KEX project.

Solving PDEs in "free space" by the use of FFTs

In many applications, one wants to numerically solve a partial differential equation (PDE) without imposing boundary conditions on an enclosing boundary and rather mimic an "infinite domain". This is the case for example when simulating electromagnetic or acoustic waves that hit an object and scatters outwards. However, discretizing the PDE with traditional grid based techniques, a finite size domain must be used, and quite advanced techniques have been developed to avoid the artificial reflection that occurs at boundaries of the computational domain.

Very recently, a new method was proposed by Vico et al. to solve PDEs without such external boundaries, in "free space". The method relies on regularizing the Fourier transform of the free-space Green's function of the PDE, and allows for a simple discretization on uniform grids and the use of the fast Fourier transform (FFT) to solve the problem. The approach completely avoids the problem of imposing artificial boundary conditions on the computational domain.

The purpose of this project is to solve a PDE with this method, both in two and three dimensions. Initially, we will consider the Poisson equation. A bit more specifically, this entails:

- Learning about how to use the FFT to solve linear PDEs for periodic problems in one dimension. Write a Matlab program to do it, and then extend it to two dimensions.
- Learn about so-called aperiodic convolutions and how FFTs can be used to compute these.
- In two dimensions, write a Matlab program to solve the free-space Poisson equation. Then extend to three dimensions (not a difficult step, although it might sound like it).
- If time permits, solve a different equation, e.g. the Helmholtz equation with the same technique.

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References:
F. Vico, L. Greengard and M. Ferrando. Fast convolution with free-space Green's functions. Journal of Computational Physics, 323:191-203, 2016.

The Wikipedia page on the FFT: https://en.wikipedia.org/wiki/Fast_Fourier_transform

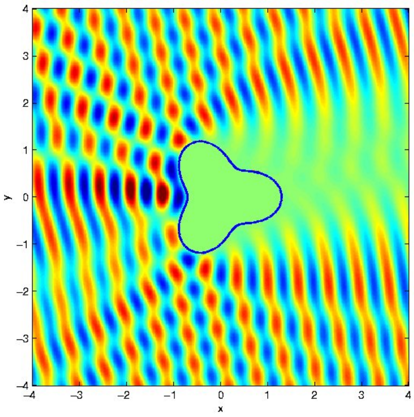

Fig: 2D wave scattering from an obstacle. Picture courtesy of Prof. Barnett at U. of Dartmouth.