the real world.[2] Economic models are interpreted by a model narrative, and thus connected to the world. This narrative is part of the model, as economists commonly use models as interpreted entities (Gibbard and Varian 1978). Further, the narrative plays a central role in the way a model user manipulates the model in order to learn from it (Morgan 2001).

Models play a similarly important role in game theory. They are the 'games' – like the 'prisoners' dilemma', the 'ultimatum game' or the 'battle of the sexes' – that most people are familiar with. It is well known that these models come with specific narratives attached, which in many cases have provided the model's vernacular name. At the same time, there is a tendency to neglect the important functions of these stories for the application of game theory to economic situations. Morgan

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(2007), in an analysis of the prisoners' dilemma, has argued against this tendency. She shows that the narrative provides an interpretation of the formal part of the model, and supports the 'solution' of the game.

This paper elaborates on Morgan's claim, and corrects some of its shortcomings. We agree with her argument that narratives are important for the application of game models, but we think that she neglects the role of theory. We therefore begin (in section 2) by discussing the architecture of game theory, identifying game structure and model narrative as part of the game model, and by distinguishing the game model from the 'theory proper'. Particular emphasis is put on the role of the theory proper as the 'toolbox' of game theory, which provides both the elements from which game structures are constructed, and a menu of solution concepts through which game structures are solved. In section 3 we present the different functions of the model narrative in the application of game models to economic situations. In particular, we show how model narratives support the choice of solution concepts from the menu provided by the theory proper. We further argue that the role of both the narrative and the theory proper in solving a game structure makes them necessary parts of the application of game models to economic situations. We conclude in section 4 that this has important implications for the interpretation of game theory: namely that game theory is not a universal theory of rationality, but only offers tools to model specific situations at varying degrees and kinds of rationality.

2 The architecture of game theory

In this section, we describe the different elements that make up game theory. We call game theory the whole complex with which economists strive to investigate and explain phenomena of strategic interaction.[3] Within game theory, we distinguish between the game model and the theory proper, and within game models we distinguish between the game structure and the model narrative.

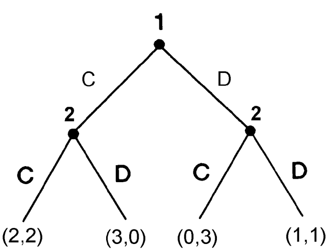
2.1 Game structure

Open almost any game theory textbook, and the part that will spring to your eye immediately is something like what is seen in Figure 1(a) and (b).

We will call these entities game structures. Figure 1(a) is a game matrix that represents a strategic form game, and Figure 1(b) is a game tree that represents an extensive form game. The labels *R1*, *R2* at the beginning of each row of a matrix represent the 'strategies' of a 'player'; the labels *C1*, *C2* at the top of each column

	C1	*C2*
R1	2,2	1,3
R2	3,1	0,0

(a)



(b)

Figure 1. Two kinds of game structures.

represent those of another. The numbers in each cell represent the 'payoff' for a combination of two players' strategies, which is called a 'strategy profile'. The first number represents the payoff for the player whose strategies are represented as rows, the second number represents the payoff for the player whose strategies are represented as columns.

A game tree consists of nodes and branches. The number at each node signifies the player who chooses a 'move', labelled *C* and *D*, at this node. A player's strategy consists of all her moves from the initial node to one of the final branches. A strategy profile is the combination of all players' strategies from the initial nodes to one final branch. The numbers at the end of the final branches represent the payoffs for each player for that strategy profile.

Game structures are the entities with which novices are commonly introduced to game theory. Students learn about the concepts of player, strategy, payoff function, simultaneous or sequential move, etc. by studying specific exemplars of game structures – e.g. the prisoners' dilemma as an exemplar of the strategic form, and the chain store game as an exemplar of the extensive form.[4] For many, therefore, the matrices and the trees are what game theory is all about.

Such a view, we think, is incorrect. First, game structures lack an interpretation. While their elements are designated 'players', 'strategies' or 'payoffs', they need to be filled with content to function as the model of something. It has been suggested that model narratives provide the content for game structures. Narratives will be discussed in the next subsection. Second, game structures are constrained by a theory. This theory defines the formal properties of possible game structures, and also provides processes to determine the strategy equilibria in specific games. We discuss this 'theory proper' in section 2.3.

