Quantum structure of holographic black holes

Elias Riedel Gårding

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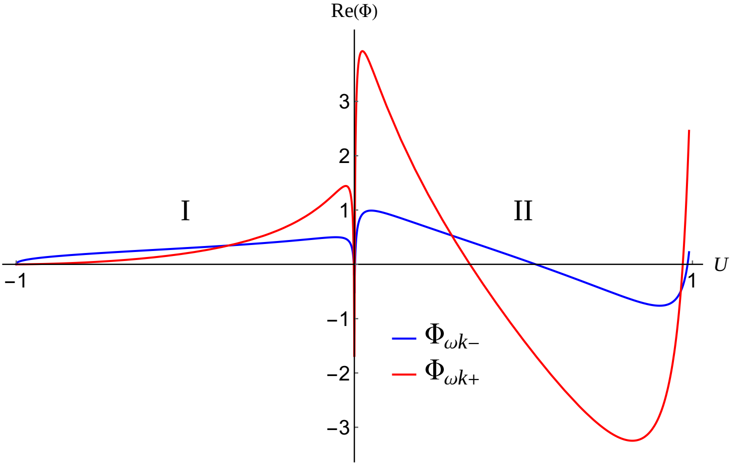
Supervisor: Adam Bzowski, UU

The thesis on a t-shirt



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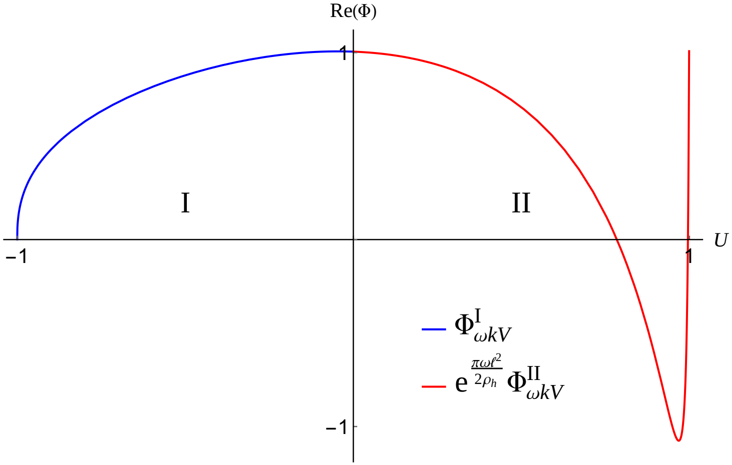
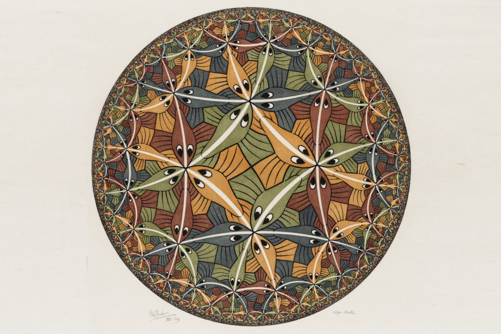
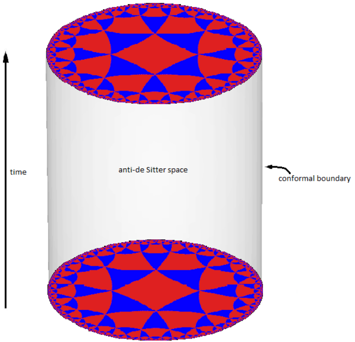


Why study this?





- AdS/CFT: QG (poorly understood) = CFT (well-understood)!
- BTZ black hole: 3D, so gravity is non-dynamical $\implies$ easier!

General relativity

Metric

Spacetime is a manifold with metric:

$$ds^2 = G_{\mu\nu}\, dx^\mu\, dx^\nu \tag{1}$$

For example, Minkowski space

$$ds^2 = -\,dt^2 + dx^2 + dy^2 + dz^2 \tag{2}$$

or the Schwarzschild black hole

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{1}{1 - \frac{2M}{r}}\, dr^2 + r^2\left(d\theta^2 + \sin^2(\theta)\, d\phi^2\right) \tag{3}$$

General relativity

Coordinate transformations

Coordinate transformations change the appearance of the metric.

