All-Optical Wavelength Conversion for NRZ-OOK Signal based on Four-Wave Mixing in a Silicon Waveguide

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Abstract

Wavelength conversion is a key technology in modern optical communication systems. All-optical methods are highly preferred to more complex optical–electrical–optical ones for cost and efficiency reasons. OOK is the simplest and most commonly used modulation format for optical signals. This project attempts to demonstrate wavelength conversion of an NRZ-OOK signal in a silicon waveguide, based on the nonlinear parametric optical process of four-wave mixing. The conversion is performed, but the eye diagram of the weak converted signal cannot be observed due to unavailable equipment.

Contents

Nomenclature

ASE Amplified Spontaneous Emission

COM Common port

CW Continuous Wave

CWDM Coarse Wavelength Division Multiplexer

DPSK/DBPSK Differential (Binary) Phase Shift Keying

EDF Erbium Doped Fibre

EDFA Erbium Doped Fibre Amplifier

EYDF Erbium–Ytterbium Doped Fibre

EYDFA Erbium–Ytterbium Doped Fibre Amplifier

FOPA Fibre Optical Parametric Amplifier

FWM Four-Wave Mixing

HNLF Highly Non-Linear Fibre

HP High Power

ISO Isolator

MMF Multi-Mode Fibre

MZM Mach-Zehnder (Interference) Modulator

NRZ-OOK Non-Return-to-Zero On-Off Keying

OA Optical Attenuator

OC Optical Coupler

OOK On-Off Keying

OPM Optical Power Meter

OSA Optical Spectrum Analyser

PBS Polarization Beam Splitter

PC	Polarization Controller
PMF	Polarization-Maintaining Fibre
PRBS	Pseudo-Random Binary Sequence
SMF	Single-Mode Fibre
SOI	Silicon-on-Insulator
TL	Tunable Laser
WDM	Wavelength Division Multiplexer

1 Introduction

In order to allow simultaneous transmission of multiple signals through a fibre by way of wavelength division multiplexing, as well as to enable the use of components with a specific band of compatible wavelengths, wavelength conversion is needed. Optical–electrical–optical wavelength conversion is a possibility, but suffers from complexity, high costs and low efficiency. All-optical wavelength conversion techniques have the potential to greatly improve on this.

As an alternative to space-inefficient processes in fibres, integrated optics is emerging as a potential solution. Processes traditionally requiring long stretches of fibre can be replaced by small silicon-on-insulator (SOI) chips.

Due to its simplicity, OOK is the most widely used modulation format in metropolitan area networks. The purpose of this project is to examine a wavelength conversion scheme for OOK signals based on the nonlinear parametric process of four-wave mixing (FWM) in an SOI waveguide.

The project was carried out in the course of three weeks with the laser group at the Centre for Optical and Electromagnetic Research at Zijingang Campus, Zhejiang University, Hangzhou, Zhejiang, China. It was a part of the Joint International Research Centre for Optics and Photonics (JORCEP), which is a cooperation between Zhejiang University, the Swedish Royal Institute of Technology and Lund University. The project was organized and supervised by Yang Xiong of the laser group and broadly supervised by professor Gao Shiming.

In addition to Yang Xiong, the authors wish to warmly thank the following people for participating as instructors: Wu Zhihang, Lou Yang, Bai Hangyu, Zhang Dan, Li Xibin, Jin Qiang, Yin Taoce, Tu Zhihua, Jiang Xiaogang and He Hongwei. You have made the project a great experience!

2 Theory

2.1 Optical fibres

To transmit optical signals, the basic component is the optical fibre. There is a large variety of fibres, but they all share some common elements. The signal travels through the innermost part, called the *core*, which is surrounded by a *cladding* with a lower refractive index. The signal is confined to the core by way of total internal reflection according to Snell's Law (as long as the fibre is not excessively bent).

In a fibre only a finite number of *modes* of propagation are possible. Each propagation angle (angle between the propagation direction and the fibre's axis) corresponds to a mode. Because light waves with large propagation angles (high-order modes) travel not only parallel to the fibre's axis, but also perpendicular to it, those modes will travel more slowly through the fibre. If multiple modes are possible, this will lead to *intermodal dispersion* of signals (they will arrive in a longer time span than they were sent in).

To avoid this problem, *single-mode fibres* (SMF) are often preferred to *multi-mode fibres* (MMF). However, multi-mode fibres are generally cheaper to produce and can carry more energy due to their thicker cores.

In both multi-mode and single-mode fibres, two other dispersion phenomena occur. One of them is due to the fact that signals are not perfectly monochromatic, but consist of light in a range of wavelengths around a peak value. Because the refractive index of a material in general depends on the wavelength of the light in question, this also gives rise to dispersion. The phenomenon is called *chromatic dispersion*.

The other dispersion phenomenon is known as *polarization dispersion*, and is caused by the fibre material having different refractive indices along different axes of polarization. This effect introduces an unpredictable difference in arrival time for different polarization angles.

Often in optical circuits, the polarization of the signal is relevant. Ordinary fibres can introduce random changes to the polarization because of small irregularities. When polarization is important, *polarization-maintaining fibres* (PMF) are used. A common form of these are called PANDA fibres, since the cross section resembles the animal.

Fibres are often *doped* with a rare-earth metal to achieve various properties, such as a specific emission spectrum or larger nonlinear coefficients.

2.2 Optical components

In optical communication systems, a variety of components are used to manipulate the signals. Some of the components have superficial similarities to common electrical components. Components may be broadly divided into *passive* and *active* components. A passive component may be loosely defined as a component that consumes no external power, while an active component is one that does.

2.2.1 Passive components

Attenuators An optical attenuator (OA) is (very roughly) analogous to the resistor in electrical engineering. Its purpose is to dissipate the power of the incident light, which is often necessary in order to avoid damage to other components in the system. Two simple designs of attenuators involve an air gap or a misalignment in the fibre.

Isolators An isolator (ISO) is roughly analogous to the diode in an electrical circuit; it only allows light to pass through it in one direction. Isolators work by a component called the *Faraday rotator*, which uses a non-reversible electromagnetic effect that rotates the polarization of light passing through it. In combination with polarization filters, this creates a one-way passage.

This simple kind of isolator destroys any information contained in one of the polarization states. More complicated arrangements can produce *polarization-independent* isolators.

Isolators are often used directly on the output of a sensitive component in order to protect it from reflected beams or other light travelling in the reverse direction.

Circulators A circulator has a number of ports which act both as inputs and outputs. A beam sent into port 1 will be output from port 2, a beam sent into port 2 will be output from port 3 and so on, but the last port will generally not connect to the first one. This design is in order to prevent reflections from going an entire cycle and come out of their input port. It works on the same principles as isolators.

Couplers/splitters Optical couplers (OC) are a family of components that combine and divide their input beams in different ways. The simplest kind of coupler, also known as a *combiner*, simply channels its input channels into one output channel.

A *splitter* serves the opposite purpose; to divide a single input into multiple output channels.

Another simple coupler, the *directional coupler*, has two input channels and two output channels (inputs and outputs are interchangeable). The coupler will act as a combiner, but split the power of the signal between the two outputs, in some predetermined proportion. By cascading multiple directional couplers one can create couplers with many inputs and outputs. When one speaks of "optical couplers", directional couplers are what is most often meant.

Polarization controllers A polarization controller (PC) is a simple device that applies an adjustable stress to the fibre in order to control the polarization state of the beam. In practice, these are tuned by trial-and-error.

Multiplexers A variety of *multiplexers*, devices that can selectively split or combine beams based on certain criteria, exist. This project makes heavy use of the *wavelength division multiplexer* (WDM). For instance, it can act as a band-pass filter on a beam input into its common (COM) port by letting a certain band of wavelengths through a *pass port* and the rest through its *reflect port*.

Modulators To insert a signal into a beam of light, a modulator is used. A *phase modulator* operates by the linear electro-optic effect (that is, the refractive index's approximately linear dependence on field strength) in a planar waveguide to change the phase of the incoming light.

One kind of phase modulator is the Mach-Zehnder interference modulator (MZM). It operates by splitting the beam and applying different voltages to the resulting beams, therefore changing the phases an equal or unequal amount. If the beams have a phase difference of π, they will destructively interfere when they are reunited through a coupler. In this sense, the MZM also works as an *intensity modulator*.

2.2.2 Lasers

A laser (Light Amplified Stimulated Emission of Radiation) is a device that outputs a (nearly) *coherent* beam of light. This means that the light is nearly monochromatic (single-wavelength) and that the phase and polarization of all the photons is almost the same.

A laser works by injecting energy into a *gain medium* to excite a large portion of its atomic electrons. The excitation energy is converted into light (of wavelength $hc/\Delta E$ where ΔE is the energy difference between the excited state and a lower-energy state) through two processes: *spontaneous emission* and *stimulated emission*. Spontaneous emission is the familiar effect of spontaneous deexcitation from the excited state. Stimulated emission is a quantum-mechanical effect in which an already existing photon passing by the atom will induce the electron to deexcite and emit a photon that is coherent with (i.e. identical to) the incident photon. Cascaded, this effect results in an amplified coherent beam.

Photons emitted through spontaneous emission subsequently participate in stimulated emission. This effect is referred to as *amplified spontaneous emission* (ASE).

