

Topics in Enumerative Geometry (Winter 2019/20)

Lecturer: Georg Oberdieck

Date: Friday 2-4pm, Endenicher Allee 60, Seminarraum 1.008

Oral Exam: In the week February 3-7 (repetition date, March 16-20)

Summary: The course is an introduction to the intersection theory of algebraic varieties with application to the enumerative geometry of algebraic curves.

Prerequisites: Algebraic Geometry (on the level of Hartshorne book Chapters I-III, or as taught in Algebraic Geometry I and II), singular cohomology.

References: The main reference for the first half of the book is Fulton's book *Intersection Theory*. There are several other lecture notes and books on intersection, for example the book *3264 and all that* by Eisenbud and Harris, or notes by Gathmann (2004 Version, Section 9, 10) <https://www.mathematik.uni-kl.de/~gathmann/en/alggeom.php>. However, Fulton's book remains essential. Below we will point out also several other references.

Schedule¹

Oct 11: Three and a half examples: Lefschetz trace formula, number of lines that meet 4 given lines in space, number of degree d rational curves in \mathbb{P}^2 passing through $3d - 1$ points, number of bitangents of a degree d plane curve.

Oct 18: Definition Chow groups, Pushforward of cycles (proof this and next lecture)

Oct 25: Pullback of cycles, characterization of rational equivalence in terms of $X \times \mathbb{P}^1$, Characterization of the rational-equivalent cycles as the smallest collection of subgroups which is stable under pullback and pushforward, and contains $[0] - [\infty]$ on \mathbb{P}^1 .

Nov 1: No class.

Nov 8: Discussion intersection product (in particular the naive attempt), intersection with Cartier divisors/line bundles. Nov 15: Localization exact sequenc $(A_k(X) \rightarrow A_k(Y) \rightarrow A_k(U) \rightarrow 0$ if $X \subset Y$ closed with complemen U), affine bundles, projective bundles, Segre classes, Chern classes, description of Chow ring of projective bundle, Grothendieck relation, example: Hirzebruch surface $\mathbb{P}(\mathcal{O} \oplus \mathcal{O}(d))$ (in particular Chow ring, classes of zero and infinity section, canonical class)

Nov 22: Proof of the description of Chow ring of projective bundle in last

¹As of February 7, 2020

lecture. Construction Gysin map for vector bundles. Normal bundle, normal cone, definition intersection product, Nov 29: Deformation to the normal cone. Proof that intersection product preserves rational equivalence, Definition Chow ring. Properties of Chern classes (Whitney sum, description of top chern class in terms of vanishing locus of regular section), splitting principle.

Dez 6: Chern character, lines on cubic surface, tritangent problem (in a general pencil of degree d plane curves, how many have a tritangent?). See notes on the website.

Dez 13: Connection of tritangent problem with Noether-Lefschetz theory (see reference: Maulik, Pandharipande, Noether-Lefschetz theory and Gromov-Witten theory); Cycle class map. Discussion of Weil conjectures leading to Grothendieck standard conjectures (Künneth decomposition of the diagonal).

Dez 20: Beauville Decomposition of Chow ring of abelian variety. A good reference is [Birkenhage, Lange - Complex abelian varieties].

Jan 10: Grothendieck-Riemann-Roch formula. K -theory.

Jan 17: Chow ring of Grassmannian, Tautological ring of moduli space of curves, examples (following the discussion of Mumford's paper 'Towards an enumerative geometry of curves', Faber-Zagier relations.

Jan 24: No class.

Jan 31: Section 2 and 3 of Yin's thesis 'Tautological cycles on curves and Jacobians'