On Symmetries and Metrics in Geometric Inference

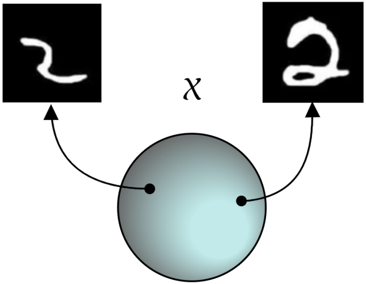
Giovanni Luca Marchetti

Supervisor: Prof. Danica Kragic
Co-Supervisor: Dr. Anastasiia Varava

I. Overview

The Geometry of Data

Data naturally carry geometric structure.



A data-set

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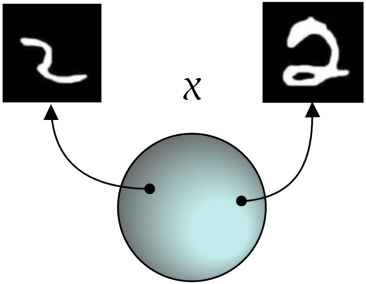

A data-~~set~~-manifold

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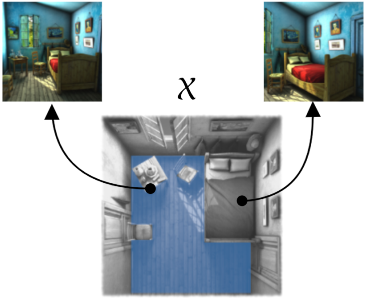


A data~~-set~~-manifold

The Geometry of Data

Data naturally carry geometric structure.

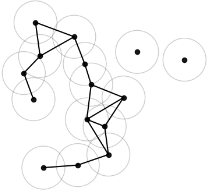
Question: How can the geometric structure be extracted statistically from data and exploited for inference?

A data-~~set~~-manifold

Geometric Methods

Geometric structure can be extracted in several forms.

- Combinatorial: graphs and simplicial complexes [4]



[4] Edelsbrunner et al., "Computational topology: an introduction", 2022
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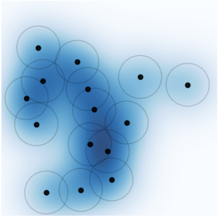

- Combinatorial: graphs and simplicial complexes [4]
- Fuzzy: density estimators [13]

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Geometric Methods

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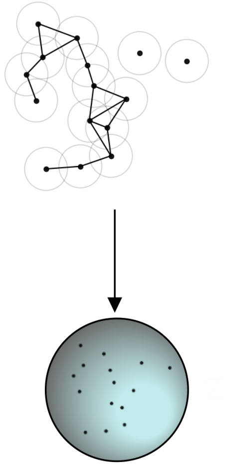
- Combinatorial: graphs and simplicial complexes [4]
- Fuzzy: density estimators [13]
- Smooth: representations [2]



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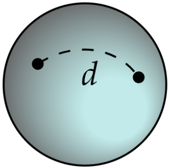
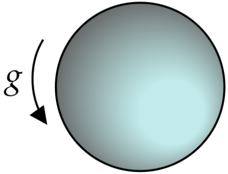
Symmetries and Metrics

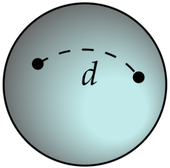


Metrics



Symmetries

Symmetries and Metrics



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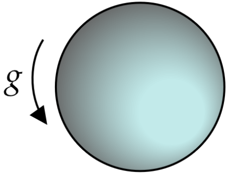


Symmetries

- Local in nature
- Can be inferred from data
- Suitable for non-parametric approaches

- Global in nature
- Unknown a priori
- Require powerful parametric approaches

Included Papers

(A) *Active Nearest Neighbor Regression Through Delaunay Refinement*
Kravberg*, Marchetti*, Polianskii* et al., ICML 2022

(B) *Voronoi Density Estimator for High-Dimensional Data: Computation, Compactification and Convergence*
Polianskii*, Marchetti* et al., UAI 2022