The effect of stimulated emission competes with that of absorption, since the wavelengths emitted from excited electrons are also easily absorbed by electrons of the lower energy state. Therefore, continuous pump power is needed to keep a large enough portion of the electrons excited to maintain the amplifying property of the gain medium (to keep sufficient *population inversion*).

To create a large enough gain, the beam must be sent through the gain medium several times. To this effect, a *resonant cavity* is used. The simplest variety consists of semi-reflective mirrors (or mirror-like components) surrounding the gain medium. The cavity will only allow the wavelengths that constructively interfere, or its *longitudinal modes*, to cascade.

Since the pump supplies a limited amount of power, the more modes there are in the cavity to share the gain medium, the less power each mode will obtain. This phenomenon is known as *modal competition*. To counteract it, a band-pass filter can be introduced into the cavity in order to filter out all but one mode.

2.2.3 Amplifiers

To compensate for power loss in fibres and other components, as well as to establish general control over signal power, optical amplifiers are needed. These can be constructed in several different ways depending on needs.

Fibre amplifiers such as the *erbium–ytterbium doped fibre amplifier* (EYDFA) work similarly to a laser, but without the cavity. By injecting a gain medium with a pump, a population inversion can be established, consequently allowing the signal to cause stimulated emission in the medium. This, however, also gives rise to some ASE light that must be filtered away.

Semiconductor optical amplifiers work similarly to fibre amplifiers, but instead of electrons transitioning between different valence excitation states, the transitions will be between the conduction and valence band of a semiconductor. A current is used to maintain a population inversion.

Raman amplifiers and *fibre optical parametric amplifiers* (FOPA) work by nonlinear effects in fibres. In Raman amplification a pump photon excites a vibrational state of a molecule, which then deexcites and emits a single photon of the same frequency (energy) as the signal photons. Depending on whether the energy of the signal photon is lower or higher, an *optical phonon* is emitted or absorbed from the material. FOPAs work with four-wave mixing (see Section 2.4 below). Both these processes use pump power in direct combination with the signal and as such, no population inversion is needed.

2.3 Modulation formats

Different kinds of modulators can alter properties of a light wave such as amplitude, frequency, phase and polarization. To use modulation to transmit data, a *modulation format* (keying scheme) must be chosen. Some relatively simple modulation formats are:

- Amplitude modulation
  - On-Off Keying (OOK): No light is `0`, light is `1`.
  - Amplitude-Shift Keying (ASK): Multiple intensity levels corresponding to symbols in a non-binary alphabet.
- Phase modulation
  - Differential (Binary) Phase Shift Keying (DBPSK/DPSK): Phase difference between adjacent bits: 0 is `0`, π is `1`.
  - Differential Quadrature Phase Shift Keying (DQPSK): Phase difference between adjacent bits can be 0, $\pi/2$, π or $3\pi/2$.

This project focuses on OOK, specifically *non-return-to-zero* OOK (NRZ-OOK). In NRZ-OOK, two adjacent `1` bits will be encoded as an uninterrupted high signal, whereas in return-to-zero OOK (RZ-OOK) the signal would briefly drop between the two high bits.

NRZ-OOK is possibly the simplest modulation format imaginable, and is therefore easy to implement. However, it has some disadvantages such as being more sensitive to attenuation than for example DPSK.

2.4 Four-wave mixing

For strong electric fields, electric polarization does not depend linearly on the field strength. Certain media are especially prone to this and are called *nonlinear media*. Silicon is an example of such a medium.

Four-wave mixing is a consequence of the nonlinearity of the medium. Specifically, it depends on the *third-order nonlinearity*, wherein a portion of the polarization is proportional to the cube of the field strength. As such, it is most prominent in materials that have no second order nonlinearity. This is true of many crystalline materials because of symmetries in the atomic structure.

Four-wave mixing is a *parametric* optical process, which means that it leaves the quantum state of the material unchanged. Practically speaking, this means that it is instantaneous (within the precision allowed by Heisenberg's uncertainty principle). It also means that no energy or momentum is exchanged between the light and the material.

In terms of the particles involved, two incident photons, of angular frequencies ω_1 and ω_2, interact to produce two new photons of angular frequencies ω_3 and ω_4. The process must conserve energy and momentum, as expressed in the relations

$$\omega_1 + \omega_2 = \omega_3 + \omega_4 \tag{1}$$
$$\vec{\boldsymbol{\beta}}_1 + \vec{\boldsymbol{\beta}}_2 = \vec{\boldsymbol{\beta}}_3 + \vec{\boldsymbol{\beta}}_4. \tag{2}$$

$\vec{\boldsymbol{\beta}}$ (the notation $\vec{\mathbf{k}}$ is also common) is the *wave vector* or *propagation constant* of the photon, with magnitude

$$\|\vec{\boldsymbol{\beta}}\| = \frac{2\pi}{\lambda} = \frac{\omega}{v} = \frac{n(\omega)\omega}{c}$$

in the photon's propagation direction ($n(\omega)$ is the medium's refractive index and the dependence on ω is chromatic dispersion). Equation 2 is known as the *phase matching condition.*

If $\omega_1 = \omega_2$, the process is known as *degenerate four-wave mixing.* A typical instance of degenerate FWM is when a strong pump beam (ω_p) and a relatively weak signal beam (ω_s) are simultaneously injected into a material. Both the incident photons will most often be pump photons (because there are many of them), so $\omega_1 = \omega_2 = \omega_p$. For reasons similar to those of stimulated emission (see section 2.2.2), the system prefers the process where $\omega_3 = \omega_s$. These conditions are enough to determine ω_4, known as the *idler* frequency (ω_i):

$$\omega_i = 2\omega_p - \omega_s. \tag{3}$$

Because the other emitted photon is of frequency ω_s, degenerate FWM can be used to amplify the signal. This is a form of *optical parametric amplification* and is leveraged in *fibre optical parametric amplifiers* (see Section 2.2.3).

The phenomenon of (degenerate) FWM is treated more rigorously, through classical electrodynamic theory, in Appendix A.

3 Preparatory experiments

3.1 Fibre fusion splicing

When two optical fibres need to be joined at the ends, one possibility is to permanently fuse them in a process called *fibre fusion splicing.* The process consists of a series of steps to fuse the fibres, verify the quality of the splice, and package it in a plastic coating. However, most of these are usually automated by electronic equipment, so that there are just three manual steps remaining:

1. Expose the ends and cut the fibres to produce flat connection surfaces,
2. Let the machine carry out the splicing operation,
3. Let the machine carry out the splice packaging operation.

As a demonstration experiment, two splices were carried out using a Fujikura ARCMaster splicing apparatus. The experiment involved the three steps mentioned above; the first step required the most interaction and was the most time-consuming. The jacket and buffer coating were removed using a Miller clamp, and the ends were then cut using a mechanical cleaver. One of the ends was threaded through a plastic cylinder to be used as packaging. The two ends were carefully inserted into the machine and an appropriate splicing program was selected. When the splicing was completed the splice was moved into the packaging slot and the machine melted the plastic cylinder into a tight protective coating around the splice.

The procedure was done two times, with a simple SMF and a PMF respectively. For the SMF the power loss of the splice was measured by the machine to be less than 0.01 dB. For the PMF the power loss was measured to approximately 0.01 dB.

3.2 Properties of passive components

3.2.1 Setup

An experiment was carried out to measure the power loss of a number of components. A tunable laser (TL) was set to output a beam of 10 mW. The beam was led through an isolator (to prevent reflected beams from damaging the TL). At this point, the power output was measured at 7.01 mW using an optical power meter (OPM). The components in question were connected one at a time, and the power output was measured at all their ports of interest. The following components were measured:

- Variable optical attenuator
- Polarization controller (PC)
- Polarization beam splitter (PBS). When measuring, the PC was also included. Only one of the two output ports was measured.
- Circulator (ports 1, 2 & 3)

- 50/50 Directional coupler (input ports 1 & 2, output ports 3 & 4)
- 10/90 Directional coupler (input port 1, output ports 2 & 3)

In addition to the above, another experiment was carried out to measure the spectrum of the output of a *coarse wavelength division multiplexer* (CWDM). The CWDM has three ports: a common (COM) port, a reflection port and a pass port. A beam is input into the COM port, and it is split between the output ports according to wavelength. Wavelengths in the interval $(1570 \pm 7)\,\mathrm{nm}$ are let through to the pass port, and other wavelengths are reflected and exit via the reflection port.

Two beams (one at a time) of wavelengths $1550\,\mathrm{nm}$ and $1565\,\mathrm{nm}$ respectively were injected into the common port, and we measured the spectrum of the outputs on the reflection and pass ports using an *optical spectrum analyser* (OSA).

3.2.2 Results

The measured power losses are listed in Table 1. The power values are in decibel-milliwatts (dBm) and the power losses are in decibels (dB), defined as follows:

$$P_{\mathrm{dBm}} = 10\log_{10}\left(\frac{P}{1\,\mathrm{mW}}\right), \qquad \Delta P_{\mathrm{dB}} = 10\log_{10}\left(\frac{P}{P'}\right) = P_{\mathrm{dBm}} - P'_{\mathrm{dBm}}.$$

ND stands for “Not Detectable” and I\O stands for “Input\Output”.

