Standard Model reference

Elias Riedel Gårding

June 4, 2018

1 Yang–Mills theory

Gauge group G
Generators $\{t^a\}$; $U = e^{it^a\theta^a}$
Coupling constant g
Gauge field A^a_μ
Covariant derivative $D_\mu = \partial_\mu + igA^a_\mu T^a$
Field strength tensor $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf^{abc}A^b_\mu A^c_\nu \quad ([D_\mu, D_\nu] = igt^a F^a_{\mu\nu})$

Gauge transformation
Field: $\psi \mapsto e^{iT^a\theta^a}\psi$
Covariant derivative: $D_\mu\psi \mapsto e^{iT^a\theta^a} D_\mu\psi$
Gauge field: $A^a_\mu \mapsto A^a_\mu - \frac{1}{g}\partial_\mu\theta^a + f^{abc}A^b_\mu\theta^c$

2 Electroweak sector

Yang–Mills theory
$G_{\mathrm{EW}} = \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
Gauge bosons: W^1, W^2, W^3, B

$$\mathcal{L}^{\mathrm{EW}}_{\mathrm{YM}} = -\frac{1}{4}F^{Wa}_{\mu\nu}F^{Wa\mu\nu} - \frac{1}{4}F^{B}_{\mu\nu}F^{B\mu\nu}$$

Covariant derivative:

$$D_\mu = \partial_\mu + igW^a_\mu T^a + ig' B_\mu Y$$

Higgs field

$$\mathcal{L}^{\mathrm{EW}}_{\phi} = (D^\mu\phi)^\dagger(D_\mu\phi) - \mu^2|\phi|^2 - \lambda|\phi|^4 \qquad (\mu^2 < 0)$$

Leptons

$$\mathcal{L}^{\mathrm{EW}}_{\mathrm{lept}} = \bar{L}^i i\not{D} L^i + \bar{R}^i i\not{D} R^i \qquad L = \left(\begin{pmatrix}\nu_e \\ e_L\end{pmatrix}, \begin{pmatrix}\nu_\mu \\ \mu_L\end{pmatrix}, \begin{pmatrix}\nu_\tau \\ \tau_L\end{pmatrix}\right)$$

$$R = (e_R, \mu_R, \tau_R)$$

Lepton–Higgs coupling

$$\mathcal{L}_{\mathrm{lept},\phi} = -\sqrt{2}\Big(\lambda^{ij}\bar{L}^i\phi R^j + \underbrace{(\lambda^\dagger)^{ij}\bar{R}^i\phi^\dagger L^j}_{\text{h.c.}}\Big)$$

Quarks

$$\mathcal{L}^{\text{EW}}_{\text{quark}} = \bar{Q}^i_L i\not{D} Q^i_L + \bar{u}^i_R i\not{D} u^i_R + \bar{d}^i_R i\not{D} d^i_R$$

$$Q_L = \left(\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \right)$$
$$u_R = (u_R, c_R, t_R)$$
$$d_R = (d_R, s_R, d_R)$$

Quark–Higgs coupling

$$\mathcal{L}_{\text{quark},\phi} = -\sqrt{2}\Big(\lambda^{ij}_d \bar{Q}^i_L \phi d^j_R + \lambda^{ij}_u \bar{Q}^i_L \phi^c u^j_R + \underbrace{(\lambda^\dagger_d)^{ij} \bar{d}^i_R \phi^\dagger Q^j_L + (\lambda^\dagger_u)^{ij} \bar{u}^i_R \phi^{c\dagger} Q^j_L}_{\text{h.c.}} \Big)$$

($\phi^c = i\sigma^2 \phi^*$)

Spontaneous symmetry breaking $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{\mathrm{EM}}$

Unitary gauge: $\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$

Physical gauge bosons:

$$W^\pm_\mu = \frac{1}{\sqrt{2}} (W^1_\mu \mp i W^2_\mu)$$
$$Z_\mu = \cos\theta_W W^3_\mu - \sin\theta_W B_\mu$$
$$A_\mu = \sin\theta_W W^3_\mu + \cos\theta_W B_\mu$$

Covariant derivative:

$$D_\mu = \partial_\mu + \frac{ig}{\sqrt{2}} (T^+ W^+_\mu + T^- W^-_\mu) + \frac{igZ_\mu}{\cos\theta_W} \big(T^3 - \sin^2\theta_W Q\big) + ieA_\mu Q$$

