Pilot-Based Bayesian Channel Norm Estimation in Rayleigh Fading Multi-Antenna Systems

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Outline

• Introduction: Channel Norm
• Bayesian Channel Norm Estimation
  • Unitary pilot signalling
  • Weighted Eigenpilot
• Simulation
• Summary
INTRODUCTION: CHANNEL NORM
Downlink communication

- Transmission from base station to users.
- Multiple antennas on the transmitter
  - Direct the signal towards the intended user
  - Multi-user transmissions (SDMA)
  - Resource allocation “require” some CSI
System model

- Narrowband channel to user $k$:
  \[ y_k(t) = h_k^H x(t) + n_k(t) \]
  - $N$ antennas at the base station
  - Arbitrarily correlated Rayleigh fading:
    \[ h_k \in \mathcal{CN}(0, R_k) \]
  - White complex Gaussian noise:
    \[ n_k(t) \in \mathcal{CN}(0, \sigma_k^2) \]
Squared Channel Norm

- Squared Euclidean norm: $\|h_k\|^2$
  - Measure of channel gain in all directions
- Many reasons to estimate and feed back:
Reasons to estimate the norm

- Scheduling and SNR estimation in correlated systems


BAYESIAN CHANNEL NORM ESTIMATION
Estimation of the channel norm

- The channel norm $\| h_k \|^2$ should be estimated at the receiver
  - Blind estimation
  - Semi-blind estimation
  - Pilot based estimation

- Why not estimate the channel first?
  - Not necessarily be unbiased and low error
  - Most approach in literature are non-Bayesian
Scalar case: Pilot-based estimation

- Estimation of $|h|^2$ from $y = h + n$ where $h \in \mathcal{CN}(0, \lambda), n \in \mathcal{CN}(0, \mu)$

- The conditional PDF is $f(|h|^2|y|^2) = \frac{\lambda + \mu}{\lambda \mu} e^{-|h|^2 \left(\frac{\lambda + \mu}{\lambda \mu}\right)} e^{-|y|^2 \left(\frac{\lambda}{\mu(\lambda + \mu)}\right)} I_0 \left(\frac{2}{\mu} |h||y|\right)$

with the modified Bessel function $I_{\nu}(\cdot)$ of the first kind.
Scalar case: MMSE estimator

- The Bayesian MMSE estimator is the mean value of $f(|h|^2||y|^2)$ and the MSE is the variance:

$$E\{|h|^2||y|^2\} = \lambda \mu \left(1 + |y|^2 \frac{\lambda}{\mu(\lambda + \mu)}\right)$$

$$V\{|h|^2||y|^2\} = \left(\frac{\lambda \mu}{\lambda + \mu}\right)^2 \left(1 + 2|y|^2 \frac{\lambda}{\mu(\lambda + \mu)}\right)$$
Vector case: Pilot-based estimation

- We transmit the columns of a matrix:
  \[ [y(0), \ldots, y(N - 1)] = h^H U + [n(0), \ldots, n(N - 1)] \]

- For a known unitary matrix \( U \)
  - Statistics are known: Whitening
  - MMSE estimator: Summation of scalar cases:
    \[
    E\{\rho_h | \tilde{\mathcal{Q}}_y\} = \sum_{j=1}^{N} \frac{\lambda_j \mu}{\lambda_j + \mu} \left( 1 + \tilde{q}_y^{(j)} \frac{\lambda_j}{\mu(\lambda_j + \mu)} \right)
    \]
Weighted eigenpilot

• The pilot matrix can be tailored to the channel statistics (of a single user)
  • Estimation along strong eigenmodes more important (we need power allocation)

• Let \( R = V\Lambda V^H \), \( \Lambda = \text{diag}(\lambda_1, \ldots, \lambda_N) \) and pilot matrix \( Q = VP \).

• We want \( P = \text{diag}(p_1, \ldots, p_N) \), \( \text{tr}(P) = P \) to minimize the average MSE.
Optimal power allocation

• Finding the optimal power allocation is a non-linear convex optimization problem
  • The Karush-Kuhn-Tucker conditions gives

\[ p_j = \max \left( 0, -\frac{\mu}{\lambda_j} + 2\Re((-1+i\sqrt{3})(\frac{27\mu^2}{\alpha} + \sqrt{\frac{27\mu^3(8\lambda_j^3 + 27\alpha\mu)}{\alpha^3}})^{1/3})) \right) \]

\[ \alpha \leq 0 \text{ chosen such that } \sum_{j=1}^{N} p_j = P. \]

• This is a waterfilling solution

• Observe: Only depends on the statistics
SIMULATION
Simulation

Comparison

- UCA, $N = 8$
- $15^\circ$ angular spread
- User at boundary of circular cell

- Typical eigenvalues:
  \[
  \{0.6693, 0.2809, 0.0450, 0.0045, 0.315 \cdot 10^{-3}, 0.952 \cdot 10^{-5}, 0.435 \cdot 10^{-6}, 0.234 \cdot 10^{-7}\}
  \]

- Corresponding power allocation:  \{4.455, 2.837, 0.708, 0, 0, 0, 0, 0\}
SUMMARY
Summary

- Resource allocation and SNR/capacity estimation is improved by norm feedback
- We have derived a closed-form Bayesian MMSE estimator of the norm
  - Unitary pilot matrix
  - Weighted eigenpilot matrix
- Algorithm for optimal power allocation
- Both methods have a low relative error