<table>
<tr><th>Component</th><th>Output power (dBm)</th><th>Loss (dB)</th></tr>
<tr><td>None (reference)</td><td>8.46</td><td>0.00</td></tr>
<tr><td>OA (min. loss)</td><td>7.91</td><td>0.55</td></tr>
<tr><td>OA (max. loss)</td><td>ND</td><td>∞</td></tr>
<tr><td>PC</td><td>2.45</td><td>6.01</td></tr>
<tr><td>PBS (min. loss)</td><td>1.35</td><td><table><tr><td>Total</td><td>7.11</td></tr><tr><td>Component</td><td>1.10</td></tr></table></td></tr>
<tr><td>PBS (max. loss)</td><td>ND</td><td>∞</td></tr>
<tr><td>Circulator</td><td><table><tr><th>I\O</th><th>1</th><th>2</th><th>3</th></tr><tr><td>1</td><td>—</td><td>0.47</td><td>ND</td></tr><tr><td>2</td><td>ND</td><td>—</td><td>4.05</td></tr><tr><td>3</td><td>ND</td><td>ND</td><td>—</td></tr></table></td><td><table><tr><th>I\O</th><th>1</th><th>2</th><th>3</th></tr><tr><td>1</td><td>—</td><td>7.99</td><td>∞</td></tr><tr><td>2</td><td>∞</td><td>—</td><td>4.41</td></tr><tr><td>3</td><td>∞</td><td>∞</td><td>—</td></tr></table></td></tr>
<tr><td>50/50 Coupler</td><td><table><tr><th>I\O</th><th>1</th><th>2</th><th>3</th><th>4</th></tr><tr><td>1</td><td>—</td><td>ND</td><td>5.03</td><td>4.78</td></tr><tr><td>3</td><td>5.18</td><td>4.73</td><td>—</td><td>ND</td></tr></table></td><td><table><tr><th>I\O</th><th>1</th><th>2</th><th>3</th><th>4</th></tr><tr><td>1</td><td>—</td><td>∞</td><td>3.43</td><td>3.68</td></tr><tr><td>3</td><td>3.28</td><td>3.73</td><td>—</td><td>∞</td></tr></table></td></tr>
<tr><td>10/90 Coupler</td><td><table><tr><th>I\O</th><th>1</th><th>2</th><th>3</th></tr><tr><td>1</td><td>—</td><td>7.07</td><td>-2.31</td></tr><tr><td>2</td><td>7.59</td><td>—</td><td>—</td></tr><tr><td>3</td><td>-3.24</td><td>—</td><td>—</td></tr></table></td><td><table><tr><th>I\O</th><th>1</th><th>2</th><th>3</th></tr><tr><td>1</td><td>—</td><td>1.39</td><td>10.77</td></tr><tr><td>2</td><td>0.87</td><td>—</td><td>—</td></tr><tr><td>3</td><td>11.69</td><td>—</td><td>—</td></tr></table></td></tr>
</table>

Table 1: The results of the measurement of passive components.

Since the couplers had multiple outputs, an extra calculation is needed to find the total amount of dissipated power. The total loss is computed in dB according to

$$\Delta P_{\mathrm{dB}}^{\mathrm{tot}} = P_{\mathrm{dBm}}^{\mathrm{ref}} - 10\log_{10}\left(\frac{\sum P_{\mathrm{out}}}{1\,\mathrm{mW}}\right).$$

The results are listed in Table 2.

Component	Total loss (dB)
50/50 Coupler (input on port 1)	0.54
50/50 Coupler (input on port 3)	0.49
10/90 Coupler (input on port 1)	0.92

Table 2: The total power losses in the directional couplers.

The output spectra from the CWDM are shown in Figure 1.

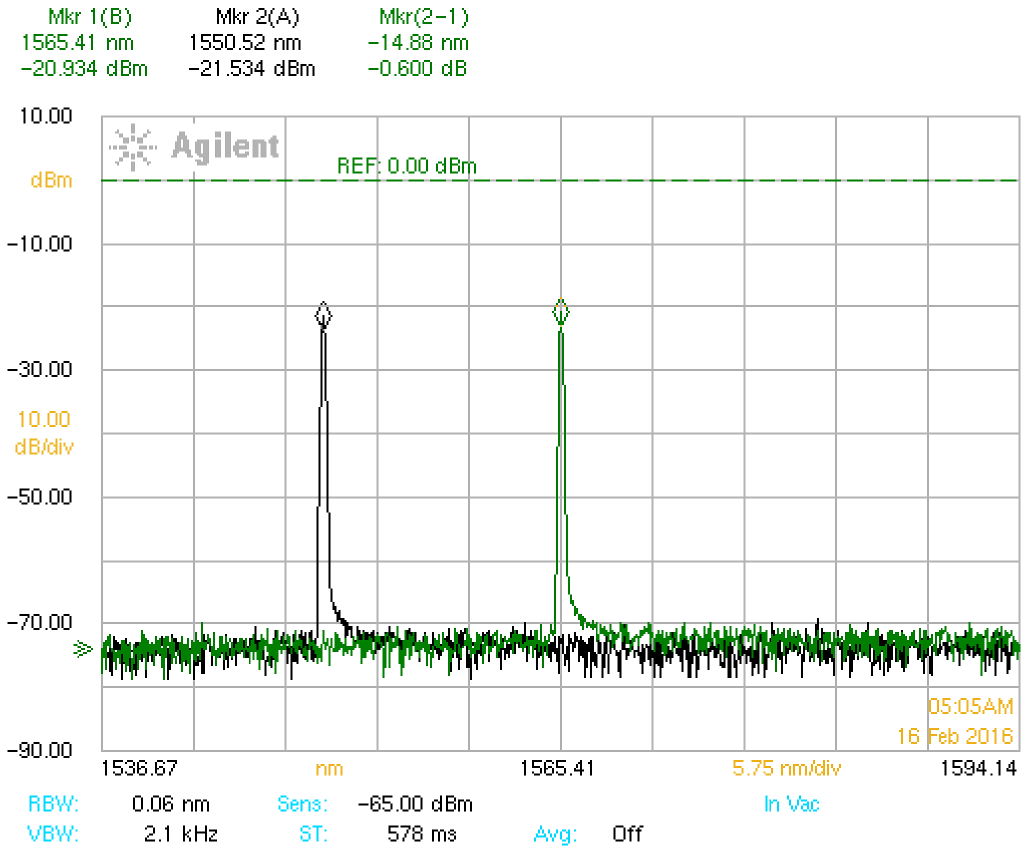


Figure 1: The output spectra of the CWDM.

3.2.3 Discussion

The components worked as expected. There were significant losses in the PC and in the 1→2 path of the circulator. The PC had an especially large loss of 6 dB.

The conclusions about the circulator are not certain, because one of the physical connectors was found to be dirty and attenuate the beam shortly after testing the circulator. The loss was high and may cast doubt on the conclusions.

3.3 Construction of a fibre laser

An experiment was carried out to construct a fibre laser based on an *erbium doped fibre* (EDF). The setup is shown in figure 2. A pump beam of wavelength 980 nm is injected into an EDF through a *wavelength division multiplexer* (WDM) whose wavelength divisions are centred at 980 nm and

1550 nm respectively. The EDF acts as the gain medium, converting the pump's energy to a beam of 1550 nm through amplified spontaneous emission. The beam is reflected back into the EDF because of the couplers, which act as the laser's resonant cavity in this case, and is further amplified through amplified stimulated emission, creating a coherent laser beam.

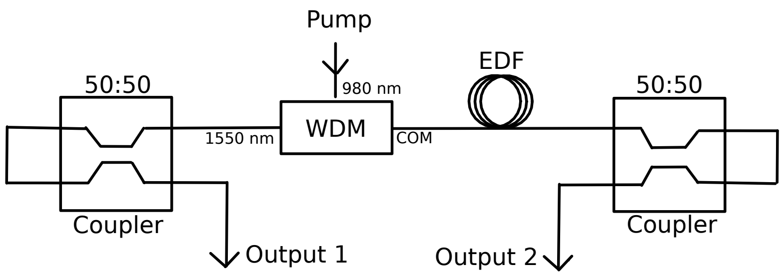


Figure 2: The setup of the EDF laser.

First, the gain medium EDF was tested, by simply injecting the pump beam into it and measuring the spectrum with an OSA. Figure 3 shows the resulting spectrum. The ASE frequency band is visible. Unfortunately only a photograph is available.

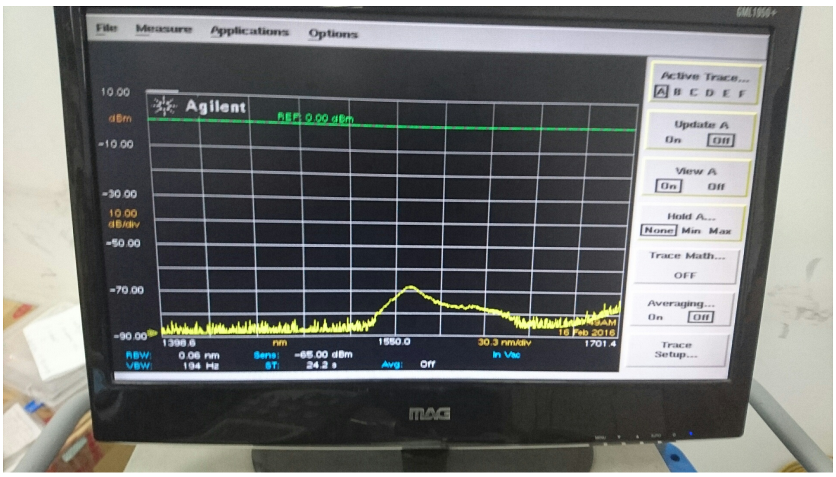


Figure 3: The ASE spectrum of the EDF.

