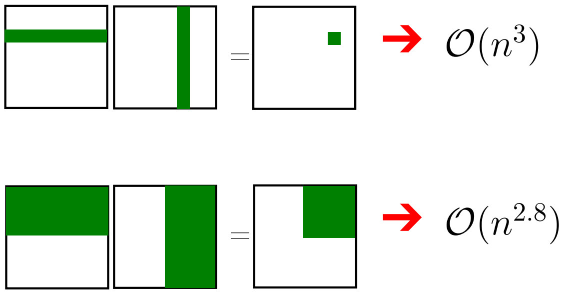
KTH Royal Institute of Technology
Numerical Analysis Group

Project proposal

Bachelor thesis project proposal:
Efficient methods for matrix multiplication

Project description. Consider two matrices $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times n}$. In your linear algebra course have you learned how you can compute the product



$$C = AB$$

with $\mathcal{O}(n^3)$ operations, by computing scalar products of rows in A with the columns of B. In many computational simulations of accurate mathematical models, large parts of the computational effort is spent on such matrix-matrix products.

In 1969 Volker Strassen discovered that the standard procedure to compute matrix-matrix multiplications is not optimal and that with a type of recursive reasoning one can derive an algorithm which computes C in $\mathcal{O}(n^\alpha)$ operations, where $\alpha = \log_2(7) \approx 2.8$. This discovery spurred an interest in finding other algorithms with lower complexity. The best algorithm has gradual improved since 1969. The current matrix-matrix multiplication world-record is an algorithm where $\alpha \approx 2.37$ - maybe you can beat this? You would have a tremendous impact in everything related to matrix computations.

The project consists of implementing the Strassen-algorithm and study:

- the computational complexity in theory and computational performance in practice.
- the numerical accuracy, impact of rounding errors and numerical stability.
- other matrix-matrix multiplication algorithms.

References and further reading:

- V. Strassen, *Gaussian Elimination is not Optimal*, Numer. Math. 13, p. 354-356, 1969
- Andrew Gibiansky, http://andrew.gibiansky.com/blog/mathematics/matrix-multiplication (blog post)
- G. Ballard, J. Demmel, O. Holtz, B. Lipschitz, O. Schwartz, *Communication-optimal parallel algorithm for Strassen's matrix multiplication*, Proceedings of the twenty-fourth annual ACM symposium on Parallelism in algorithms and architectures, p. 193-204, 2012

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