Problem List I, Topics in Enumerative Geometry

Lecturer: Georg Oberdieck

1. Determine the number of lines on a very general quintic threefold $X \subset \mathbb{P}^4$.

(You may assume the locus of these lines in the Grassmannian is a finite set of reduced points).

2. In a general pencil of degree d plane curves, how many are singular?

3. Let $G = G(2,4)$ be the Grassmannian of lines in $\mathbb{P}^3$. Let $U \subset \mathbb{C}^4 \otimes \mathcal{O}_G$ be the tautological subbundle and let $c_i = c_i(U)$. We have (see later)

$$A^*_{\mathbb{Q}}(G) = \mathbb{Q}[c_1, c_2]/(c_1^3 = 2c_1c_2, c_2^2 = c_1^2c_2, c_2^3, c_1^5).$$

Find the classes $[Z] \in A^*(G)$ of the following loci $Z \subset G$ in terms of c_1, c_2:

a) $\{[\ell] \in G | \ell \cap \ell_0 \neq \varnothing\}$ where $\ell_0 \subset \mathbb{P}^3$ is a fixed line.
b) The class of a point $p \in G$
c) $\{[\ell] \in G | p \in \ell\}$ where $p \in \mathbb{P}^3$ is a fixed point.
d) $\{[\ell] \in G | \ell$ is contained in $Q\}$ where $Q \subset \mathbb{P}^3$ is a fixed quadric.
d) Let $S \subset \mathbb{P}^3$ be a quartic surface. The loci F of bitangents to S,

$$\{\ell \in G | \ell \text{ is tangent to } S \text{ at 2 points } \}.$$

e) (Bonus) Can you calculate the topological Euler characteristic of F? (Hint: Use

$$0 \to T_F \to T_G|_F \to E|_F \to 0$$

where E is a certain bundle on G.)

4. In a general pencil of degree d curves, how many have a 4-flex? (I.e. a line that is tangent to the curve with order 4)

5. In a general pencil of quartic surfaces $S_\lambda \subset \mathbb{P}^3$, how many contain a line? How many are singular?

(Bonus: Compare your answer with the Noether-Lefschetz numbers in [Maulik-Pandharipande, Gromov–Witten theory and Noether-Lefschetz theory, Thm 2/Cor2)

6. In a general pencil of cubic 4-folds $Y \subset \mathbb{P}^5$, how many contain a plane?

7. Let $S \subset \mathbb{P}^3$ be a general degree d surface. How many tritangent planes does S have? (This is called Salmon's formula but might be night so easy).

Due date: If you hand in solutions, we will grade them. Due date for Problems 1-3 is December 13. Due date for Problems 4-7 is January 10.