When the gain medium had been verified to work, the laser was assembled according to Figure 2 and the output spectrum from one of the outputs was measured using the OSA. Figure 4 shows the spectrum, with the highest peak at −40 dBm.

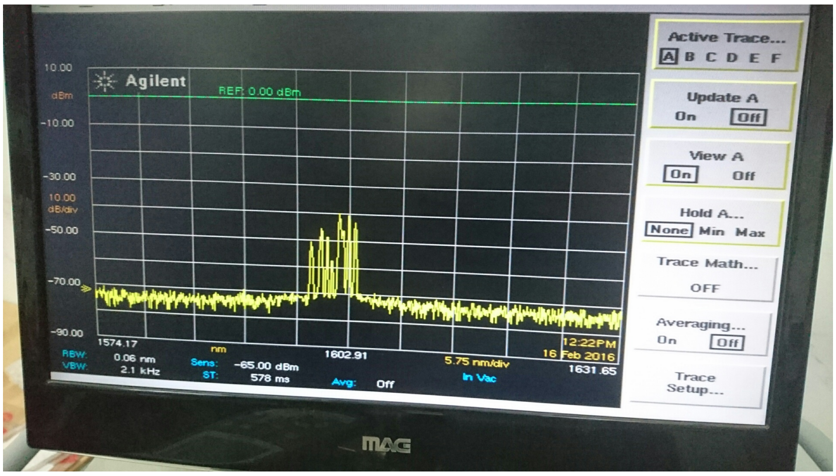


Figure 4: The output spectrum of the (unfiltered) laser.

In Figure 4 many peaks are seen, corresponding to different longitudinal modes retained by the cavity. To eliminate modal competition, a filter set to 1600 nm is introduced into the cavity. Figure 6 shows the resulting single peak at -35 dBm.

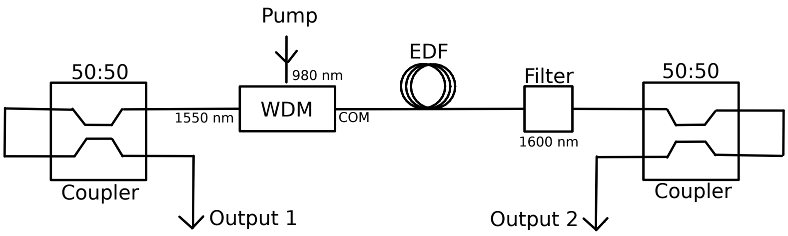


Figure 5: The setup of the filtered EDF laser.

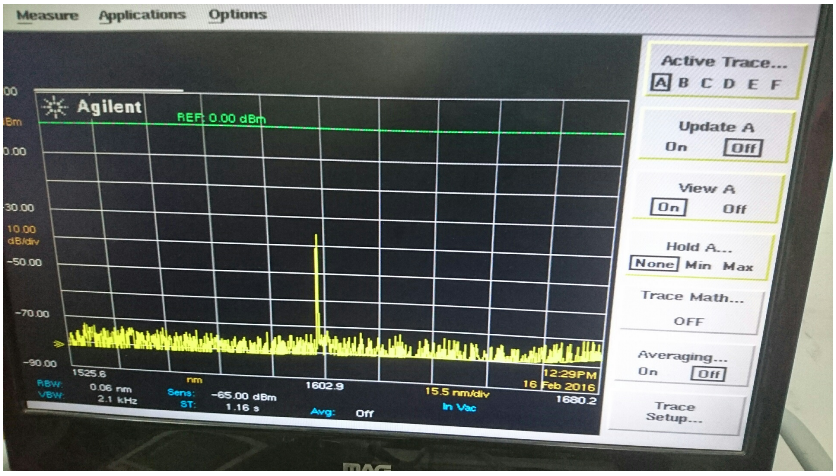


Figure 6: The output spectrum of the filtered laser.

3.4 Properties of an optical amplifier

An experiment was carried out to examine a pre-constructed EYDFA. The construction is shown in figure 7. A tunable laser (used as the signal to be amplified) is set to 1550.9 nm at 10 mW. An isolator is used to protect the TL. The signal enters an *erbium–ytterbium doped fibre* (EYDF) of 14 m, which acts as a gain medium. The pump power (915 nm, between 2 and 8 W) is introduced into the opposite end of the fibre through a WDM. Another isolator is used to maintain the unidirectional nature of the amplifier. Finally the output is directed through a filter (1550 nm, loss < 0.5 dB: ± 5 nm) to remove noise originating from ASE. 99/1 optical couplers are used to probe the power and spectra at various points in the system.

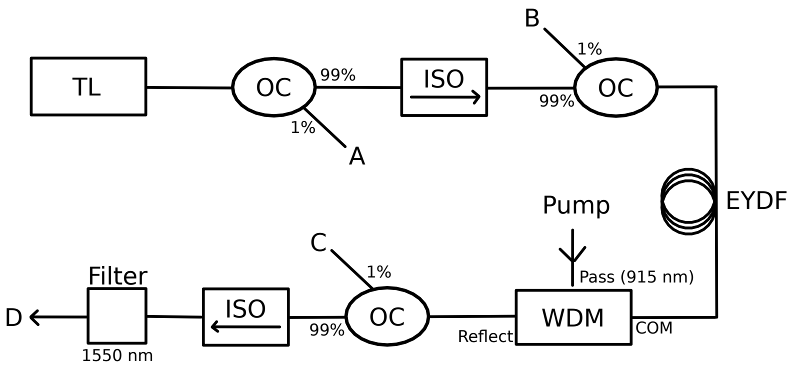


Figure 7: The construction of the EYDFA.

An initial measurement was made to find the power output of the pump (not exactly equal to its power setting) for power settings varying from 2 W up to 8 W in integer steps. The results are found in Table 3.

After this, the output power of the TL was measured to 8.9 mW. The spectrum was also measured; see Figure 9a.

The amplifier's response to the power settings was then measured. The amplified signal's output power is shown in Table 3. A linear model was fitted to the data, resulting in the approximate equation

$$P_{\text{out}} \approx 0.192 P_{\text{pump}} - 237\,\text{mW}$$

with an r^2 coefficient of determination of 0.997. The relation is plotted in Figure 8.

Finally, the spectra were measured at points A, B, C and D (of Figure 7). They are shown in Figure 9.

Pump setting (mW)	2000	3000	4000	5000	6000	7000	8000
Pump power (mW)	2033	3095	4155	5193	6207	7020	8080
Output power (mW)	187	347	538	727	973	1130	1320

Table 3: The measured output powers as a function of the pump power.

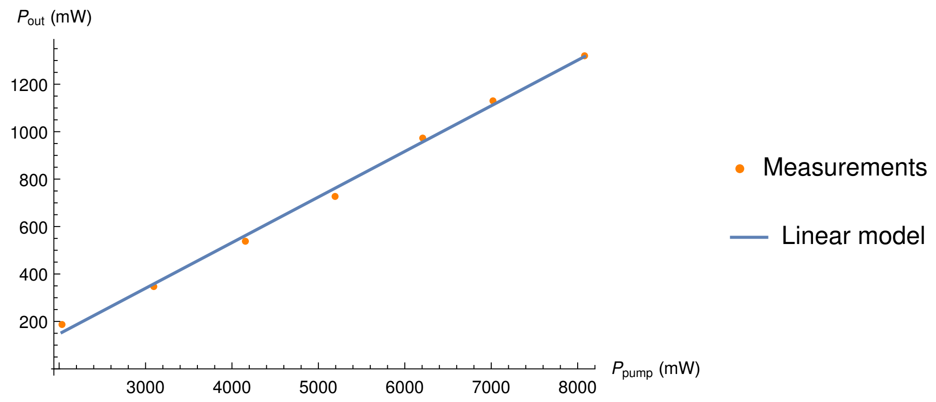
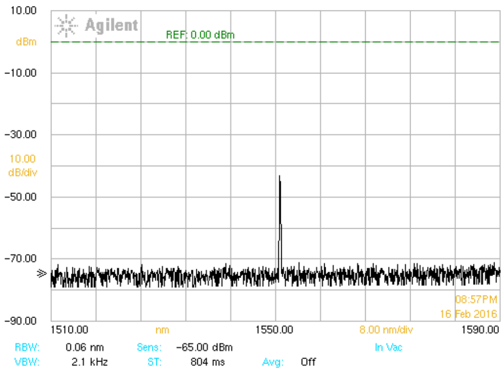
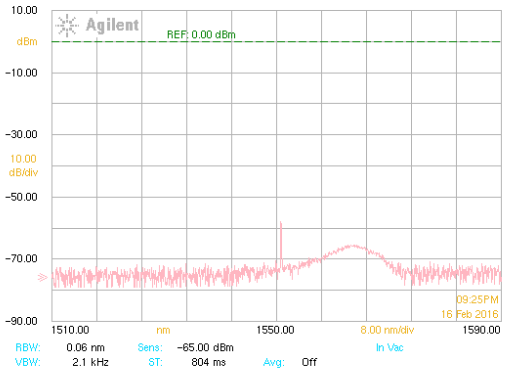


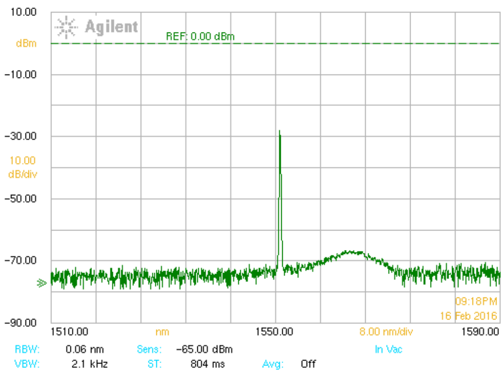
Figure 8: The linear model of the relationship between pump power and output power in the EYDFA.