2.2 Model narratives

To function as economic models, game theory has to represent real or hypothetical economic situations. A game structure is characterized only by formal properties. By itself, it does not connect in any way to an economic situation either real or imagined. To account for the use of games in economics, a game structure has to somehow acquire an interpretation. It has been proposed that formal models include a narrative that provides such an interpretation:

> In economists' use of models, there is always an element of interpretation: the model always tells a story … we can think of the story as telling what kind of extension each predicate has and what kind of domain each quantifier has. (Gibbard and Varian 1978, p. 666)[5]

In game theory, stories are also employed in this way, as Morgan (2007) has shown in her discussion of the prisoners' dilemma. To give another example, the game matrix of Figure 1(a) often comes with the following story attached: 'Two players drive cars at each other. The first to swerve away and slow down loses and is humiliated as the "chicken"; if neither player swerves, the result is a potentially fatal head-on collision.' This very simple mini-narrative characterizes the two players as human agents, who intentionally participate in an interaction. It also specifies the strategies open to both players: 'stay on course' or 'swerve'. They are simple strategies, which consist of only one action, which the players have to choose simultaneously. Further, the story characterizes the 'payoffs' for each player:

'winning' is better than 'being humiliated', which in turn is better than 'dying in a head-on collision'. By describing (more or less implicitly) the setting as a test of courage, the story also excludes other motivations as relevant for the payoffs. Despite its simplicity, the narrative provides a full interpretation of the specific game structure depicted in Figure 1(a).

Economists and game theorists have been reluctant to acknowledge the importance of stories. This may be because the notion of narrative or story is notoriously vague. We characterize a model narrative as follows: It accounts for a series of events in a certain order, and establishes connections between them. In the sense used here, it does so in a natural language: it is a verbal account, which uses terms that have a commonly understood meaning in that language. Further, it is a verbal account with a clear beginning and an explicit end. Last, it only accounts for those facts and events that act as an interpretation of some term of the game structure, or that bear relevance as background conditions to an interpretation. We will argue in section 3 that this account of background condition is of particular importance.

2.3 Theory proper

Game theory, even though it clearly operates with models, is also comprised of a sophisticated body of theoretical work, which cannot be accurately characterized as a set of models. Instead, this work betrays game theory's roots as a mathematical discipline. A lot of its effort is put, for example, into the development of equilibrium concepts or existence proofs. For many game theorists, this work is the core of their discipline. As Reinhard Selten once put it succinctly, 'game theory is for proving theorems, not for playing games' (quoted in Goeree and Holt 2001, p. 1419). We call this part of game theory the *theory proper*.

The theory proper has a curious status. By itself, it is empirically empty. In applying game theory (understood as the whole theoretical complex) to specific situations, however, it plays a central role; and this application results in empirically meaningful statements.

> Game theory is not a theory which has as output a set of refutable statements, but merely a syntax articulating the vocabulary of interdependent rationality. Game 'theory' per se is no more empirically verifiable than an alleged translation from English into an unspoken language. Yet its application to specific economic political or social situations produces many testable statements. (Bianchi and Moulin 1991, pp. 187–188)

The theory proper is empirically empty, because its terms do not have an interpretation. This is no surprise, giving its mathematical and theorem-proving nature. Although it uses predicates like 'players', strategies' or 'utility', one has to be careful not to associate these terms with any intuitive meaning.[6] Retaining this uninterpreted state has the advantage that the theory proper is widely applicable, from the social sciences through population genetics to logic. It has the disadvantage that the theory proper itself does not produce any refutable statements.[7]

That the theory proper is empirically empty can be seen in how it is used. If it could be a direct representation, students would spend their time studying how to connect real-world phenomena with the mathematical formulae of theory proper. Textbooks would be full of real-world situations and of the games that represent them. This is not the case. Textbooks that teach the theory proper instead 'focus on presenting concepts and general results, with more "toy examples" than detailed applications' (Fudenberg and Tirole 1991, p. xxi). Concrete situations are rarely discussed. Students learn game theory by learning the theory proper, and by learning a taxonomy of games.

It may therefore seem that the theory proper is largely irrelevant when it comes to applying game theory to specific situations. Such an impression, we think, is mistaken. First, the theory proper acts as a source and as a constraint of game structures. It defines the formal properties of the elements of the game structures. For example, it stipulates that a game tree has only one initial node, that each strategy profile has a payoff vector characterizing the payoffs for each player, and that players can choose probability-weighted mixes of pure strategies. When applying game theory to a situation, the theory proper is the 'toolbox' from which the elements of a game structure are taken and according to whose rules these elements are assembled.