Example (Rindler space is Minkowski space)

Rindler metric:

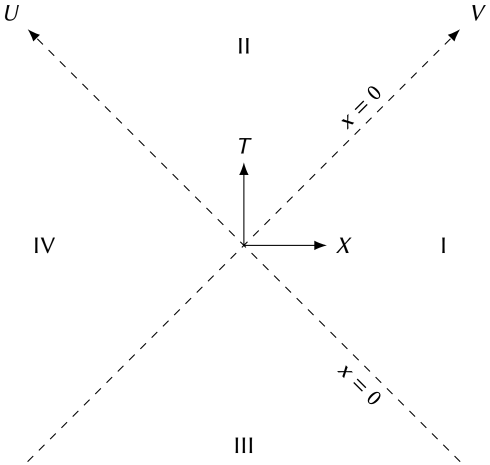
$$\mathrm{d}s^2 = -x^2\,\mathrm{d}t^2 + \mathrm{d}x^2 \tag{4}$$

Coordinate transformation

$$x = \sqrt{-UV} \qquad t = \frac{1}{2}\ln\left(\frac{V}{-U}\right) \tag{5}$$

shows that $\mathrm{d}s^2 = -\,\mathrm{d}U\,\mathrm{d}V$. With $U = T - X$ and $V = T + X$,

$$\mathrm{d}s^2 = -\,\mathrm{d}T^2 + \mathrm{d}X^2\,. \tag{6}$$



$$ds^2 = -x^2\,dt^2 + dx^2 = -\,dU\,dV = -\,dT^2 + dX^2$$

BTZ black hole

Standard metric

Vacuum Einstein equations with negative cosmological constant:

$$R_{ab} - \frac{1}{2} R G_{ab} + \Lambda G_{ab} = 0 \tag{7}$$

Black hole solution in $2+1$ dimensions:

$$ds^2 = -\frac{\rho^2 - \rho_h^2}{\ell^2}\, dt^2 + \frac{\ell^2}{\rho^2 - \rho_h^2}\, d\rho^2 + \rho^2\, d\phi^2 \tag{8}$$

where $\ell^2 = -\dfrac{3}{\Lambda}$.

Horizon at $\rho = \rho_h$, singularity at $\rho = 0$.
(Not a curvature singularity, but more subtle.)

BTZ black hole

Kruskal extension

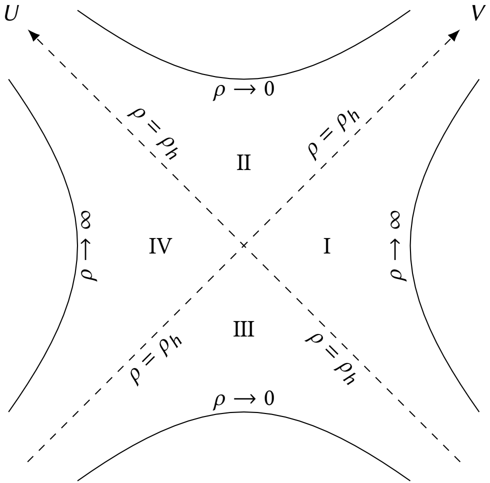
New coordinates (U, V) given by

$$\frac{\rho}{\rho_h} = \frac{1 - UV}{1 + UV} \qquad t = \frac{\ell^2}{2\rho_h} \ln\left|\frac{V}{U}\right| \qquad |UV| < 1 \tag{9}$$

transform the metric to

$$\mathrm{d}s^2 = \frac{4\ell^2}{(1+UV)^2}\left(-\,\mathrm{d}U\,\mathrm{d}V + \frac{\rho_h^2}{4\ell^2}(1-UV)^2\,\mathrm{d}\phi^2\right). \tag{10}$$

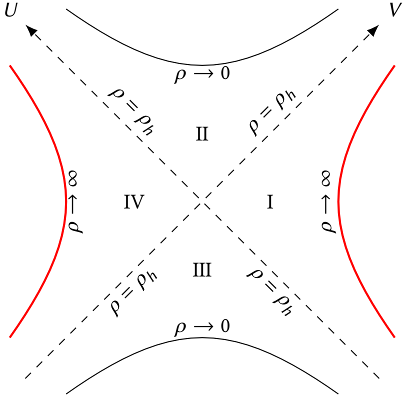
The maximal analytic extension is a “wormhole”.



$$ds^2 = \frac{4\ell^2}{(1+UV)^2}\left(-\,dU\,dV + \frac{\rho_h^2}{4\ell^2}(1-UV)^2\,d\phi^2\right)$$

AdS/CFT correspondence

Holographic boundary



Attach a boundary to the spacetime.