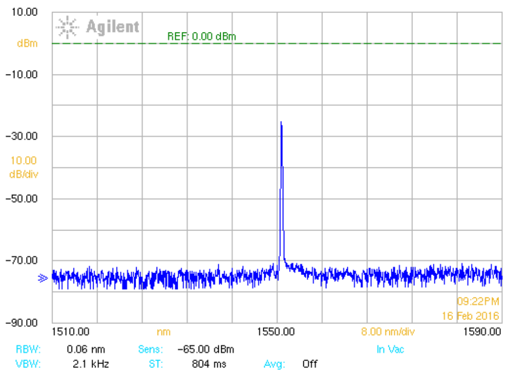
(a) Measured at A (input).



(b) Measured at B.



(c) Measured at C.



(d) Measured at D (output).

Figure 9: The spectra measured at points A, B, C and D.

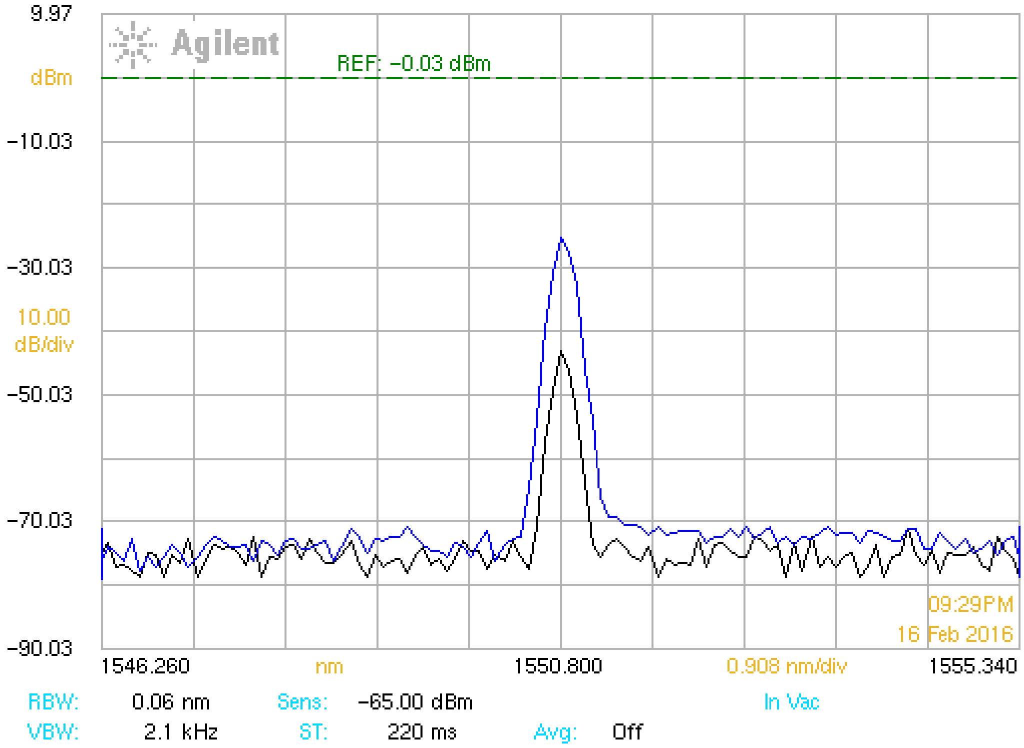


Figure 10: A comparison between the spectra of the input signal and the amplified output.

As seen clearly in Figure 10, the constructed EYDFA does indeed amplify the input signal without notably broadening its bandwidth. A comparison between the spectra at points B and C (figures 9b and 9c) illustrates that while the ASE light is generated in both directions and reaches B and C equally, only a reflection of the signal beam reaches point B. As such the signal is weaker at point B than at point C.

Note that the interpolated linear fit of Figure 8 cannot be successfully extrapolated to small values of the pump power; then the model gives negative values of the output power. This is not strange, since the linear relationship is only expected to hold for relatively strong pumps. A weak pump would not be able to keep sufficient population inversion to counteract the gain medium's absorption of the signal, so there exists a threshold value (depending on the signal power) for the pump power below which the signal is completely absorbed. For very high pump powers on the other hand, the population inversion is expected to saturate, and so the output power should reach a maximum value.

3.5 Continuous wave wavelength conversion

To prepare for wavelength conversion of OOK signals, wavelength conversion by FWM for a continuous wave (CW) was performed. Figure 11 shows the setup. A pump wave of 1553.3 nm was led through a variable EDFA set to the current 2 A. The ASE light from the EDFA was filtered away through a WDM. It was then joined with a signal of wavelength 1560 nm through another WDM. The combined signal and pump was led through a *highly non-linear fibre* (HNLF) of 500 m, causing FWM. Finally, the output spectrum was measured with an OSA as shown in Figure 12.

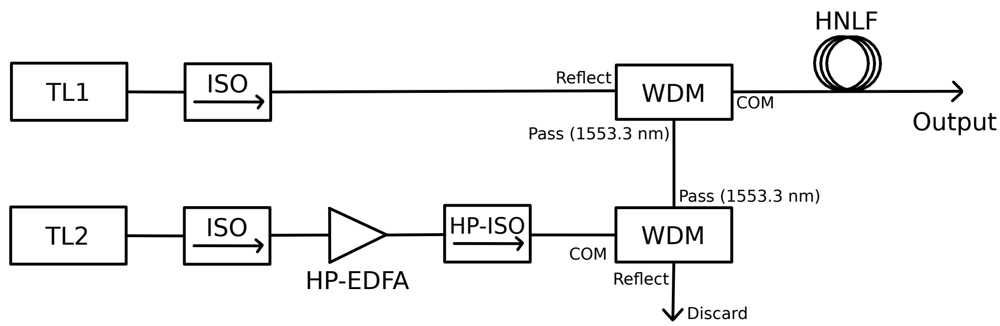


Figure 11: The construction of the CW wavelength converter.

To verify the function of this wavelength converter further, the signal wavelength was varied between 1547 nm and 1552 nm (Figure 13a), as well as between 1557 nm and 1562 nm (Figure 13b).

In Figure 12 two idlers are seen, one of them substantially weaker than the other. The frequencies of the idlers are, with f_P and f_S being the pump and signal frequencies respectively, $2f_P - f_S$ (the stronger idler) and $2f_S - f_P$ (the weaker idler). This means that they are products of two separate degenerate FWM processes between the pump and the signal. In the latter case the conventional roles of the pump and the signal are reversed. This process is only noticeable when the pump and signal are not too unequal in power.

Table 4 shows the measured idler wavelengths, compared with those computed from the measured pump and signal wavelengths. The agreement is good.

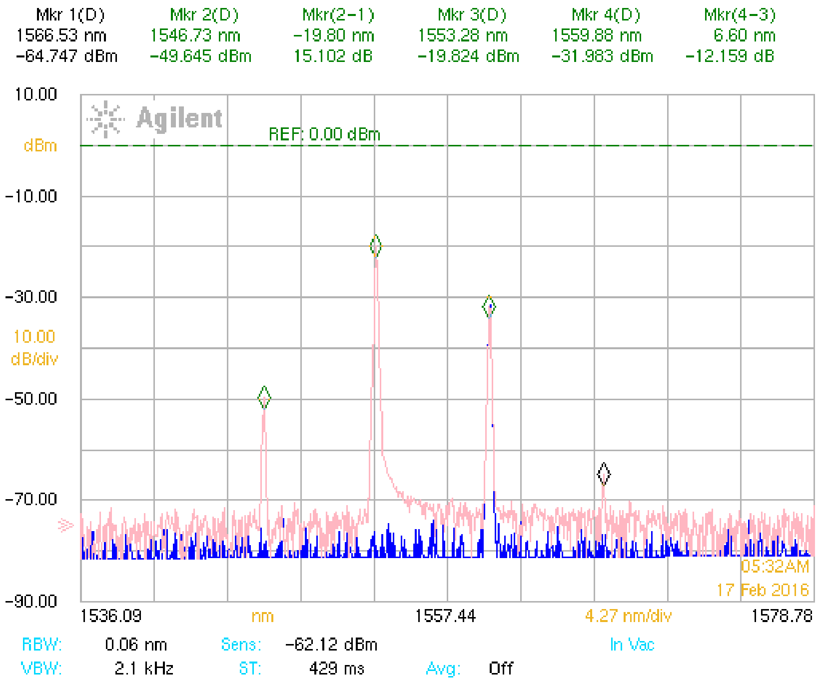
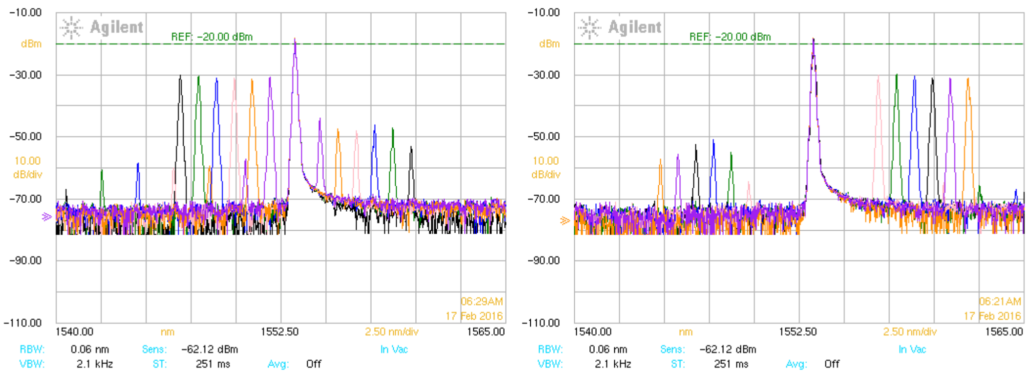