(C) *An Efficient and Continuous Voronoi Density Estimator*
Marchetti et al., AISTATS 2023, **Notable Paper Award**

(D) *Equivariant Representation Learning via Class-Pose Decomposition*
Marchetti*, Tegnér* et al., AISTATS 2023

(E) *Equivaraint Representation Learning in the Presence of Stabilizers*
Rey*, Marchetti* et al., ECML 2023

(F) *Back to the Manifold: Recovering from Out-of-Distribution States*
Reichlin, Marchetti et al., IROS 2022

(G) *Harmonics of Learning: Universal Fourier Features Emerge in Invariant Networks*
Marchetti et al., preprint 2024

*Equal contribution

II. Metric-Based Methods

Papers A, B and C

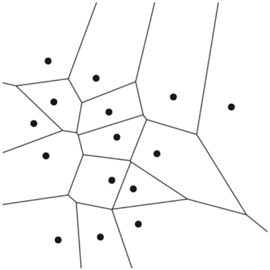
Voronoi Tessellations

Definition

Let $\mathcal{X}$ be a metric space and $P \subseteq \mathcal{X}$ finite. The Voronoi cell of $p \in P$ is

$$C(p) = \{x \in \mathcal{X} \mid \forall q \in P \; d(x, q) \geq d(x, p)\}.$$

The Delaunay triangulation is dual to the Voronoi tessellation.



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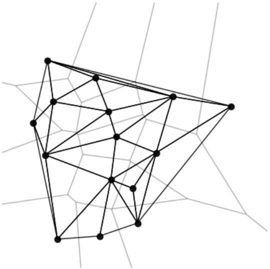

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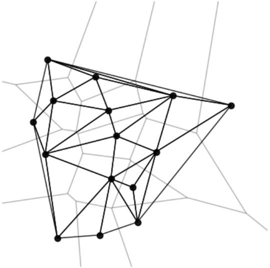
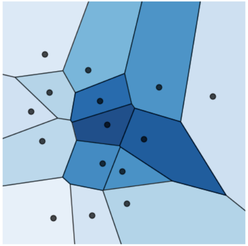


- ✓ Arbitrary convex polytopes
- ✓ Locally adaptive
- ✗ Expensive to compute

Nearest Neighbor Regressor (NNR)

NNR [5] approximates an unknown function f by the value at the closest datapoint. It is locally constant on Voronoi cells.



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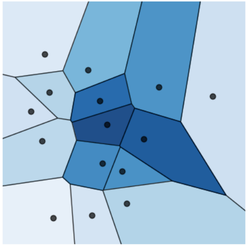

We upgrade NNR to an active regressor by querying a novel datapoint p_{t+1} at step t based on the current dataset P_t.

[5] Fix et al., "Discriminatory analysis. Nonparametric discrimination: Consistency properties", 1989

Active Nearest Neighbor Regressor (ANNR)

Work A

Our querying strategy exploits the geometry of the graph of f discretized via the Delaunay triangulation:

[A] *Kravberg*, Marchetti*, Polianskii* et al., "Active Nearest Neighbour Regression Through Delaunay Refinement", 2022.*

Active Nearest Neighbor Regressor (ANNR)

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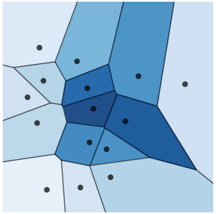
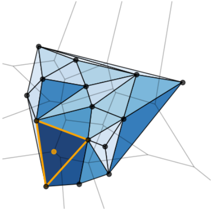
Our querying strategy exploits the geometry of the graph of f discretized via the Delaunay triangulation:

$$p_{t+1} = \mathrm{Circ}(\overline{\sigma}), \qquad \overline{\sigma} = \mathrm{argmax}_{\sigma \in \mathrm{Del}_{P_t}} \mathrm{Vol}(\hat{\sigma}).$$

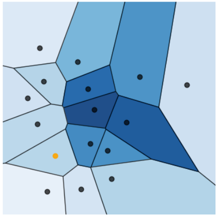
Here $\hat{\sigma}$ is the lifting of σ to the graph of λf, where λ controls exploration-exploitation.