The theory proper also provides 'solution concepts' for the particular game structures. Such a solution concept identifies a game's strategy equilibria, on the grounds of the game structure's formal properties. For instance, the theory proper defines the solution concepts of 'Nash equilibrium' and 'iterated (strict) dominance' for strategic form games; and it defines 'subgame perfection' for extensive form games. In addition, a host of alternative and refinement solution concepts have been made available. The strategy equilibria, once identified, are used normatively to prescribe how players should play, and descriptively to predict how players will play. Notably, the theory proper typically provides a menu of solution concepts for each game structure: in applying game theory to a specific situation, one often has to choose the solution concept from the menu that the theory proper offers.

Because the theory proper is not interpreted, it cannot be judged empirically. Instead, it is evaluated by a number of non-empirical criteria. These include criteria pertaining to any (applied) mathematical theory: consistency, simplicity, and aesthetic criteria. Further, the theory proper is assessed by the variety of the models that it allows to build, and by the solutions it offers for these game models.

> … a solution concept should be judged by its performance in the application, by the quantity and quality of the relations that it engenders, not by 'armchair' philosophizing about its definition. (Aumann 1985, p. 43)

To match this criterion, the theory proper is continuously enriched in order to construct more relevant game structures. In the development from the original zero sum games of Borel or von Neumann and Morgenstern to the current state, the theory proper has been expanded to include mathematical structures that allow modelling uncertainty about own rewards, incomplete information on the other players' payoffs, reputation effects stemming from repeated actions and so forth. Particularly this second criterion emphasizes the pragmatic aspects of the theory proper. For this reason, we suggest viewing the theory proper as a *toolbox for defining, constructing and solving a game*.[8]

2.4 How theory proper, game structure and model narrative relate

The architecture of game theory, we have argued, can be analysed into three parts. When game theory is used to model economic situations, these parts are arranged as depicted in Figure 2.

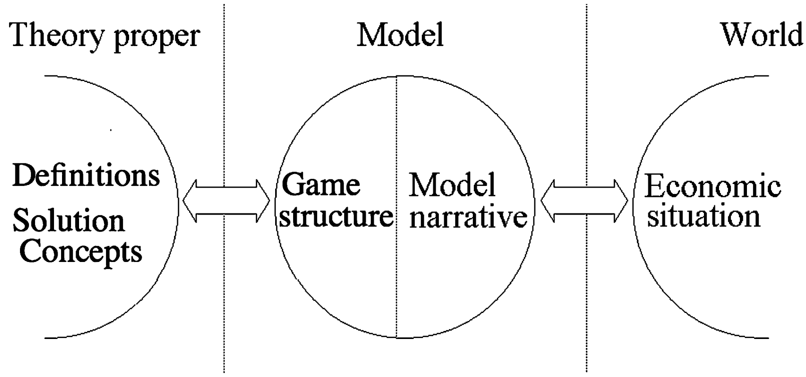


Figure 2. The architecture of game theory.

The *theory proper* (on the left hand side of Figure 2) specifies the concept of a game, it provides the mathematical elements that are needed for the construction of a game structure, and it offers solution concepts for the thus constructed game forms. The *game structure* (left half of the central circle) is constructed from elements of the theory proper. The *model narrative* (the right half of the central circle) provides an account of a real or hypothetical economic situation. Its account of the situation interprets the game structure. It further motivates the application of specific solution concepts to the thus interpreted game structure. We will argue that the game structure and the model narrative *together* constitute a model of an economic situation. We call this the game model.

3 The functions of stories

While there is widespread agreement that game theorists engage in the practice of telling stories when teaching or presenting new game models, it is more controversial whether this narrative is part of the game model in the more substantial sense claimed above. In this section, we discuss the different functions that narratives have when using game theory to model economic situations. We argue that each of these functions is *necessary* for the use of game theory as an economic model, and that therefore the narrative is a necessary part of the game model.

3.1 From game structure to situation: interpretation

If a game structure is intended to represent a situation, a connection must be established between its formal elements and that situation. The establishment of such a connection can take place in at least two ways. Either a game structure may have been specified (in accordance with the theory proper), and it now is in search for an interpretation; or a situation may be deemed interesting to explore with game theory, and hence the construction of a game structure based on that situation is desired. In this section, we focus on the first case. It is a very common case, because game theorists often work out interesting problems for the theory proper by developing a new game matrix or tree. To see whether such a game structure has any relevance, a story is then invented to give an interpretation of this new structure.[9]

We illustrated the first function of the model narrative in this with the 'chicken' game in section 2.2. Its function is to fill all elements of the game structure with content. In contrast to the theory proper, which constrains the terms formally, the narrative gives a verbal, commonly understood account, which puts empirical flesh on the formal bones. In other words, it specifies who the players are, what their choices consist in, and how the ensuing consequences are evaluated. In the case of hypothetical situations, the story characterizes imaginary entities with commonly understood properties; in the case of a real situation, it strives to provide definite descriptions of the objects.