Figure 12: The FWM spectrum, showing the pump, signal and the two idlers. The blue spectrum shows just the signal (when the pump is turned off).

	Strong idler	Weak idler
Predicted (nm)	1546.93	1566.13
Measured (nm)	1546.73	1566.53

Table 4: The predicted and measured idler wavelengths.



(a) 1547 nm to 1552 nm

(b) 1557 nm to 1562 nm

Figure 13: The spectra resulting from FWM for a variety of signal wavelengths.

3.6 Modulation of an OOK signal

In preparation of wavelength conversion for an OOK signal, an experiment was carried out to generate an OOK signal and examine its quality. An electrical OOK signal was generated by an Anritsu MT1810A signal generator and input into a Mach-Zehnder modulator. A tunable fibre laser was set to 1550.00 nm and connected through an isolator to a PC to ensure proper polarization. The PC, in turn, was connected to the MZM, and the modulated signal was amplified by an EDFA. To test the signal's information-carrying properties, it was then sent through a 50 km SMF and the resulting waveform was observed through an oscilloscope. The detailed setup is shown in Figure 14.

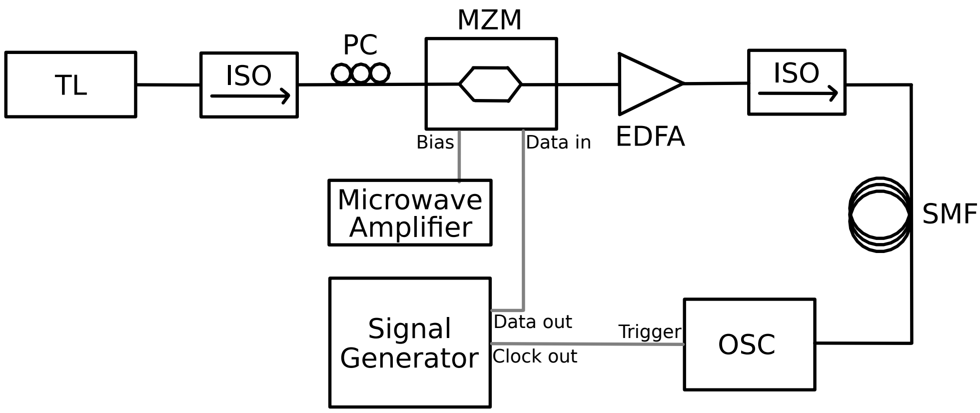


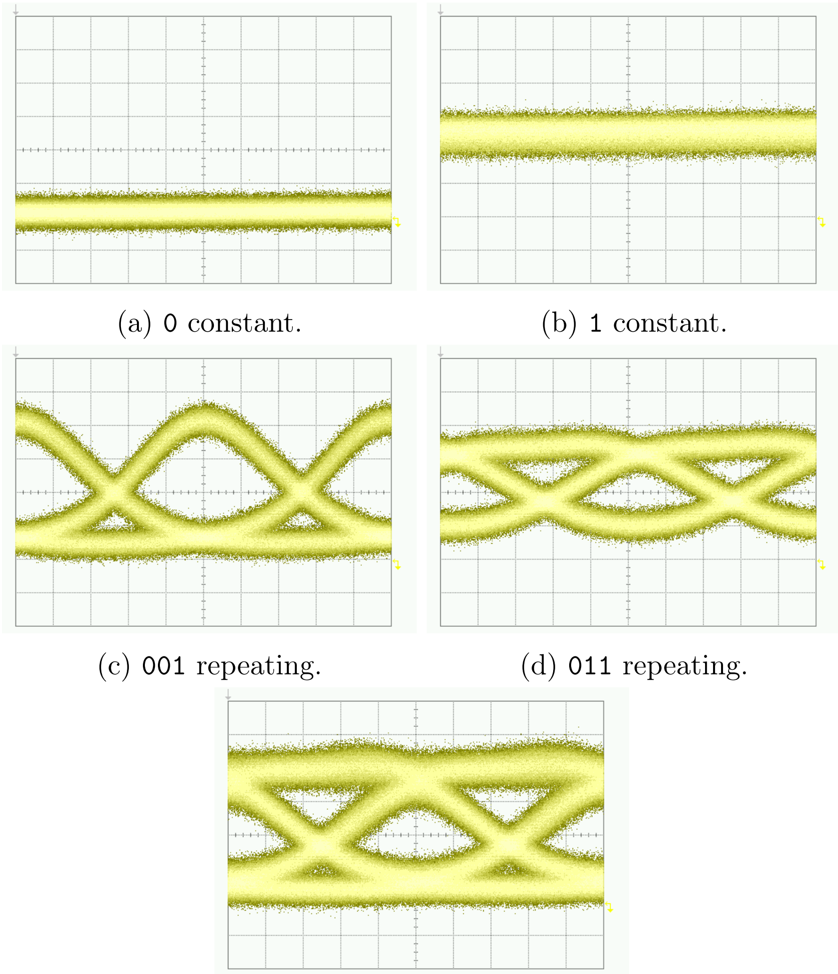
Figure 14: Setup of the OOK modulation experiment.

The PC and the *bias voltage* of the MZM, provided by a *microwave amplifier*, were carefully adjusted to produce the most accurate signal waveform, by observing it through the oscilloscope.

The power was measured with an OPM before and after the fibre, giving 8.1 dBm before and −2.5 dBm after. The fibre's attenuation was specified as 0.2 dB/km, which for 50 km becomes 10 dB. This agrees well with the measured loss of 10.6 dB (the excess is likely due to the physical connectors at the ends).

The signal generator produced a signal of bitrate 10 GHz. The signal pattern was varied; the repeating sequences `0`, `1`, `001` and `011` were tested along with a pseudo-random binary sequence (PRBS). The resulting eye diagrams are listed in Figure 15.

The waveforms are generally of good quality, especially concerning the height of the "eye" (though less so for the `011` signal). There is a notable



(a) `0` constant.

(b) `1` constant.

(c) `001` repeating.

(d) `011` repeating.

(e) PRBS.

Figure 15: The eye diagrams for the OOK signals after the SMF.

asymmetry interchanging the `0` and `1` bits, causing the crossing point between rising and falling signals to be lower than expected. A plausible explanation is that a signal of many `1` bits causes the population inversion in the EDFA's gain fibre to reach an unsaturated equilibrium, due to the larger amount of light being amplified. The explanation holds well when considering the constant `1` signal (Figure 15b), where the transmitted intensity is noticeably lower and noisier than the peaks of the other signals.

4 Main experiment

When the preparatory experiments were completed, a final experiment was carried out to perform wavelength conversion of an OOK signal in an SOI waveguide.

4.1 Setup

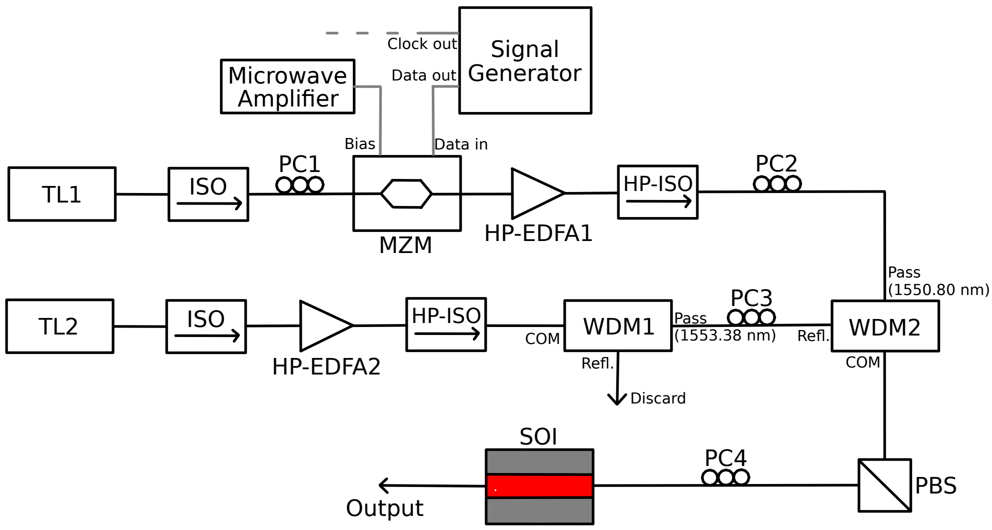


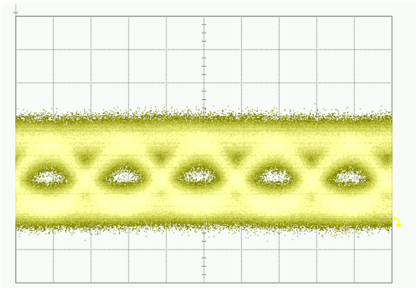
Figure 16: Setup of the main experiment.

