Solution framework for Adaptive Finite Element methods

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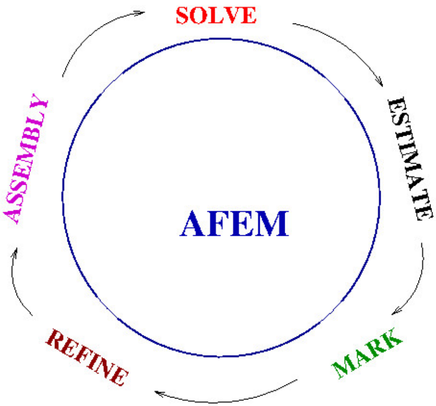
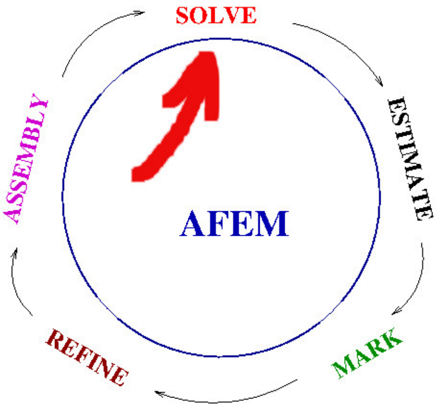
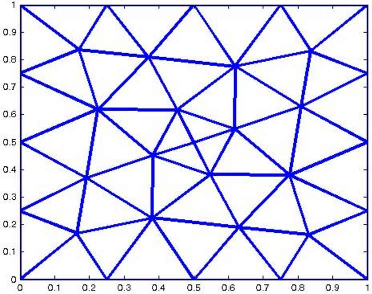
AFEM framework

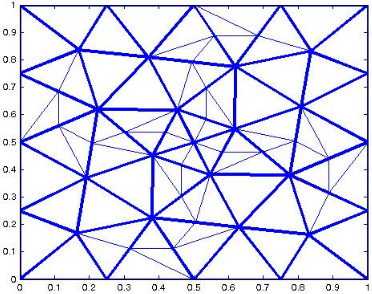


AFEM framework

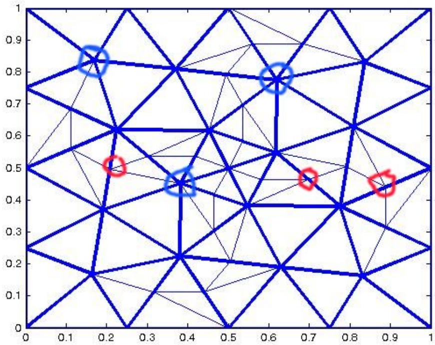


Mesh refinement





Mesh refinement



In blue - old points, in red - new points
The devision in new'-'old' imposes a two-by-two block structure on the finite element stiffness matrix.

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By construction, $A^{(\ell)}$ has a two-by-two block structure

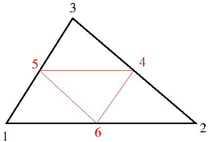
$$A^{(\ell)} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{matrix} \}\text{red} \\ \}\text{blue} \end{matrix}$$

It can be preconditioned by a block-factorized form by approximating some of the matrix blocks.

$$B^{(\ell)} = \begin{bmatrix} A_{11} & 0 \\ A_{21} & S^{(\ell)} \end{bmatrix} \begin{bmatrix} I_1 & A_{11}^{-1} A_{12} \\ 0 & I_2 \end{bmatrix}$$

where $S^{(\ell)}$ is some approximation of the exact Schur complement $S_A^{(\ell)}$ of $A^{(\ell)}$, $S_A^{(\ell)} = A_{22} - A_{21} A_{11}^{-1} A_{12}$.

What to do with $S_A^{(\ell)}$?

For regular refinements 

and Hierarchical basis functions (HBF) it is well known that $A^{(k-1)}$ is a good approximation of $S^{(k)}$. Thus, the preconditioner is

$$B^{(\ell)} = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A^{(\ell-1)} \end{bmatrix} \begin{bmatrix} I_1 & A_{11}^{-1}A_{12} \\ 0 & I_2 \end{bmatrix}$$

In practice:

$$B^{(\ell)} = \begin{bmatrix} A_{11} & 0 \\ A_{21} & B^{(\ell-1)} \end{bmatrix} \begin{bmatrix} I_1 & A_{11}^{-1}A_{12} \\ 0 & I_2 \end{bmatrix}$$

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Take-with-you message:

- In the AFEM framework for a reasonable broad class of problems (*anisotropic, convection-diffusion-advection, parabolic problems, discontinuous coefficients*) the matrix at the previous refinement level is a good approximation of the Schur complement.
- No construction costs!
- Recursively applicable.
- Mesh-independent estimates can be shown.

Problem 1 (nonsymmetric)		Problem 2 (anisotropy)		Problem 3 ('Moon'-like domain)	
Problem size	Iter.	Problem size	Iter.	Problem size	Iter.
545	15	537	15	1035	1
2112	18	2037	19	3952	9
8278	20	7504	23	15239	12
32030	23	26770	26	54998	14
118720	25	96779	29	176234	16
403811	28	275989	31	546851	17
		486307	32		

Work to do:

- Estimate the spectral equivalence constant for AFEM
- Test more problems