But the narrative accomplishes more than specifying the extension of the game structure predicates and the range of its quantifiers. Its second function is to embed the interpreted terms in a coherent account of a strategic situation. The story tells us that something is happening or has happened, and that this event involves two or more agents who are (at least rudimentarily) aware of each other's deliberations. It is understood that the game structure represents this initial situation. The narrative drive of the story then indicates that something will happen as a consequence of this initial event. In other words, the narrative prepares the ground for the 'solution of the game', i.e. the identification of equilibrium strategy profiles, by applying the theory proper to the game structure. It does this in both a descriptive and a normative sense. Descriptively, the story makes us expect that the players will make a choice in reaction to the situation. Normatively, it makes us form an intuitive notion of how the players *should* choose.

Because stories are the only devices that perform these two functions, they are necessary to interpret a game structure as the representation of a specific situation.

3.2 Situation to model: construction

Suppose now that we have an informal account of a situation, and we want to investigate it with the help of game theory. To illustrate, let's imagine we are interested in constructing a game-theoretic representation of Rousseau's stag hunt, in which members of a hunting party are confronted with the following choice:

> If a deer was to be taken, every one saw that, in order to succeed, he must abide faithfully by his post: but if a hare happened to come within the reach of any one of them, it is not to be doubted that he pursued it without scruple, and, having seized his prey, cared very little, if by so doing he caused his companions to miss theirs. (Rousseau 1754/1992)

A game structure construction needs to answer at least the following questions. Who are the players? What are their available choices? Do they choose simultaneously or in a sequence? What are their payoffs for each of their strategy profiles? Fortunately, game theorists have (as a textbook exercise) constructed a game structure from this narrative, so that we can study in more detail how they go about constructing a model:

> To make this into a game, we need to fill in a few details. Suppose that there are only two hunters, and that they must decide simultaneously whether to hunt for stag or for hare. If both hunt for stag, they will catch one stag and share it equally. If both hunt for hare, they will catch one hare. If one hunts for hare while the other tries the stag, the former will catch the hare and the latter will catch nothing. Each hunter prefers half a stag to one hare. – This is a simple example of a game. The hunters are the players. Each

> player has the choice between two strategies: hunt stag and hunt hare. The payoff to their choice is the prey. If, for instance, a stag is worth 4 'utils' and a hare is worth 1, then when both players hunt stag each has a payoff of two utils. A player who hunts hare has payoff 1, and a player who hunts stag by himself has payoff 0. (Fudenberg and Tirole 1991, p. 3)

The construction process, as quoted, exhibits three distinguishable levels. On the first level, Rousseau recounts the situation of interest. His narrative identifies an unspecified number of hunters as the players, specifies the choices available (hunt stag or hare), and clarifies the relevant knowledge of the players (that it requires all hands to get the stag). Curiously, while he is unclear what the payoffs for the players involved are for each strategy profile, he already presumes the outcome of the interaction, namely that the collective stag hunt will break down. Even so, the account raises our interest: why would the hunt break down? What are the mechanisms that lead to the breakdown?

On the second level, the phenomenon is manipulated into a more precise story. The authors' claim that this manipulation is only 'to fill in a few details' is misleading, because various kinds of manipulation are involved in this process, some much stronger than mere 'filling in'. First, the account is disambiguated: relevant information, which was ambiguous, is added to the initial account without distorting it. The preference for half a stag over one hare, for example, was not explicit in the original situation. However, if a preference like this one had not existed, the situation would have been strategically trivial. Hence a preference of this sort was implicit in the original account of the situation, and was only made explicit by the disambiguation. Second, the account abstracts from the phenomenon by reducing its complexity. For example, the account assumes that only the prey is relevant for the hunters' preferences, and hence abstracts from any ethical or reputation concerns. In particular, it remains silent about possible repetitions of the hunt with the same group of players – so that those who hunted hare last time may be punished in the future. Because it does not assume such a long-term perspective, it also remains silent about the evaluation of future payoffs in comparisons to present ones. Further, the authors assume that the hunters must decide simultaneously, excluding the possibility that hunters might observe each other's choices. All of these aspects would increase the complexity of the situation, but may also have considerable influence on the solution of the game. Third, the account idealizes the phenomenon through deliberate distortion; in this case, for example, it turns the hunting party into only two hunters. Here the authors do not just abstract from factors whose contribution could be analysed and hence separated; rather, it changes the situation in its substance to achieve a story that can be handled with the available theoretical tools. All these manipulations have the goal of presenting the situation in a way that allows it to be mapped into a game structure.