Figure 16 describes the experimental setup. An pseudo-random OOK signal from the signal generator was modulated into the beam from tunable fibre laser (TL1) set to 1550.8 nm by an MZM in the fashion described in Section 3.6. It was then combined with a pump beam of wavelength 1553.38 nm in the same way as in Section 3.5, with the exception that the signal beam was amplified (a higher power was needed due to the short effective length of the waveguide) and the combiner WDM (WDM2) also acted as a band-pass filter eliminating the ASE noise from HP-EDFA1. Polarization controllers PC2 and PC3 were added and individually fine-tuned to maximize the throughput of the PBS. The combined beam was sent, via PC4 for final tuning, through the silicon-on-insulator waveguide (SOI). The resulting beam was analysed with an OPM, an OSA and an oscilloscope.

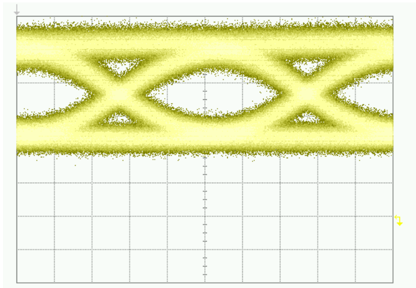
First, the function of wavelength conversion was verified for a continuous wave, with the setup shown in Figure 16 but with the MZM omitted. Afterwards, the MZM and signal generator were connected and tuned using the oscilloscope.

4.2 Measurements

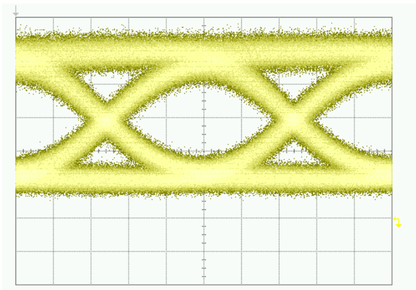
The spectra (Figure 18) and power levels (Table 5) were measured in both cases. Before and after EDFA1, as well as after WDM1, an eye diagram of the pseudo-random signal was taken with the oscilloscope (connected through a variable attenuator). They are found in Figure 17.



(a) Before EDFA1.



(b) After EDFA1.



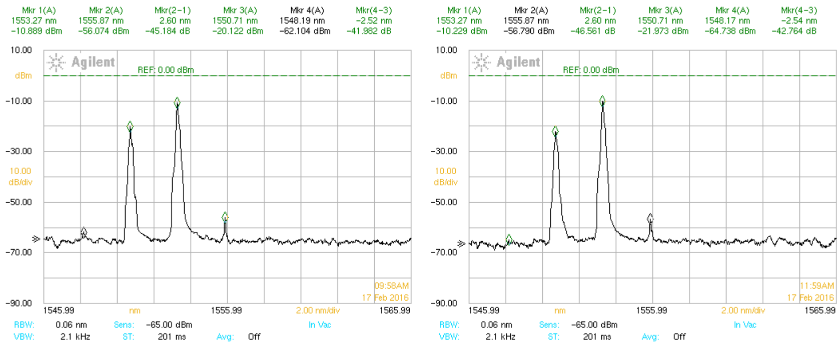
(c) After WDM1 (the filter).

Figure 17: The eye diagrams for the signal at various points in the system (the yellow arrow on the right edge shows the zero level).

(dBm)	CW	OOK
Signal after PBS	17.84	15.7
Pump after PBS	25.1	26.16
Combined after SOI	23.3	23.71

Table 5: Measured power values with and without the OOK modulation.

After having verified (through Figure 18b) that FWM occurred, and having confirmed the converted signal wavelength at 1555.87 nm (compare with 1555.84 nm, predicted from the measured pump and signal), the SOI output



(a) The spectrum for CW. (b) The spectrum for OOK.

Figure 18: The spectra for the combined signal and pump after the SOI waveguide, showing FWM.

was filtered through a band-pass WDM. The resulting spectrum is shown in Figure 19. Unfortunately the converted signal's power was very weak, around −20 dBm. This resulted in the oscilloscope not being able to detect it. This is a normal problem and is usually solved by amplifying the signal using a *pre-amplifier* (an amplifier capable of amplifying very weak signals to moderate power levels). Unfortunately, at the experiment site, no such device was available. A normal amplifier was tried, but it did not pick up the signal. As a result, no eye diagram of the converted signal could be created.

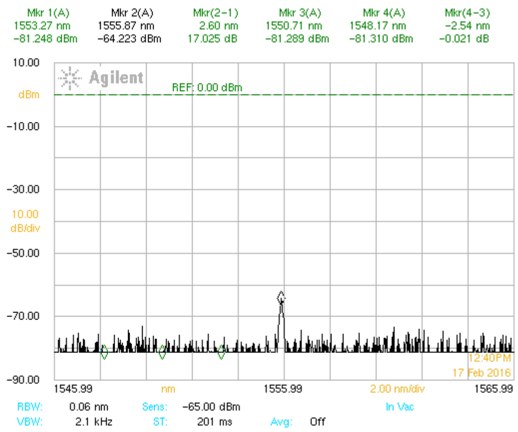


Figure 19: The filtered spectrum showing the weak converted signal.

4.3 Discussion

Because no eye diagram could be created, no conclusion can be drawn about the quality of the converted signal. However, because FWM is a parametric (instantaneous) process, there ought to be no temporal distortion of the signal. Distortion in the power dimension might conceivably ruin the distinction between `1` and `0`, although this problem would likely have been solved by the pre-amplifier. For these reasons, we feel confident that the goal of preserving the signal under wavelength conversion would have been reached if an appropriate pre-amplifier had been available.

4.4 Conclusion

Four-wave mixing in a silicon waveguide seems promising as a wavelength conversion scheme. Its relative simplicity is a great advantage compared to optical–electrical–optical processes, and an integrated chip could be made very small. It is not susceptible to temporal distortion of the signal because of its parametric nature. It should be noted, however, that while wavelength conversion of OOK signal in a silicon waveguide is possible, it is by no means a power efficient process, in our case losing 35 dB of signal power.

A Classical derivation of four-wave mixing

This appendix gives a detailed theoretical demonstration of degenerate FWM in the simplest reasonably complete way deemed possible.

A.1 Maxwell's equations in nonlinear media

Classical electromagnetic theory can be summarized in Maxwell's equations

$$\begin{aligned} \boldsymbol{\nabla} \times \vec{\mathbf{E}} &= -\frac{\partial \vec{\mathbf{B}}}{\partial t} & \qquad \boldsymbol{\nabla} \cdot \vec{\mathbf{D}} &= \rho_f \\ \boldsymbol{\nabla} \times \vec{\mathbf{H}} &= \vec{\mathbf{J}}_f + \frac{\partial \vec{\mathbf{D}}}{\partial t} & \qquad \boldsymbol{\nabla} \cdot \vec{\mathbf{B}} &= 0 \end{aligned}$$

where $\vec{\mathbf{J}}_f$ and ρ_f are the free current and free charge densities and the auxiliary fields $\vec{\mathbf{D}}$ and $\vec{\mathbf{H}}$ are defined, using the polarization $\vec{\mathbf{P}}$ and magnetization $\vec{\mathbf{M}}$, by

$$\vec{\mathbf{D}} = \varepsilon_0 \vec{\mathbf{E}} + \vec{\mathbf{P}} \qquad \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{H}} + \vec{\mathbf{M}}.$$

Consider only the cases where ρ_f and $\vec{\mathbf{J}}_f$ are both zero. Also make the simplifying assumption that $\vec{\mathbf{M}} = \vec{\mathbf{0}}$ (i.e. $\mu_r = 1$). These assumptions hold well in optical fibres and in silicon.

