Proposal for a bachelor thesis project

October 3, 2017

1 Title

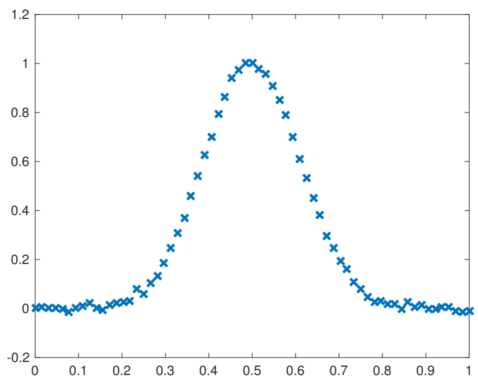
How accurate can you estimate the derivative from sampled data?

The problem In real world applications one is often confined to measured values. This measurements are subject to errors that are inherent in the physical process observed and the method to measure data. Therefore, the errors cannot be made as small as we wish. Since numerical differentiation is very sensitive with respect to errors in the data, special methods must be used in order to obtain meaningful approximations.

2 Background

An interesting application is wound healing. This is a fascinating process where a quantitative understanding is making more and more progress. A particular problem is to understand how forces drive the process. Cells (including blood cells) generate force whenever they encounter another cell or a substrate. Similar to tensile testing in rods of metal, cells pull on the surrounding to determine its stiffness. Depending on the stiffness the cell may react by generating more or less force. It may be important to determine the time which it takes to increase or decrease the observed force. The speed of change is obtained as the derivative of the observed data. The time history of the forces is measured at discrete points in time and thus subject to a certain uncertainty. So we will need numerical methods for estimating the derivative under noise.

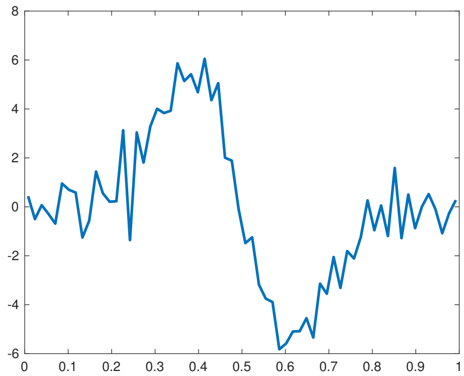
In order to understand the numerical problem, consider as a simple example the discrete data (x_i, y_i) as provided in the figure below:



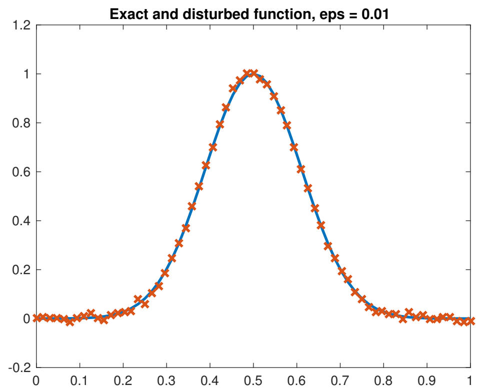
Here, $y_i \approx g(x_i)$ is a measured sample of the underlying unknown function g. We are interested in an estimation of the derivative of this function. A simple finite difference approximation of the type

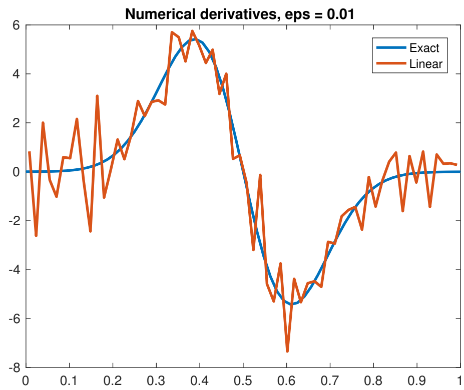
$$g'(x_{i+1/2}) \approx \frac{y_{i+1} - y_i}{x_{i+1} - x_i}$$

with $x_{i+1/2} = (x_{i+1} + x_i)/2$ give the following result:



This behavior seems far from being true. In fact, the following two figures show plots of the hiden function, its exact derivative and the numerical approximation:





3 General Task

Numerical differentiation is a simpe instance of a so-called ill-posed problem where the results does not depend continuously on the perturbations of the data. Such problems can lead to desastrous results unless specially designed numerical methods are applied. Your task is to implement methods for the numerical differentiation and to compare their properties at a number of examples. For that, no biological knowledge is necessary.

4 Steps

- Theory: What does ill-posedness mean? What are the consequences? How can they be overcome?
- Investigation of numerical methods for numerical differentiation.
- Development and implementation of numerical methods for numerical differentiation. Compare their merits.

5 References

- Andreas Kirsch: An Introduction to the Mathematical Theory of Inverse Problems, Springer 2011
- Martin Hanke, Otmar Scherzer: Inverse problems light: Numerical differentiation. Amer. Math. Monthly 108(2001)6, 512–521

For those who are interested in the biological background:

- L. Trichet et al.: Evidence of a large-scale mechanosensing mechanism for cellular adaptation to substrate stiffness. PNAS 109(2012), 6933–6938