KTH Royal Institute of Technology
Numerical Analysis Group

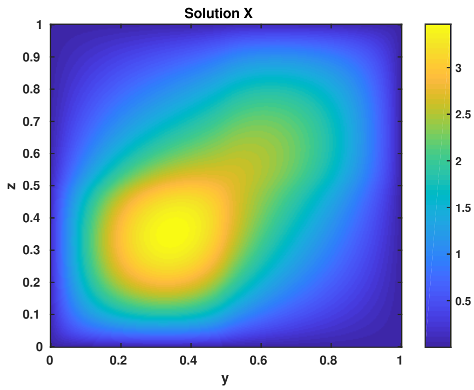
Project proposal

Bachelor thesis project proposal:
Numerical integration based methods for Sylvester equations

Project description. The Sylvester equation is formulated as

$$AX + XB^T = C, \quad (1)$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{m\times m}$, and $C \in \mathbb{R}^{n\times m}$ are given matrices. The matrix $X \in \mathbb{R}^{n\times m}$ is an unknown that we want to determine. The Sylvester equation arises in a variety of different contexts, such as dynamical systems and partial differential equations (but you do not need any prior knowledge about this to complete the project).



It is known that the solution to (1) can be expressed as any of the two integrals below,

$$X = -\frac{1}{4\pi^2}\int_{\Gamma_1}\int_{\Gamma_2}\frac{(\lambda I - A)^{-1}C(\mu I - B^T)^{-1}}{\lambda+\mu}d\mu d\lambda,$$

$$X = -\int_0^\infty e^{At}Ce^{B^T t}dt.$$

In basic courses you have learned how to compute integrals numerically (for instance trapezoidal rule), this is called quadrature. Based on the integral formulations above, the aim of this project is to develop new methods for computing approximative solutions to (1).

The project consists of tasks:

- Formulating the solution to the Sylvester equation as an integral.
- Identify numerical quadrature as a way to compute the solution. Investigate different quadrature rules. Compare computation costs and approximation properties.
- Implement an test some selected methods. Evaluate applicability of the methods by comparing with existing algorithms.

References:

- The lecture notes *Methods for Lyapunov equations* by E. Jarlbering for the course SF3580. `https://people.kth.se/~eliasj/NLA/matrixeqs.pdf`
- The survey paper *Computational methods for linear matrix equations* by V. Simoncini. `http://www.dm.unibo.it/~simoncin/public_matrixeq_rev.pdf`

Supervision: Emil Ringh, doktorand
Co-supervisor: Elias Jarlebring, docent
Supervision available in Swedish or English
contact: eringh@kth.se