Boundary metric $g_{ab} = \frac{z^2}{\ell^2} G_{ab}$ defined up to conformal transformations.

AdS/CFT:

$$\boxed{\text{bulk string theory} \leftrightarrow \text{boundary CFT}}$$

Here: string theory $\approx$ free scalar field

AdS/CFT correspondence

Statement

Free scalar field:

$$S[\Phi] = -\int \mathrm{d}^3x \sqrt{|G|}\left(\frac{1}{2}G^{ab}\nabla_a\Phi\nabla_b\Phi + \frac{1}{2}m^2\Phi^2\right) \tag{11}$$

AdS/CFT principle: $\mathcal{Z}_\Phi = \mathcal{Z}_{\text{CFT}}$ where

$$\mathcal{Z}_\Phi = \int \mathcal{D}\Phi\, \mathrm{e}^{\mathrm{i}S[\Phi]} \tag{12}$$

$$\mathcal{Z}_{\text{CFT}} = \int \mathcal{D}\chi\, \mathrm{e}^{\mathrm{i}S_{\text{CFT}}[\chi]} \tag{13}$$

and S_{CFT} is some (interacting) CFT action.

Classical approximation: $\mathcal{Z}_{\text{CFT}} \approx \mathrm{e}^{\mathrm{i}S[\Phi_{\text{cl}}]}$.

AdS/CFT correspondence

Boundary conditions

Actually

$$\mathcal{Z}_\Phi[\phi_{(0)}] = \int\limits_{\Phi|_{\partial\mathrm{BTZ}} \sim \phi_{(0)}} \mathcal{D}\Phi\, \mathrm{e}^{\mathrm{i}S[\Phi]} \tag{14}$$

and

$$\mathcal{Z}_{\mathrm{CFT}}[\phi_{(0)}] = \int \mathcal{D}\chi\, \mathrm{e}^{\mathrm{i}\left[S_{\mathrm{CFT}}[\chi] - \int_{\partial\mathrm{BTZ}} \mathrm{d}^d x \sqrt{|g|}\, \mathcal{O}(\mathbf{x})\phi_{(0)}(\mathbf{x})\right]} \tag{15}$$

$\boxed{\text{Boundary conditions on } \Phi \leftrightarrow \text{source for some CFT operator } \mathcal{O}(\mathbf{x}).}$

Eventual goal: Relate the CFT operators on both sides of the wormhole.

Classical equation of motion

Extremum of $S[\Phi]$ is given by the covariant Klein–Gordon equation

$$\left(-G^{ab}\nabla_a\nabla_b + m^2\right)\Phi_{\mathrm{cl}} = 0 \tag{16}$$

which expands to

$$\left[\left(\rho^2 - \rho_h^2\right)\partial_\rho^2 + \frac{3\rho^2 - \rho_h^2}{\rho}\partial_\rho - \frac{\ell^4}{\rho^2 - \rho_h^2}\partial_t^2 + \frac{\ell^2}{\rho^2}\partial_\phi^2 - \ell^2 m^2\right]\Phi_{\mathrm{cl}} = 0. \tag{17}$$

Classical equation of motion

Exact solution in terms of hypergeometric functions:

$$\Phi_{\omega k\pm}(\rho, t, \phi) = \mathrm{e}^{-\mathrm{i}\omega t+\mathrm{i}k\phi}\left(\frac{\rho^2}{\rho^2-\rho_h^2}\right)^a\left(\frac{\rho_h^2}{\rho^2-\rho_h^2}\right)^{\Delta^\pm/2} {}_2\mathbf{F}_1\left(\begin{matrix}\frac{\Delta^\pm}{2}+a+b, \frac{\Delta^\pm}{2}+a-b \\ \Delta^\pm\end{matrix};\frac{\rho_h^2}{\rho_h^2-\rho^2}\right) \tag{18}$$

where

$$\Delta^\pm = 1 \pm \sqrt{1+m^2\ell^2}. \tag{19}$$

Expand field operator in terms of classical modes:

$$\hat{\Phi}(t,\rho,\phi) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \sum_{k\in\mathbb{Z}} \left(\hat{a}_{\omega k} u_{\omega k}(t,\rho,\phi) + \hat{a}^\dagger_{\omega k} u^*_{\omega k}(t,\rho,\phi) \right) \tag{20}$$

Canonical commutation relations $\iff$ $u_{\omega k}$ appropriately normalised.