Charges

$\mathrm{SU}(2)_L$: Weak isospin T^1, T^2, T^3 ($T^\pm = T^1 \pm T^2$)

$\mathrm{U}(1)_Y$: Weak hypercharge Y

$\mathrm{U}(1)_{\mathrm{EM}}$: Electric charge $Q = T^3 + Y$

	$\mathrm{SU}(2)_L$ T^i	$\mathrm{U}(1)_Y$ Y	$\mathrm{U}(1)_{\mathrm{EM}}$ Q	
ϕ	τ^i	½	1, 0	$-$, h
ℓ_L	τ^i	-½	0, -1	(ν_e, ν_μ, ν_τ), (e_L, μ_L, τ_L)
ℓ_R	0	-1	-1	e_R, μ_R, τ_R
Q_L	τ^i	⅙	⅔, -⅓	(u_L, c_L, t_L), (d_L, s_L, b_L)
u_R	0	⅔	⅔	u_R, c_R, t_R
d_R	0	-⅓	-⅓	d_R, s_R, b_R

($\tau^i = \sigma^i/2$; fundamental representation of $\mathrm{SU}(2)$)

Coupling constants

$\mathrm{SU}(2)_L$ coupling: g

$\mathrm{U}(1)_Y$ coupling: g'

Weinberg angle: $g = \sqrt{g^2 + g'^2} \cos\theta_W$, $g' = \sqrt{g^2 + g'^2} \sin\theta_W$

Fundamental electric charge: $e = g \sin\theta_W = \dfrac{gg'}{\sqrt{g^2 + g'^2}}$

Currents
Lepton–gauge boson interactions from $\mathcal{L}_{\text{lept}}^{\text{EW}}$:

$$\mathcal{L}_{\text{lept}}^{\text{int}} = -\frac{g}{2\sqrt{2}}\left(W_\mu^+ J^\mu + W_\mu^- J^{\mu\dagger}\right) - \frac{g}{2\cos\theta_W} Z_\mu J_n^\mu - eA_\mu J_{\text{EM}}^\mu$$

Charged weak current $J^\mu = \sum_{\ell=e,\mu,\tau} \bar{\nu}_\ell \gamma^\mu (1-\gamma^5)\ell$ (subject to flavour mixing)

Neutral weak current $J_n^\mu = \sum_{\ell=e,\mu,\tau} \bar{\nu}_\ell \gamma^\mu \nu_\ell + \bar{\ell}\left(\frac{1-\gamma^5}{2} - 2\sin^2\theta_W\right)\ell$

Electric current $J_{\text{EM}}^\mu = \sum_{\ell=e,\mu,\tau} -\bar{\ell}\gamma^\mu \ell$

(similarly for quarks)

Flavour mixing
Left-handed quarks: $V_{\text{CKM}} = K_u^\dagger K_d$ where $\lambda_u = K_u \Lambda_u S_u^\dagger$ and $\lambda_d = K_d \Lambda_d S_d^\dagger$
Left-handed leptons: V_{PKNS}

Fermi effective theory
Fermi weak Lagrangian

$$\mathcal{L}_{\text{W}}^{\text{eff}} = -\frac{G_F}{\sqrt{2}}\left(J_\mu^\dagger J^\mu + \rho J_{n\mu}^\dagger J_n^\mu\right) \qquad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2} = \frac{1}{2v^2}, \quad \rho = \frac{m_W^2}{m_Z^2\cos^2\theta_W}$$

3 QCD sector

Yang–Mills theory
$G_{\text{QCD}} = \text{SU}(3)_C$
Gauge bosons: A^a, $a \in \{1, \dots, 8\}$ (gluons)
Quarks q_f, $f \in \{u, d, c, s, t, b\}$

$$\mathcal{L}^{\text{QCD}} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \sum_f \bar{q}_f (i\not{D} - m_f) q_f$$

(m_f comes from the Higgs coupling $\mathcal{L}_{\text{quark},\phi}$)

Renormalization For $\text{SU}(N)$ Yang–Mills theory with N_f quarks in the fundamental representation,

$$\beta(g) = -\frac{g^3}{16\pi}\left(\frac{11}{3}N - \frac{4}{3}N_f\right) + \mathcal{O}(g^5)$$