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May 28, 2014
• Phd student in Computational Mathematics at Linköping University
  supervisor: Jan Nordström
  **Topic of research:** Numerical methods for geophysical problem (climate and oceanographic modelling, earthquakes, etc.)

• **Master in Applied mathematics at “La Sapienza” University of Rome** in collaboration with ENEA (Italian national agency for New Technologies, Energy and Sustainable Economic Development)
  **Thesis:** “Simulations of Quasi-Geostrophic Flows with Finite Difference Schemes”
Interface problem for the advection equation with discontinuous coefficient and solution

\[ u_t + au_x = 0, \quad -1 \leq x \leq 0 \]
\[ v_t + bv_x = 0, \quad 0 < x \leq 1 \]

**Interface jump condition:** \( v(0, t) = cu(0, t), \quad c \in \mathbb{R} \)

**Application:**
- Wave propagation in different materials
- Earthquakes
The continuous problem inspires the scheme’s construction

The energy method

\[
\int_{-1}^{0} u [u_t + au_x] \, dx + \int_{0}^{1} \alpha c \nu [\nu_t + b\nu_x] \, dx = 0
\]

\[
\frac{d}{dt} (\|u\|^2 + \alpha c \|v\|^2) = \text{BoundaryTerm} + \text{InterfaceTerm}
\]

gives the following guidelines:

- **Well-posedness** \( \forall a, b > 0 \) and \( c \in \mathbb{R} : BT + IT \leq 0 \) in some specific norm defined by \( \alpha c \)

- **Boundary conditions**

- **Conservation** \( c = a/b \)
Semidiscretization with SBP\(+\)SAT

Spatial discretization

\[ u_x \approx Du = P^{-1}Qu, \quad P > 0 \text{ and diagonal}, \quad Q + Q^T = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \]

ignoring the boundary term

\[ u_t + aP_l^{-1}Q_l u = P_l^{-1}\sigma_L(cu_N - v_0)e_N \]
\[ v_t + bP_r^{-1}Q_r v = P_r^{-1}\sigma_R(v_0 - cu_N)e_0, \]

The energy method

\[ \frac{d}{dt} (\|u\|^2 + \alpha_d \|v\|^2) = \text{Boundary Term} + \text{Interface Term} \]

\[ \text{Interface Term} = \begin{pmatrix} u_N \\ v_0 \end{pmatrix}^T H \begin{pmatrix} u_N \\ v_0 \end{pmatrix} \quad \text{where } H \text{ is symmetric} \]
The core of the study

Choose $\sigma_L, \sigma_R$ and $\alpha_d$ such that

- **Stability:** $\Im T \leq 0$
- **Conservation:** continuous and semidiscrete?
- **High order accuracy**
- **Spectral Analysis:** convergence of the semidiscrete spectrum to the continuous one

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**Convergence of conservative scheme with dissipation**

**Convergence rate of the discrete spectrum without dissipation**