P_t



Query



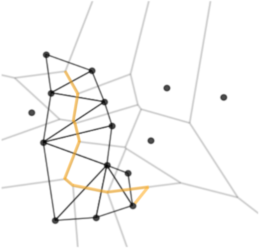
P_{t+1}

[A] *Kravberg*, Marchetti*, Polianskii* et al., "Active Nearest Neighbour Regression Through Delaunay Refinement", 2022.*

Computation and Theory

Work A

We compute the Delaunay triangulation approximately via a random walk over the Voronoi boundaries [11].



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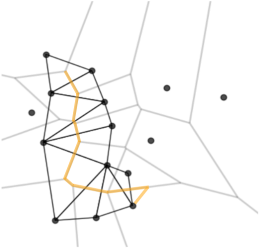

We prove halting guarantees:

Theorem

If f is Lipschitz then
$\lim_{t\to\infty} \max_{\sigma \in \mathrm{Del}_{P_t}} \mathrm{Vol}(\hat{\sigma}) = 0.$

and provide a Riemannian interpretation:

Theorem

If $f \in C^1(\Omega)$ then
$\log \mathrm{Vol}(\Gamma_{\lambda f}) \gtrsim C\lambda^2 \left\| f - \fint_\Omega f \right\|_2^2 + \log \mathrm{Vol}(\Omega).$

[A] Kravberg*, Marchetti*, Polianskii* et al., "Active Nearest Neighbour Regression Through Delaunay Refinement", 2022.
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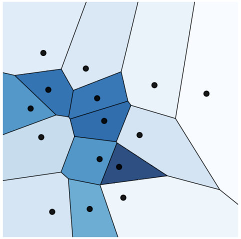
Voronoi Density Estimator (VDE)

VDE [9] is inversely proportional to the volume of Voronoi cells.



VDE

[9] Ord, "How many trees in a forest", 1978
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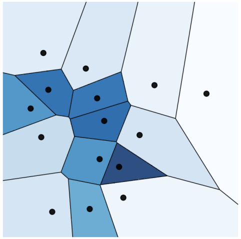

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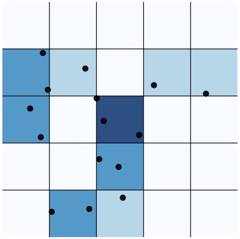
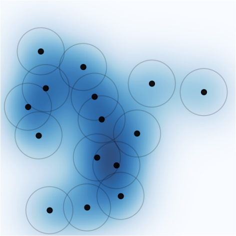


Histograms [10]



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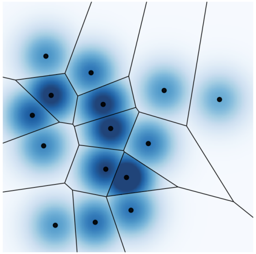
Compactified Voronoi Density Estimator (CVDE)

Work [B]

We compactify (unbounded) cells via a kernel K:



CVDE

$$\hat{\rho}(x) = \frac{K(x-p)}{|P| \int_{C(p)} K(y-p)\mathrm{d}y}, \quad x \in C(p).$$

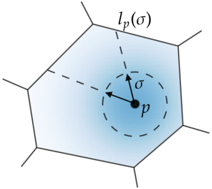
[B] *Polianskii*, Marchetti* et al., "Voronoi Density Estimator for High-Dimensional Data", 2022.*

Computation and Convergence

Work [B]

We compute the volumes via Monte-Carlo spherical integration:

$$\int_{\mathbb{S}^{n-1}} \int_0^{l_p(\sigma)} K(t\sigma) t^{n-1} \mathrm{d}t \mathrm{d}\sigma$$



[B] Polianskii*, Marchetti* et al., "Voronoi Density Estimator for High-Dimensional Data", 2022.

Computation and Convergence

Work B

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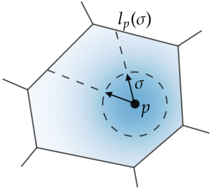


We prove convergence of CVDE to the ground-truth density ρ.