Disambiguation, abstraction and distortion of a situation can often be justified by pointing out that these manipulations are in accordance with how the players actually perceive the situation.

> … a good model in game theory has to be realistic in the sense that it provides a model for the perception of real life social phenomena … It should incorporate a description of the relevant factors involved, as perceived by the decision makers. These need not necessarily represent the physical rules of the world … It is not meant to be isomorphic with respect to 'reality' but rather with respect to our perception of regular phenomena in reality. (Rubinstein 1991, p. 910)

In order to judge how the players may perceive a situation, the model builder needs to get a 'feel' for the situation. A narrative that accounts for a situation in a certain way supports getting such a 'feel', and at the same time constrains what perceptions can plausibly be attributed to the players.

The third step in the construction process combines the story and the tools of the theory proper to construct the game structure. The account is sufficiently manipulated if formal elements provided by the theory proper are clearly identifiable: the players, the strategies, the payoffs, etc. At this stage, the narrative provides a blueprint for the game structure. The story dictates which structure must be chosen, and how the concrete game structure needs to be constructed; i.e. the story determines whether to choose a strategic or an extensive game structure, how many players are involved, what the strategies are, which preferences the agents have over the possible outcomes of their strategic interaction, etc. Aumann compares this process of constructing the model with an artistic expression through a difficult and resistive medium:

> The resistiveness of the medium imposes a kind of discipline that enables – or perhaps forces – the artist to think carefully about what he wants to express, and then to make a clear, forthright statement … In game theory … the resistive medium is the mathematical model. (Aumann 1985, p. 42)

The construction process therefore affects both game structure and narrative: the story determines the assembling of formal elements into game structure, and the formal constraints on the game structure provide the resistive medium through which the narrative is refined. In the end, both game structure and narrative should have converged to form a unified account of the situation. Once this is achieved, the model construction process is complete. It is then ready to be processed by one of the solution concepts of the theory proper.

Because stories and stories' refinement are the only devices that allow the manipulation of an initial account into the narrative blueprint for a game structure, stories are necessary for constructing a game structure from a specific situation. This and the previous function of narratives have been treated by previous authors; our discussion elaborates on their claims. In the following section, we make the novel claim that narratives also play a crucial role in applying the theory proper's solution concepts to game structures.

3.3 Game solutions

In section 3.2 we argued that the narrative account of a situation determines how the tools provided by the theory proper are assembled into a particular game structure that represents this situation. The toolbox view of the theory proper, we argued in section 2.3, also extends to the solution strategies it offers. The theory proper typically provides a variety of solution concepts, which can be applied to the same game structure, but which may yield substantially different results. The existence of a solution concept menu requires criteria for the choice of the 'right' solution for a particular game. Here, the model narrative also plays a decisive role: it supports the choice of a solution concept to solve the game structure representing a specific situation.

Before discussing this function of narratives in game models, we sketch the recent work on the role of stories in 'solving' economic models more generally. Morgan

(2001) offers a good account of this. She argues that an important aspect of model use in economics is the manipulation of the formal part of the model. With the help of a story, the structure has been interpreted as representing an economic situation. By manipulating this structure in certain ways and recording the results, one can hope to learn about the dynamics of the represented situation. One may ask a question of the sort: 'Suppose something changes while other things remain constant, what will be the effects?', manipulate the structure accordingly, and answer the question with the result from the manipulation. However, as Morgan (2001) argues, instructions of how to manipulate the structure do not ensue automatically from that question. For example, to know which other factors will remain constant, the causal history of the envisaged change must be elaborated. Further, it may be the case that the envisaged event does not have a direct correlate in the structure's interpretation. Take for example the demand–supply model. If individuals' endowments change, the model structure will only be affected if a causal connection to the demand curve is specified. The graph itself does not contain a parameter dimension for endowment; hence it is not directly affected by an endowment change. However, a common narrative that accounts for the demand behaviour of agents in a given market will relate endowment to demand. Crucially, this relationship cannot be captured by a general theory: as the cases of inferior and Giffen goods show, the effect of endowment changes on demand depend on specific characteristics of the market context. Rather, establishing the correct relationship requires an account of the specific and local context. A story provides the informal account of the causal relations in the economic situation under scrutiny. A narrative account of the situation is therefore a necessary part in manipulating a model structure with the goal of answering a question about the represented situation.

Thus, although a model's structure may be its most visible part, its story has at least three crucial roles. It provides an interpretation of the structure as the representation of some economic situation, it allows connecting a question one is interested in with specific manipulations of the model structure, and, finally, it incorporates the results from the manipulation, thus yielding a narrative account of events that is backed up with the exact and rigorous power of a formal structure. Using an economic model, therefore, never means only using a formal structure. Rather, it means using a composite object consisting of a formal structure and a story, both of which play crucial parts in the scientific use of models.