Taking the curl of the first of Maxwell's equations, one can use the vector identity $\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \vec{\mathbf{E}} = \boldsymbol{\nabla}\left(\boldsymbol{\nabla} \cdot \vec{\mathbf{E}}\right) - \nabla^2 \vec{\mathbf{E}}$. Here one last assumption is needed: that the total net charge density ρ is zero (or small), so $\boldsymbol{\nabla} \cdot \vec{\mathbf{E}} = \rho/\varepsilon_0 = 0$. Then, after using the equations and assumptions, one arrives at the simplified wave equation

$$\nabla^2 \vec{\mathbf{E}} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2} = \mu_0 \frac{\partial^2 \vec{\mathbf{P}}}{\partial t^2}. \tag{4}$$

For every material there is a *constitutive equation* relating $\vec{\mathbf{P}}$ and $\vec{\mathbf{E}}$. It is written as the tensor Taylor expansion

$$\vec{\mathbf{P}} = \overbrace{\varepsilon_0 (\boldsymbol{\chi}^{(1)} \vec{\mathbf{E}}}^{\vec{\mathbf{P}}^{\mathrm{L}}} + \overbrace{\boldsymbol{\chi}^{(2)} \vec{\mathbf{E}}\vec{\mathbf{E}} + \boldsymbol{\chi}^{(3)} \vec{\mathbf{E}}\vec{\mathbf{E}}\vec{\mathbf{E}} + \dots)}^{\vec{\mathbf{P}}^{\mathrm{NL}}}. \tag{5}$$

Here $\boldsymbol{\chi}^{(n)}$ is the n-th order *susceptibility tensor* and the terms denote tensor contractions; for example (with Einstein's summation convention):

$$\left(\boldsymbol{\chi}^{(3)} \vec{\mathbf{E}}\vec{\mathbf{E}}\vec{\mathbf{E}}\right)^i = \left(\boldsymbol{\chi}^{(3)}\right)^i_{jkl} E^j E^k E^l.$$

Assuming an isotropic medium, $\boldsymbol{\chi}^{(1)} = \chi_{\mathrm{e}}$ is a scalar.

Writing $\vec{\mathbf{P}} = \vec{\mathbf{P}}^{\mathrm{L}} + \vec{\mathbf{P}}^{\mathrm{NL}} = \varepsilon_0 \chi_{\mathrm{e}} \vec{\mathbf{E}} + \vec{\mathbf{P}}^{\mathrm{NL}}$ in Equation 4, and using $1 + \chi_{\mathrm{e}} = n^2$ (n is the refractive index) and $\mu_0 \varepsilon_0 = 1/c^2$, the nonlinear wave equation is reached:

$$\nabla^2 \vec{\mathbf{E}} - \frac{n^2}{c^2} \frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2} = \mu_0 \frac{\partial^2 \vec{\mathbf{P}}^{\mathrm{NL}}}{\partial t^2}. \tag{6}$$

A.2 Four-wave mixing in waveguides

Four-wave mixing relies on the third-order nonlinearity, which is most prominent when there is no second-order one, that is $\boldsymbol{\chi}^{(2)} = \mathbf{0}$. This is the case in many waveguide materials because of symmetries in the crystal structure.

To illustrate the phenomenon of (degenerate) four-wave mixing in a simple way, imagine a straight single-mode polarization-maintaining fibre (made out of third-order nonlinear material), such that the light propagates in the z direction and its $\vec{\mathbf{E}}$-field is in the x direction ($\vec{\mathbf{E}} = E(z,t)\hat{\mathbf{x}}$ in the middle of the fibre). Then the constitutive equation (5) becomes (ignoring higher-order nonlinearities)

$$\vec{\mathbf{P}}^{\mathrm{NL}} = \varepsilon_0 \vec{\mathbf{e}}_i \big(\boldsymbol{\chi}^{(3)}\big)^i_{xxx} E^3 = \varepsilon_0 \vec{\boldsymbol{\chi}}^{(3)} E^3$$

or in an isotropic medium simply

$$\vec{\mathbf{P}}^{\mathrm{NL}} = \varepsilon_0 \chi^{(3)} E^3 \hat{\mathbf{x}}. \tag{7}$$

The nonlinear wave equation (6) then becomes

$$\frac{\partial^2 E}{\partial z^2} - \frac{n^2}{c^2} \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P^{\mathrm{NL}}}{\partial t^2}. \tag{8}$$

Now consider what happens when two beams of light, at angular frequencies ω_1 and ω_2, are injected into the fibre. Equation 8 is difficult to solve analytically, so in the spirit of *perturbation theory* (but lacking the theoretical rigour), first solve the simpler, linear, equation

$$\frac{\partial^2 E}{\partial z^2} - \frac{n^2}{c^2} \frac{\partial^2 E}{\partial t^2} = 0. \tag{9}$$

For the two light beams, the general solution is

$$E = E_1 \cos(\omega_1 t - \beta_1 z) + E_2 \cos(\omega_2 t - \beta_2 z - \varphi)$$

where $\beta = n(\omega)\omega/c$ is the *propagation constant* of the beam (chromatic dispersion means that n is not a constant but a function of ω) and φ is the phase difference. At some point in time and space, the waves will be in phase. This can be formalized by the change of coordinates

$$\begin{cases} z' = z - \dfrac{\omega_1 \varphi}{\omega_2 \beta_1 - \omega_1 \beta_2} \\ t' = t - \dfrac{\beta_1 \varphi}{\omega_2 \beta_1 - \omega_1 \beta_2} \end{cases}$$

which simplifies the solution to

$$E = E_1 \cos(\omega_1 t' - \beta_1 z') + E_2 \cos(\omega_2 t' - \beta_2 z').$$

At $z' = 0$, the solution is especially simple:

$$E(z' = 0) = E_1 \cos(\omega_1 t') + E_2 \cos(\omega_2 t'). \tag{10}$$

Now the linear solution to (9) can give a good approximation to the full solution to (8). Inserting (10) into the constitutive equation (7) gives

$$P^{\mathrm{NL}}(z' = 0) = \varepsilon_0 \chi^{(3)} \left(E_1 \cos(\omega_1 t') + E_2 \cos(\omega_2 t')\right)^3$$

which expands (with the help of a computer algebra system) to

$$\begin{aligned} P^{\mathrm{NL}} = \frac{\varepsilon_0 \chi^{(3)}}{4} \Big[& 3(E_1^3 + 2E_1 E_2^2) \cos(\omega_1 t') + E_1^3 \cos(3\omega_1 t') \\ & + 3(2E_1^2 E_2 + E_2^3) \cos(\omega_2 t') + E_2^3 \cos(3\omega_2 t') \\ & + 3E_1^2 E_2 \cos((2\omega_1 - \omega_2) t') + 3E_1 E_2^2 \cos((2\omega_2 - \omega_1) t') \\ & + 3E_1^2 E_2 \cos((2\omega_1 + \omega_2) t') + 3E_1 E_2^2 \cos((2\omega_2 + \omega_1) t') \Big]. \end{aligned} \tag{11}$$

The frequencies of P^{NL} are seen to be ω_1, ω_2, $3\omega_1$, $3\omega_2$, $2\omega_1 - \omega_2$, $2\omega_2 - \omega_1$, $2\omega_1 + \omega_2$ and $2\omega_2 + \omega_1$. Because the polarization oscillates with these frequencies, it will emit light of the same frequencies.

The four frequencies ω_1, ω_2, $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$ are equally spaced and close to each other compared to the rest. Moreover, an examination of (11) shows that the amplitude of ω_1 depends on E_2 and vice versa; the energy of the two frequencies are mixed, hence the name "four-wave mixing".

A.3 Phase matching

In (11), P^{NL} was evaluated at $z' = 0$ to simplify the calculations. This does not give the full picture of the spatial dependence. In the full expansion of

$$P^{\mathrm{NL}}(z', t') = \varepsilon_0 \chi^{(3)} \left(E_1 \cos(\omega_1 t' - \beta_1 z') + E_2 \cos(\omega_2 t' - \beta_2 z')\right)^3$$

every term will depend on the space coordinate z' as well as on time. Take the term of frequency $2\omega_1 - \omega_2$ as an example (corresponding to Equation 3 with $\omega_1 = \omega_p$ and $\omega_2 = \omega_s$). The computer algebra system gives this term as

$$\frac{3\varepsilon_0 \chi^{(3)}}{4} E_1^2 E_2 \cos((2\omega_1 - \omega_2)t' - (2\beta_1 - \beta_2)z').$$

This component of the nonlinear polarization describes a travelling wave of speed

$$v_{\mathrm{pol}} = \frac{2\omega_1 - \omega_2}{2\beta_1 - \beta_2}.$$

As discussed in the previous section, at a fixed value of z' this will oscillate at the frequency $\omega_3 = 2\omega_2 - \omega_1$, thus emitting light of the same frequency. This light will travel through the medium at speed

$$v_{\mathrm{light}} = \frac{\omega_3}{\beta_3} = \frac{2\omega_1 - \omega_2}{\beta_3}.$$

If these velocities are different, eventually the emitted light waves will interfere destructively with each other. If they are the same, however, they will interfere constructively indefinitely in what might be likened to a periodic version of the "sonic boom" effect in acoustics, resulting in a strong generated wave.

For the velocities v_{pol} and v_{light} to be the same it is required that

$$2\beta_1 - \beta_2 = \beta_3, \tag{12}$$

which is called the *phase matching condition*. It expands to

$$2\omega_1\, n(\omega_1) - \omega_2\, n(\omega_2) = (2\omega_1 - \omega_2)\, n(2\omega_1 - \omega_2).$$

In the absence of chromatic dispersion, this would be automatically fulfilled.

B References

Most of the information presented in this document was given to us orally, with the aid of digital slideshows, by the instructors of the laser group. We are told that most of this information, if not all of it, is to be found in Govind P. Agrawal's *Optical fibre nonlinearity* (2007. Elsevier, Singapore. ISBN: 978-0-12-369516-1).

The theoretical derivation in Appendix A is based on a less detailed one found in the Wikipedia article on *Nonlinear optics*, in the *Parametric processes* section. (`https://en.wikipedia.org/wiki/Nonlinear_optics#Parametric_processes`. Checked on 2016–07–14).