KEX project.

Numerical quadrature for singular integrals.

Integrals with singular integrands occur for example when solving integral equations. In this project we will consider the numerical evaluation of integrals where the integrand is singular at some point within the integration interval, but where the integral is well defined. This requires special numerical quadrature schemes, and we will investigate some strategies to design such schemes. To get started, we will:

- Use a standard quadrature rule to evaluate a singular 1D integral. Analyze why the accuracy of the numerical integration degenerates for this case, and what accuracy can be expected.
- Explore the idea of so-called singularity subtraction to improve on the results.
- Investigate how the situation can be improved by using adaptive quadrature.

We will then investigate one or two techniques for designing appropriate quadrature schemes. One approach is to locally modify weights in the quadrature rule, according to the type of singularity. Another approach is to combine a polynomial approximation of the smooth function f(x) with analytical integration.

The integral

$$\int_{-1}^{1} \frac{f(x)}{\sqrt{x}}\, dx$$

is well-defined even though the integrand is singular at $x = 0$.
(Here, $f(x)$ *is a smooth function).*

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References:

The Wikipedia page on numerical integration:
(Does not discuss singular integrals though)
http://en.wikipedia.org/wiki/Numerical_integration

Chapter 5 from Dahlquist and Björck, “Numerical Methods in Scientific Computing” (SIAM).
http://www.siam.org/books/ot103/OT103%20Dahlquist%20Chapter%205.pdf