We believe that Morgan's discussion of models is essentially correct. Can it also be applied to game models and their model narratives in particular? In a recent paper, Morgan (2007) discusses this question for the special case of the prisoners' dilemma. Her initial reaction is sceptical. She found that

> … narratives filled in the middle space between a set of individually rational actors with a matrix of numbers and an equilibrium solution. The narratives gave accounts of the situations, but as stories they remained curiously unsatisfactory, for their endings were already pre-supposed, and the whole problem was how to get there. (Morgan 2007, p. 169)

Identifying a specific situation as being a prisoners' dilemma, she seems to have thought, already implies the solution to that situation: if two players are in a PD situation, then they will 'defect'. In this reading the story does not contribute to the 'solution' of the model. She then reconsiders her doubts:

> It is the interpretative text, not the matrix, that implicitly contains economists' traditional assumptions about individual rationality (i.e. which characterize the players) and about the necessity of equilibrium outcomes or solutions (a requirement of 'good' economic theories) and explicitly contains the rules of the game (i.e. noncollaboration, simultaneous moves, etc). All these, taken together, characterize the game situation. Thus the interpretative text is as necessary as the matrix of payoffs to the narratives that enable economists to use the game to reason in economic terms about another case at hand. (Morgan 2007, pp. 11–12)

We think that this claim needs further elaboration. As we have argued in section 2.3, it is the theory proper that specifies the various solution concepts and details the necessary conditions for their application. Hence we disagree with the claim that it is the interpretative text that contains the rationality assumptions and that entails the game solutions. As we have also argued, however, the theory proper provides a menu of solutions, and it requires extra-theoretical reasons to choose between them. In our view, then, the model narrative gives reasons for choosing a solution concept for the specific situation represented by the model. We will illustrate our claim with the following four examples.

The simplest solution concept is the elimination of dominated strategies (EDS). A player's strategy is dominated if she is better off choosing another available strategy, whatever the other players do. EDS eliminates all dominated strategies of all players. If each player has only one non-dominated strategy, a unique solution of the game ensues. Only relatively weak conditions have to be satisfied for EDS to be applicable. Each player has to know all strategies available to her, know the consequence of each strategy profile and make preference comparisons over them. Although weak, these conditions are not trivial. It may for example be that a certain situation is describable as a two-person game with only one non-dominated strategy for each player, but that the players are not aware of their evaluation of the consequences. If a model user wants to employ a unique solution ensuing from EDS in order to predict how players will choose, she will have to show that the conditions for EDS are satisfied. This requires information about the situation, which is found in the narrative account.

Most games cannot be solved by simple elimination of dominated strategies. In the game matrix of Figure 3, for example, only the player 'Row' has a dominated strategy, *R1*. Eliminating *R1* will not yield a unique solution. A more powerful solution strategy in such a case is the iterated elimination of dominated strategies. The idea is that player 'Column' anticipates that Row will not choose the dominated strategy *R1*, and hence eliminates *R1* from the set of possible strategy profiles. This in turn may convince Row that Column will not choose *C2*, because these are dominated by *C1* and *C3* in the reduced set of strategy profiles. This in turn may convince Column that Row will not choose *R3*, because it is dominated by *R2* in the twice-reduced set of strategy profiles. Finally, this may convince Row that Column

	C1	*C2*	*C3*
R1	3,2	1,3	1,1
R2	5,4	2,1	4,2
R3	4,3	3,2	2,4

Figure 3. Iterated elimination of dominant strategies.

will not choose *C3*, because it is dominated by *C1* in the thrice-reduced set of strategic profiles. Hence *R2, C1* emerges as the only non-dominated strategy.

The legitimate application of iterated elimination to a specific game matrix requires that the players have knowledge sufficient to perform such a reasoning process. In particular, it is required that the players know that the other players are rational, and that they know that the others know that they are rational. Such epistemic conditions are without question satisfied in many interactive situations. But crucially, whether the conditions are satisfied depends on the specific situation. In a good game model, this information will be found in the narrative: it justifies the application of iterated elimination to this specific game model.