Theorem

Suppose that ρ has support in the whole $\mathbb{R}^n$. For any $K \in L^1(\mathbb{R}^n \times \mathbb{R}^n)$, CVDE, seen as a random measure, converges to ρ in distribution w.r.t x and in probability w.r.t. P as $|P| \to \infty$.

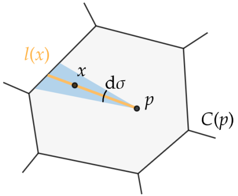
[B] Polianskii*, Marchetti* et al., "Voronoi Density Estimator for High-Dimensional Data", 2022.

An Improved Voronoi Density Estimator

CVDE is:

- ✓ Adaptive and convergent
- ✓ More efficient than VDE but less than KDE
- ✗ Discontinuous

An Improved Voronoi Density Estimator



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- ✓ Adaptive and convergent
- ✓ More efficient than VDE but less than KDE
- ✗ Discontinuous

We improve this by exploiting the radial geometry of Voronoi tessellations: $l(x)$ is continuous and computable in $O(|P|)$.

Radial Voronoi Density Estimator (RVDE)

Work C

For a kernel K, we define a radial bandwidth $\beta = \beta(I(x))$ implicitly via an integral equation:

[C] *Marchetti et al., "An Efficient and Continuous Voronoi Density Estimator", 2023.*

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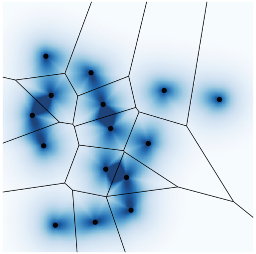

RVDE

$$\hat{\rho}(x) \propto \frac{K(\beta d(x,p))}{|P|}, \quad x \in C(p),$$

$$\underbrace{\int_0^{l(x)} t^{n-1} K(\beta t)\, \mathrm{d}t}_{\text{Conical integral}} = \text{const}$$

In particular, $\int_{C(p)} \hat{\rho}(x)\ \mathrm{d}x$ is constant, implying convergence.

[C] Marchetti et al., "An Efficient and Continuous Voronoi Density Estimator", 2023.

Modes and Performance

Work [C]

The map $I \mapsto \beta(I)$ generalizes Lambert's function. In particular:

Theorem

The modes of RVDE are located among nodes and edges of the Gabriel graph.

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Modes and Performance

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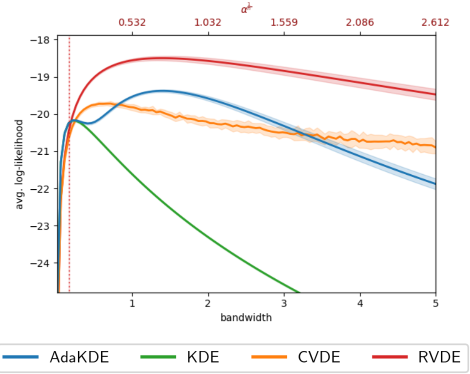
RVDE performs well:



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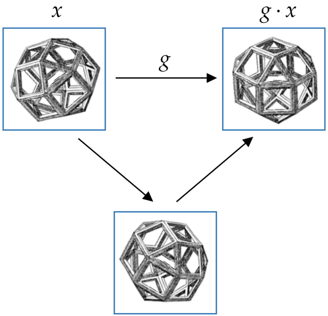
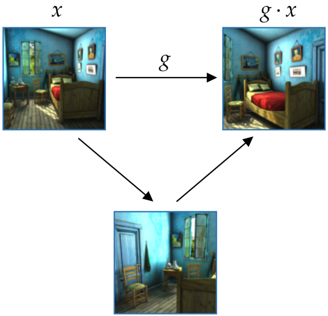
III. Symmetry-Based Methods

Papers D, E, F and G

Symmetries of Data

Data naturally exhibit symmetries.