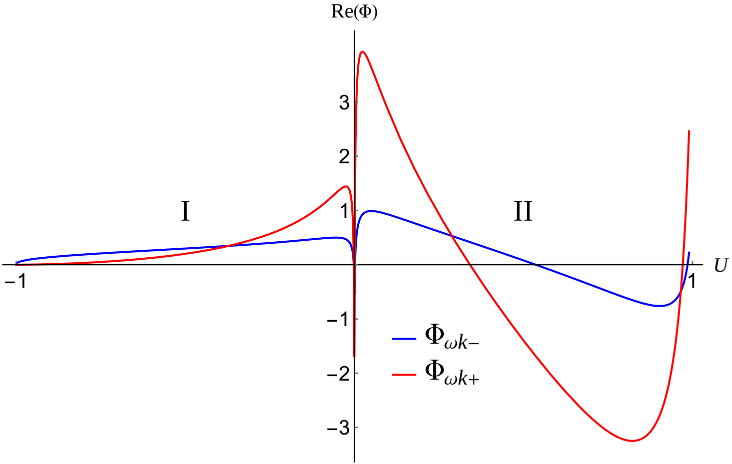
Holographic renormalisation

Problem: $S[\Phi_{\text{cl}}]$ is infinite (IR divergence as $\rho \to \infty$). Ill-defined theory.
Solution: Add counterterms that cancel the divergent parts.

After a (long) computation based on a series expansion of Φ, we find:

- The right counterterm is $S_{\text{ct}}[\Phi]$ $\left(= -\lim_{\epsilon\to 0}\int_{z=\epsilon} \mathrm{d}^2x\,\sqrt{|\gamma|}\,\frac{\Delta^-}{2}\Phi^2\right)$.
- Two solutions for Φ_{cl}, which go as $\Phi_\pm \overset{\rho\to\infty}{\sim} \rho^{-\Delta^\pm}$.
- Φ_- has source but no VEV.
- Φ_+ has VEV but no source.

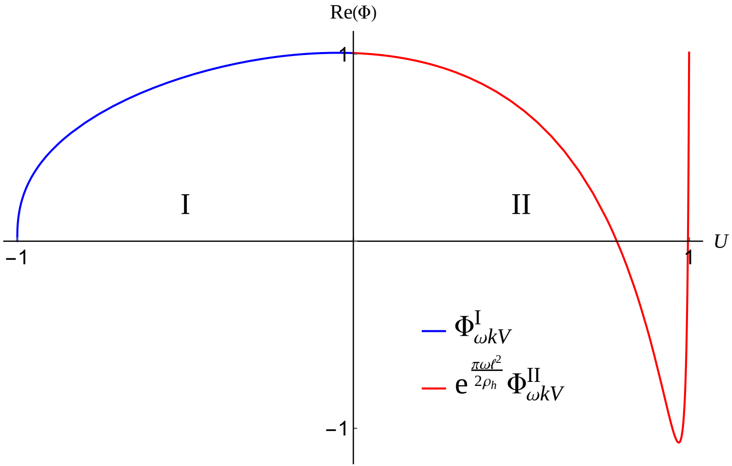
Solutions in global coordinates



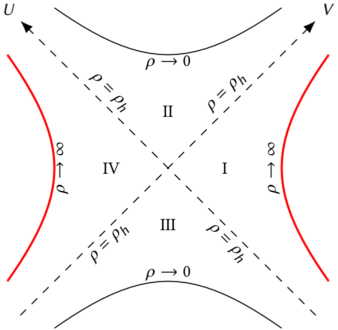
Solutions $\Phi^{\pm}_{\omega k}(U, V)$ on the line $V = 1$.

Solutions in global coordinates

One linear combination $\Phi_{\omega k V}$ continues smoothly across the horizon:



Conclusions & outlook



Now we can connect source and VEV of the boundary CFTs to solutions inside the black hole.

Next: Connect the two boundaries together?

This requires boundary conditions at $\rho = 0$, which some models give rise to. Future research!

Eventually, compute correlators $\langle \mathcal{O}_{\mathrm{I}}(\mathbf{x}) \mathcal{O}_{\mathrm{IV}}(\mathbf{y}) \rangle \ldots ?$

Contact

Elias Riedel Gårding (Uppsala University)

Email: eliasrg@kth.se
elias.riedel_garding@math.uu.se

Thesis and slides available at https://people.kth.se/~eliasrg