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Bachelor thesis project proposal

Wave propagation in a heterogenous medium

Kandidatexjobb

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1. Background and general goal

In this project we consider the acoustic/elastic wave equation. In the most simple case (in 1D) we seek a wave function $u(x,t)$ (where $x \in \mathbb{R}$ is the location in space and $t \in [0,T]$ is the time) with initial values $u(x,0) = f(x)$ and $\partial_t u(x,0) = g(x)$ that solves

$$\partial_{tt} u(x,t) - \partial_x \left(A(x) \partial_x u(x,t) \right) = F(x,t). \tag{1}$$

The wave speed $A(x)$ is a function that changes it's values quickly (different materials) and we call it a multiscale coefficient. A source term is given by F.

Elastic wave equations have a significant relevance for determining subsurface formations in the earth's crust. For instance, in order to recover oil or petroleum from geologic formations, to sequestrate carbon dioxide (i.e. storing liquid CO2 in the subsurface) or to predict earthquakes, scientists need a clear image of the geologic underground, i.e. its composition concerning rock formations, soil, faults, groundwater, oil, petroleum and so on. Practically, an image of the underground is determined by generating an energy impulse (or vibrating source) that sends a seismic wave into the ground. A fraction of the wave is reflected, where the reflection pattern depends on the structures in the subsurface. When the reflected wave is measured by microphones, so called seismometers, it is possible to reconstruct the subsurface structures from the measured data by solving an inverse problem. Solving this so-called inverse problem requires to solve the elastic wave equation (forward problem) several times. In this project we want to study, in a very simple model example, how this can be solved with a numerical method and what kind of problems we will expect. One issue is for instance how the choice of initial values (in the wave equation) can trigger fast time oscillations or dispersive effects. We shall investigate the sensitivity of the finite element method with respect to these phenomena.

2. Tasks

The following list is preliminary and can change upon the students personal interests and preferences.

- Description of the model and its physical background.
- Stating a few numerical methods for solving the wave equation based on finite elements.
- Implementing one (or more) of the numerical methods.
- Simulating wave propagation for a heterogenous function A (wave speed).
- Investigating the accuracy of the method over (possibly long) time.
- Studying how the choice of the initial values influences the performance and the accuracy of the method. Stating explanations for the observed effects.

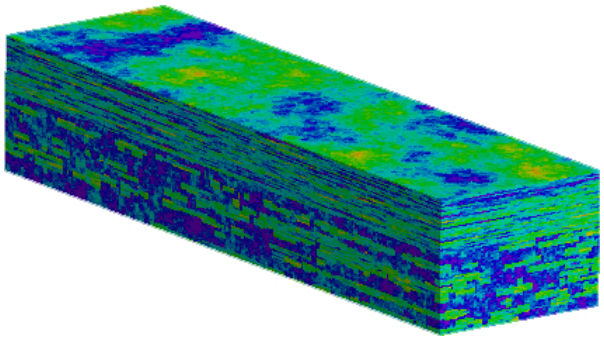

Figure 1: Geological structure of a subsurface shallow-marine sandstone formation taken from the SPE10 dataset (*www.spe.org*).