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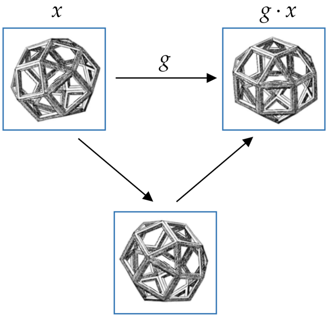
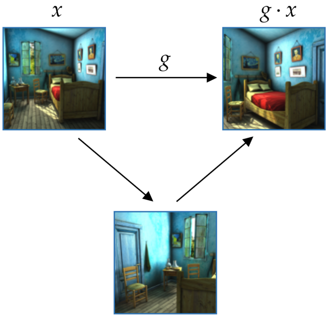
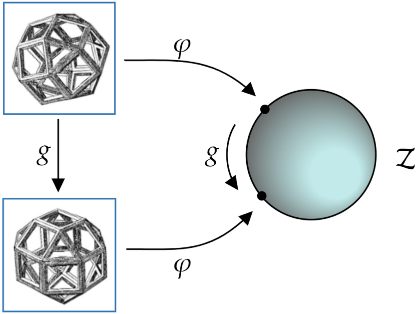




Symmetries are modelled as an unknown action $G \times \mathcal{X} \to \mathcal{X}$ by a group G on $\mathcal{X}$.

Equivariant Representations

A representation respecting symmetries is deemed equivariant.



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[7] Kipf et al., "Contrastive learning of structured world models", 2020
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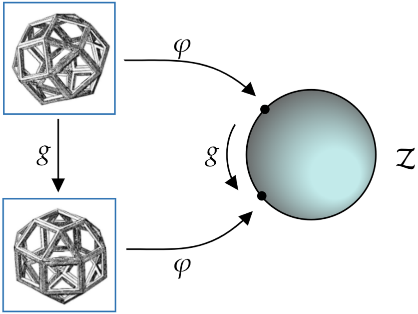

They are related to:

- Convolutional and graph neural networks [3]
- World models, incorporating interactions into representations [7, 1]

[3] Bronstein et al., "Geometric deep learning: Grids, groups, graphs, geodesics, and gauges", 2021
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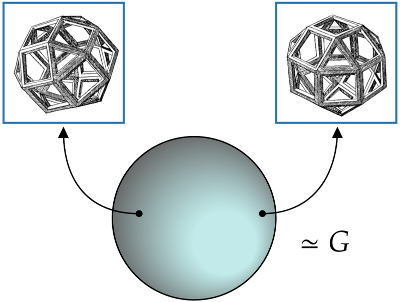
Geometry from Symmetries

Work D



- Group actions determine classes deemed orbits $\mathcal{E} = \mathcal{X}/G$

[D] *Marchetti*, Tegner* et al., "Equivariant Representation Learning via Class-Pose Decomposition", 2023.*

Geometry from Symmetries

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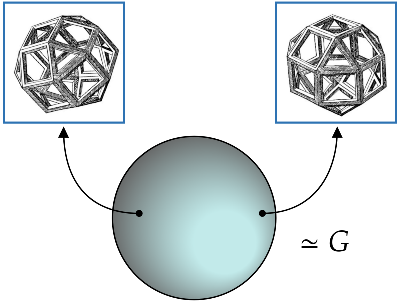

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- For free group actions, each orbit is isomorphic to G and thus $\mathcal{X} \simeq \mathcal{E} \times G$

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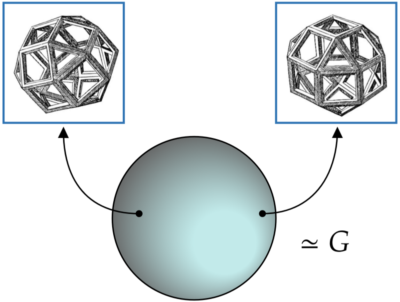

- Group actions determine classes deemed orbits $\mathcal{E} = \mathcal{X}/G$
- For free group actions, each orbit is isomorphic to G and thus $\mathcal{X} \simeq \mathcal{E} \times G$
- Every orbit-preserving equivariant map $\varphi : \mathcal{X} \to \mathcal{E} \times G$ is an isomorphism

