

List of presentations for the lecture Algebraic Stacks

Presentation 1: (Nov 27, 30 min) Example of an algebraic space that is not a scheme.

Speaker: Felix Jäger

Present Example 5.3.2 in Olsson on an algebraic space that is not a scheme. In case of interest you could also talk about Section 4.4.2 as an example of failure of descent for smooth morphism. See also [Hartshorn, Algebraic Geometry, App.B. 3.4.2].

Presentation 2: (Nov 27, 20 min) Failure of descent for non-polarized schemes.

Speaker:

Present the example by Raynaud that shows that descent for proper morphisms can fail. It also shows that the fibered category \mathcal{M}_1 as defined in Vistoli, Example 4.39, does not satisfy effectiveness of descent and hence is not a stack. Reference: [2, XIII 3.2] (in French!).

Presentation 3: (Dec 4, 30 min) De Rham Cohomology of differentiable stacks.

Speaker: Mingjia Zhang

Follow the first pages of [1] and introduce de Rham cohomology for differentiable stacks (Definition 9).

Presentation 4: (Dec 23, 30 min) Cohomology of BG and Group cohomology.

Speaker: Mingyu Ni

Let G be a finite group. Relate the group cohomology of finite groups to the sheaf cohomology of quasi-coherent sheaves on BG . This can be done in two ways: Either by showing that both theories give the right derived functor of the functor taking invariants, or by Čech cohomology. Unfortunately, I don't know a great reference for this. On the group cohomology side a nice source is the book 'Cohomology of groups' by Brown. Section III.1. On the stack side there is a discussion about this connection in Section 11.6.3 of Olsson.

Presentation 5: (Jan 15, 30 min) Lefschetz trace formula for algebraic stacks.

Speaker: Ali Barkhordarian

This talk is about the analog of the trace formula relating the number of \mathbf{F}_q -points of a smooth variety with the trace of Frobenius over the cohomology. This has been conjectured and then later proven by Behrend. The formula related the number of \mathbb{F}_q points of a smooth algebraic stack of finite type over \mathbf{F}_q to the trace of Frobenius on its ℓ -adic cohomology. The goal of the talk is very modest: State the formula in the stacks context, and calculate the example of $B\mathbb{G}_m$.

References: The introductions to the following papers:

Behrend, The Lefschetz trace formula for algebraic stacks, *Inventiones*

Behrend, Derived l-adic categories for algebraic stacks, *Mem. AMS* 774,
available:

<https://www.math.ubc.ca/~behrend/ladic.pdf>

Presentation 6: The stack $\overline{M}_{1,1}$ and modular forms

Speaker: Thiago Solovera e Nery (Feb 5)

The goal of the talk should be to interpret the space of modular forms Mod_k of weight k for the full modular group $\text{SL}_2(\mathbb{Z})$ as sections of a certain line bundle on the stack $\overline{M}_{1,1}$. If somebody wants to work this out, one can go also further and calculate the dimensions of Mod_k via Grothendieck-Riemann-Roch for DM stacks. Reference: Course notes of Milne, <https://www.jmilne.org/math/CourseNotes/MF.pdf>

REFERENCES

- [1] Behrend, Cohomology of Stacks, Lecture Notes, ICTP <https://www.math.ubc.ca/~behrend/CohSta-1.pdf>
- [2] Michel Raynaud, *Faisceaux amples sur les schmas en groupes et les espaces homogènes*, Lecture Notes in Mathematics, Vol. 119 Springer-Verlag, Berlin-New York 1970 ii+218 pp.