Cristina la Cognata - Computational mathematics - MAI - Linköping University

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Linköping University - MAI Computational mathematics

May 28, 2014

- **Phd student in Computational Mathematics at Linköping University**
  supervisor: Jan Nordström
  Topic of research: Numerical methods for geophysical problem (climate and oceanographic modelling, earthquakes, etc.)
- **Master in Applied mathematics at "La Sapienza" University of Rome** in collaboration with **ENEA** (Italian national agency for New Technologies, Energy and Sustainable Economic Development)
  Thesis: "Simulations of Quasi-Geostrophic Flows with Finite Difference Schemes"

LINKÖPINGS UNIVERSITET

SAPIENZA
UNIVERSITÀ DI ROMA

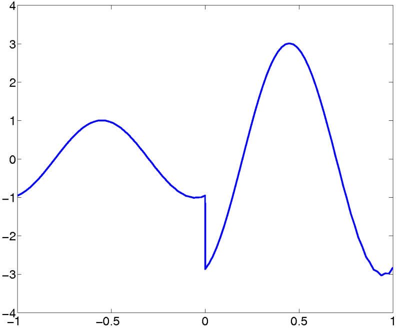
ENEA
Agenzia nazionale
per le nuove tecnologie, l'energia
e lo sviluppo economico sostenibile

"Study reality with a simple model"

Interface problem for the advection equation with discontinuous coefficient and solution

$$u_t + au_x = 0, \quad -1 \leq x \leq 0$$
$$v_t + bv_x = 0, \quad 0 < x \leq 1$$

Interface jump condition: $v(0,t) = cu(0,t), \quad c \in \mathbb{R}$



Application:

- Wave propagation in different materials
- Earthquakes

The continuous problem inspires the scheme's construction

The energy method

$$\int_{-1}^{0} u\left[u_t + a u_x\right] dx + \int_{0}^{1} \alpha_c v\left[v_t + b v_x\right] dx = 0$$

$$\frac{d}{dt}\left(\|u\|^2 + \alpha_c \|v\|^2\right) = BoundaryTerm + InterfaceTerm$$

gives the following guidelines:

- Well-posedness $\forall\ a, b > 0$ and $c \in \mathbb{R}$: $BT + IT \leq 0$ in some specific norm defined by α_c
- Boundary conditions
- Conservation $c = a/b$

Semidiscretization with SBP+SAT

Spatial discretization

$$u_x \approx D\mathbf{u} = P^{-1}Q\mathbf{u}, \qquad P > 0 \text{ and diagonal }, Q + Q^T = \begin{bmatrix} -1 & & \\ & 0 & \\ & & 1 \end{bmatrix}$$

ignoring the boundary term

$$\begin{aligned} \mathbf{u}_t + aP_l^{-1}Q_l\mathbf{u} &= P_l^{-1}\sigma_L(cu_N - v_0)e_N \\ \mathbf{v}_t + bP_r^{-1}Q_r\mathbf{v} &= P_r^{-1}\sigma_R(v_0 - cu_N)e_0, \end{aligned}$$

The energy method

$$\frac{d}{dt}\left(\|\mathbf{u}\|^2 + \alpha_d\|\mathbf{v}\|^2\right) = BoundaryTerm + InterfaceTerm$$

$$\textbf{Interface Term} = \begin{pmatrix} u_N \\ v_0 \end{pmatrix}^T H \begin{pmatrix} u_N \\ v_0 \end{pmatrix} \qquad \textbf{where H is symmetric}$$

The core of the study

Choose σ_L, σ_R and α_d such that

- **Stability:** $IT \leq 0$
- **conservation:** continuous and semidiscrete?
- **High order accuracy**
- **Spectral Analisys:** convergence of the semidiscrete spectrum to the continuous one