[D] Marchetti, Tegner* et al., "Equivariant Representation Learning via Class-Pose Decomposition", 2023.*

Equivariant Isomorphic Networks (EquIN)

Work D

We propose to learn a representation $\varphi\colon \mathcal{X} \to \mathcal{E} \times G$ by optimizing:

$$\mathcal{L}(x, g, y = g \cdot x) = \underbrace{d_{\mathcal{E}}(\varphi_{\mathcal{E}}(y),\ \varphi_{\mathcal{E}}(x))}_{Invariant/Contrastive} + \underbrace{d_G(\varphi_G(y),\ g\varphi_G(x))}_{Multiplication-Equivariant}$$

[D] Marchetti*, Tegner* et al., *"Equivariant Representation Learning via Class-Pose Decomposition"*, 2023.
[6] Higgins et al., *"Towards a definition of disentangled representations"*, 2018

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Work D

We propose to learn a representation $\varphi \colon \mathcal{X} \to \mathcal{E} \times G$ by optimizing:

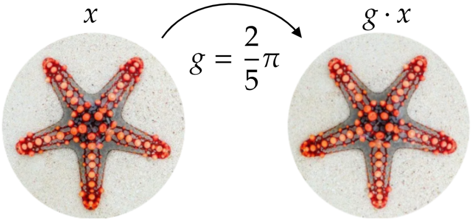
$$\mathcal{L}(x, g, y = g \cdot x) = \underbrace{d_{\mathcal{E}}(\varphi_{\mathcal{E}}(y),\ \varphi_{\mathcal{E}}(x))}_{Invariant/Contrastive} + \underbrace{d_G(\varphi_G(y),\ g\varphi_G(x))}_{Multiplication-Equivariant}$$

At the optimum, φ is an isomorphism. In particular, it is:

- ✓ Lossless
- ✓ Disentangled [6]
- ✗ Based on the assumption that G known a priori

[D] *Marchetti*, Tegner* et al., "Equivariant Representation Learning via Class-Pose Decomposition", 2023.*
[6] *Higgins et al., "Towards a definition of disentangled representations", 2018*

Non-Free Group Actions



In general, it is necessary to consider stabilizer subgroups:

$$G_x = \{g \in G \mid g \cdot x = x\}.$$

EquIN with Stabilizers

Work E

- Each orbit $O \subseteq \mathcal{X}$ is isomorphic to the coset space G/G_{x_O}, $x_O \in O$.

[E] *Rey*, Marchetti* et al., "Equivariant Representation Learning in the Presence of Stabilizers", 2023.*

EquIN with Stabilizers

Work E

- Each orbit $O \subseteq \mathcal{X}$ is isomorphic to the coset space G/G_{x_O}, $x_O \in O$.
- Any orbit-preserving equivariant map

$$\varphi : \mathcal{X} \to \coprod_{O \in \mathcal{X}/G} G/G_{x_O}$$

is an isomorphism.

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[E] Rey, Marchetti* et al., "Equivariant Representation Learning in the Presence of Stabilizers", 2023.*

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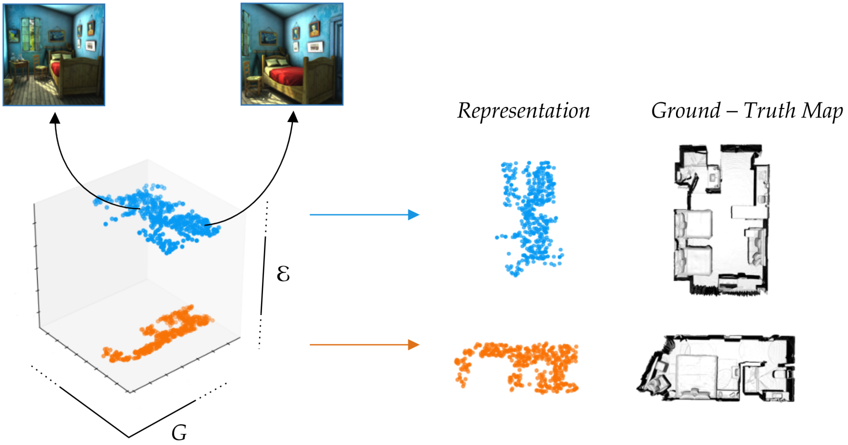
We generalize EquIN to $\varphi\colon \mathcal{X} \to \mathcal{E} \times 2^G$, with an entropy loss term for minimality.