A powerful and widely used solution concept of game theory is the Nash equilibrium. A Nash equilibrium is defined as a profile of strategies such that each player's strategy is an optimal response to the other players' strategies. Whether the Nash equilibrium is a good predictor of how players choose in a given situation is again a question dependent on the specificities of that situation. Kreps, for example, suggests that Nash is applicable only if there is some 'mutually self-evident way to play' (Kreps 1990, p. 31) and then explicates the mutual self-evidence as 'cues':

> … individuals who have never faced each other and who have never been in a situation very much like the one they now face may be able to see in the current situation cues that they think will be suggested to all concerned parties. … It is here that the notion of Nash equilibrium enters; if the cues are meant to suggest themselves to all parties concerned, then they should suggest a mode of behaviour that constitutes a Nash equilibrium. (Kreps 1990, p. 35)

Such cues are not available in many situations. Kreps mainly addresses the issue of finding the Nash equilibrium in general; but similar considerations apply to the selection of an equilibrium in symmetric coordination games with multiple Nash equilibria. An exemplary game structure of these types of games is depicted in Figure 4.

The game structure has two Nash equilibria: *R1, C1* and *R2, C2*. Because both players are equally well off in either equilibrium, neither does 'suggest [itself] to all parties concerned': the necessary clue is lacking. For a unique solution of the game, information is necessary that is not contained in the game structure. Schelling's (1960) theory of *focal points* suggests that the cues that tell us which strategy to coordinate on may be found in the specifics of the situation, or more correctly in the narrative account of the situation. Binmore gives an example of such a narrative:

> Two saboteurs are parachuted into enemy territory. During the jump, they unexpectedly get separated, but it is essential to their mission that they meet up again. If you were such a saboteur, seeking the rendezvous with your colleague, where would you go on the map shown in Figure [5]? (Binmore 1992, p. 296)

Again, it is the narrative that contains crucial information that may justify the application of a solution concept to obtain a prediction about how the game will be

	C1	*C2*
R1	1,1	0,0
R2	0,0	1,1

Figure 4. A symmetric coordination game.

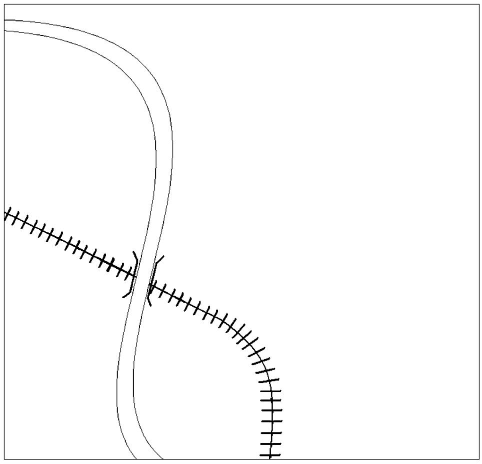


Figure 5. A non-verbal narrative.

played. Having information about the agents, their training, their goals, and their information (as detailed in the map of Figure 5) may help determining that both agents will choose the bridge from the continuum of possible meeting points.

Finally, backward induction is an often-used solution concept for extensive form games. Reinhard Selten, in his famous 'chain store paradox' paper (Selten 1978), argued that for specific situations, backward induction is not the appropriate solution concept.

The situation in question concerns an interaction between a single retail market leader (the chain store) and a finite number of potential local competitors, each one in a separate town. Applying backward induction yields the result that the chain store is always better off accommodating a local entrant than starting a price war, as price wars will be more costly than increased competition. Against this, Selten argues that the chain store may be able to establish a 'tough' reputation, by starting price wars against early entrants. Because potential entrants take the chain store's past behaviour into consideration, it is plausible that the chain store will be successful in deterring potential competitors from entering the market. This deterrence theory stands in marked contrast to the backwards induction solution concept: it takes different information into account, and it yields a very different solution.

> … the deterrence theory is much more convincing. If I had to play a game in the role of [the chain store], I would follow the deterrence theory. I would be very surprised if it failed to work. From my discussion with friends and colleagues, I get the impression that most people share this inclination. In fact, up to now I met nobody who said that he would behave according to the [backwards] induction theory. My experience suggests that mathematically trained persons recognize the logical validity of the induction argument, but they refuse to accept it as a guide to practical behaviour. (Selten 1978, pp. 132–133)

Selten argues on the basis of his and other people's intuitions, against the logical validity of the induction argument. Crucially, these intuitions depend on the account of a specific situation, in which the chain store and the potential entrants have specific characteristics not captured by the game structure.