[E] Rey*, Marchetti* et al., "Equivariant Representation Learning in the Presence of Stabilizers", 2023.

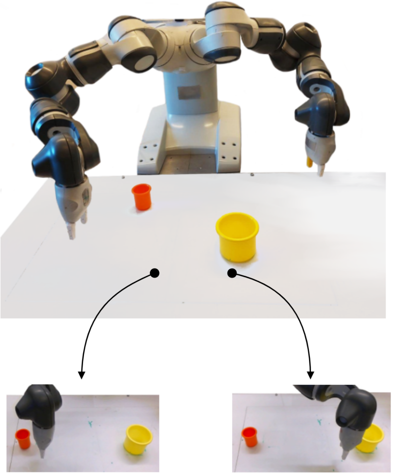
Extracting Geometry

EquIN extracts isometric maps of the world.



Back to the Manifold

Work F

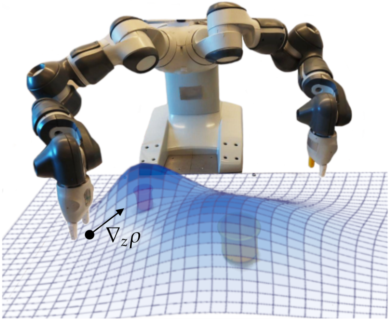


The extracted geometry can be exploited to address problems via classical methods.

[F] *Reichlin, Marchetti et al., "Back to the Manifold: Recovery from Out-of-Distribution States", 2022.*

Back to the Manifold

Work [F]



The extracted geometry can be exploited to address problems via classical methods.

We propose to stabilize a policy by using the estimated latent density ρ as a potential:

$$\widetilde{\pi}(x) = \nabla_z \rho(\varphi(x)).$$

[F] *Reichlin, Marchetti et al., "Back to the Manifold: Recovery from Out-of-Distribution States", 2022.*

Abstract Harmonic Analysis

Suppose that G is unknown. Is it possible to discover symmetries from data?

Abstract Harmonic Analysis

Group theory is intimately related to harmonic analysis.

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Definition

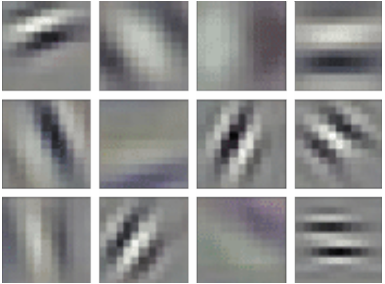
The Fourier transform is a linear isometric isomorphism of the form:

$$\mathbb{C}^G \to \bigoplus_{\rho\colon G \to \mathrm{U}(V)} \mathrm{End}(V),$$

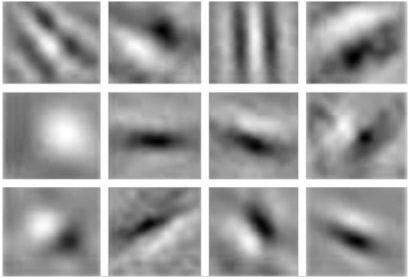
where ρ is an irreducible unitary representation (irrep) of G in a Hilbert space V.

Emergence of Harmonics

Harmonics are ubiquitous in both biological and artificial networks.



AlexNet



Macaque

[8] Krizhevsky et al., "Imagenet classification with deep convolutional neural networks", 2012
[14] Zylberberg et al., "A sparse coding model with synaptically local plasticity and spiking neurons can account for the diverse shapes of V1 simple cell receptive fields", 2011

Harmonics of Learning

Work G

We show that, under certain conditions, invariance of $\varphi(W, x)$ in x w.r.t. to a finite group G implies emergence of harmonics in W.

[G] *Marchetti et al., "Harmonics of Learning: Universal Fourier Features Emerge in Invariant Networks", 2024.*

Harmonics of Learning

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We show that, under certain conditions, invariance of $\varphi(W, x)$ in x w.r.t. to a finite group G implies emergence of harmonics in W.

Informal Theorem

If φ is invariant then each component of W is an irrep of G up to a linear transformation. In particular, if W is orthonormal, it (almost) coincides with the Fourier transform.

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Informal Theorem

If φ is invariant then each component of W is an irrep of G up to a linear transformation. In particular, if W is orthonormal, it (almost) coincides with the Fourier transform.

Informal Theorem

If φ is 'almost invariant' and the W is 'almost orthonormal', then G can be recovered from W up to isomorphism.

[G] Marchetti et al., *"Harmonics of Learning: Universal Fourier Features Emerge in Invariant Networks"*, 2024.

Future Work

Extensions to other metric spaces, e.g. Riemannian manifolds.

- Spheres for directional statistics.
- Hyperbolic spaces for hierarchical data.
- Complex projective spaces: Kendall shape space.

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Extensions to other metric spaces, e.g. Riemannian manifolds.

- Spheres for directional statistics.
- Hyperbolic spaces for hierarchical data.
- Complex projective spaces: Kendall shape space.

Extensions to algebraic structures beyond groups:

- Groupoids for local symmetries.
- C^*-algebras for more general transformations.

Tack!

References I

[1] Kartik Ahuja, Jason Hartford, and Yoshua Bengio. "Properties from mechanisms: an equivariance perspective on identifiable representation learning". In: *ICLR* (2022).

[2] Yoshua Bengio, Aaron Courville, and Pascal Vincent. "Representation learning: A review and new perspectives". In: *IEEE transactions on pattern analysis and machine intelligence* 35.8 (2013), pp. 1798–1828.

[3] Michael M Bronstein et al. "Geometric deep learning: Grids, groups, graphs, geodesics, and gauges". In: *arXiv preprint arXiv:2104.13478* (2021).

[4] Herbert Edelsbrunner and John L Harer. "Computational topology: an introduction". In: (2022).

References II

[5] Evelyn Fix and Joseph Lawson Hodges. “Discriminatory analysis. Nonparametric discrimination: Consistency properties”. In: *International Statistical Review/Revue Internationale de Statistique* 57.3 (1989), pp. 238–247.

[6] Irina Higgins et al. “Towards a definition of disentangled representations”. In: *arXiv preprint arXiv:1812.02230* (2018).

[7] Thomas Kipf, Elise van der Pol, and Max Welling. “Contrastive learning of structured world models”. In: *ICLR* (2020).

[8] Alex Krizhevsky, Ilya Sutskever, and Geoffrey E Hinton. “Imagenet classification with deep convolutional neural networks”. In: *Advances in neural information processing systems* 25 (2012).

[9] JK Ord. “How many trees in a forest”. In: *Mathematical Scientist* 3 (1978), pp. 23–33.

References III

[10] Karl Pearson. "Contributions to the mathematical theory of evolution". In: *Philosophical Transactions of the Royal Society of London. A* 185 (1894), pp. 71–110.

[11] Vladislav Polianskii and Florian T Pokorny. "Voronoi Graph Traversal in High Dimensions". In: *KDD*. 2020.

[12] Murray Rosenblatt. "Remarks on Some Nonparametric Estimates of a Density Function". In: *The Annals of Mathematical Statistics* 27.3 (1956), pp. 832–837. ISSN: 00034851. URL: http://www.jstor.org/stable/2237390.

[13] David W Scott. "Multivariate density estimation: theory, practice, and visualization". In: (2015).

References IV

[14] Joel Zylberberg, Jason Timothy Murphy, and Michael Robert DeWeese. "A sparse coding model with synaptically local plasticity and spiking neurons can account for the diverse shapes of V1 simple cell receptive fields". In: *PLoS computational biology* 7.10 (2011), e1002250.