In all four cases, the choice of which solution concept to apply to a given game structure in order to arrive at a prediction of the strategy profile played depended on considerations that lay outside the game structure. This choice problem is often described as a reliance on intuition:

> Does the solution concept exclude all equilibria that seem intuitively unreasonable? Does the intuitive logic of the solution concept seem compelling as a characterization of rational behaviour? … [The best we can do is] to formalize part of our intuitive criteria about how rational intelligent players might behave in a game. (Myerson 1991, pp. 240–241)

Crucially, the intuitions that justify the application of a solution concept are derived from the situation in question. The theory proper cannot be seen as a theory of universal rationality, which is applied to situations across the board; rather, it offers different solution concepts that characterize rational behaviour for different types of situation. The narrative accounts provide us with the intuition about the players' rationality in the respective situations. It is from the narrative that we draw the intuitions Myerson speaks about. Therefore, stories are necessary to choose and to apply a solution concept to a specific game structure.

4 Conclusion

Game-theoretic models consist of a formal game structure and an informal model narrative. When game theory is used to model economic situations, the narratives play a central role in interpreting, constructing and solving game structures. Narratives interpret existing game structures by specifying the extension of their predicates and the domain of their quantifiers, and by embedding these interpretations in a coherent account of an interactive event that calls for a solution. Further, stories are necessary to construct game structures from situations. They account for a situation in such a way that on its basis the formal elements provided by the theory proper can be assembled, effectively providing a narrative blueprint of the game structure. Last, narratives are necessary for the solution of games, in conjunction with the theory proper. By providing information about the rationality and epistemic state of the players in a specific situation, they legitimize the application of a solution concept to the game structures representing that situation. Without the narrative, model users lack justification for which solution concept to choose from the menu offered by the theory proper.

Model narratives, as well as the theory proper, are therefore necessary parts of the application of game models to economic situations. That they play a necessary part in the application of the solution concept has an important implication for game theory. Each solution concept enshrines a specific notion of rationality. If their application depends on the characteristics of a particular situation, game theory offers tools to model specific situations at varying degrees and kinds of rationality. It cannot be seen as a universal theory of rationality.

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Notes

1. 'Many economists describe the fifties as the era of *general equilibrium*, the sixties as the era of *growth* and the seventies as the era of *economics of information*. The eighties are the years in which economics has been revolutionized by *game theory*' (Rubinstein 1990, p. 11); and: 'One cannot find a field of economics … in which understanding the concept of Nash equilibrium is not nearly essential to the consumption of the recent literature' (Kreps 1990, p. 1).
2. Model-based theories contrast with direct theoretical representations. Economists commonly do not construct a direct representational system of the real world: they do not proceed by identifying patterns and structures of a phenomenon, working out which of these properties are essential and which ones can be abstracted away. Typical examples of direct theoretical representations are Mendeleev's periodic system and Darwin's theory of coral reefs. In those theories, researchers identify patterns in the data, develop these patterns to organizing hypotheses, and try to predict similar phenomena on the basis of the thus-formed hypothesis (Weisberg 2007, p. 14).
3. In principle, our analysis should be applicable to any field that applies game theory such as biology or sociology. Since we lack expertise in these, however, we restrict our domain to economics.
4. As every game represented in strategic form can also be represented in extensive form, the relation between the prisoners' dilemma and the form it is represented in is not necessary, but a matter of convention.
5. See also McCloskey (1983, p. 505): 'The word "story" has in fact come to have a technical meaning in mathematical economics … It means an extended example of the economic reasoning underlying the mathematics, often a simplified version of the situation in the real world that the mathematics is meant to characterize.'
6. The application of game theory to population genetics may substantiate such a warning: there one sees how animals, or even plants, can be 'players', and strategies are nothing more than their pre-programmed behavioural dispositions.
7. Guala (2006) recently argued that game theory is refutable and has been refuted, because it does not allow reciprocity to be modelled. Modelling reciprocity would require that the history of chosen strategies would *endogenously* modify the payoffs. This modelling move is ruled out in standard game theory, due to the way the expected utility function is characterized. If Guala's argument is correct, the theory proper may be interpreted as claiming that reciprocity does not exist. This is clearly an empirical claim (which may well be false). We think, however, that Guala's argument more appropriately applies to the class of models that can be constructed from the theory proper to model reciprocal behaviour. It is those models that claim to represent a specific situation, and which may be empirically inadequate if reciprocity is a significant factor in it.
8. Compare with the discussion of the 'toolbox of science' in Cartwright, Shomar, and Suarez (1995).
9. This is apparently how the famous prisoners' dilemma came into existence. Two mathematicians at RAND, upon receiving notice from Nash's equilibrium concept, designed an experiment in which each player had a dominant strategy to defect, but both would earn more if they both used the cooperative strategy. One of Nash's professors, Al W. Tucker, saw the payoffs for this game written on the blackboard, and he invented the story. He later used it in a lecture on game theory in the psychology department at Stanford, from where it started its parade through all game theory textbooks to come (Tucker 1980).

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