Motion-Adaptive Transforms based on Vertex-Weighted Graphs

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Motivation

• Limitation of motion-compensated predictive coding for packet-based networks

• Our approach:
  – Design motion-adaptive temporal transforms
  – Use vertex-weighted graphs to represent the motion
Outline

• Motion and vertex-weighted graphs
• Constrained energy compaction
• Motion-adaptive transforms
• Experimental results
• Conclusion
Motion and Vertex-Weighted Graphs

- $x_1, x_2, \ldots, x_n$ are pixels connected by block-based motion estimation
- A graph is formed by the connection of motion vectors
- Vertex weights is given by the values of $x_1, x_2, \ldots, x_n$
- Vector of actual pixel values: $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T$
Ideal Motion

- Energy compaction changes the magnitude of the pixels
- Ideal motion implies constant intensity of connected pixels
- Energy compaction + ideal motion
  - Use actual values for the vertex-weighted graph
  - Use scale factors to accommodate energy compaction\(^1\)
    - \( x_k = c_k x_k' \) for \( k = 1, 2, \ldots, n \)
      where \( x_k' \) is the original pixel value, \( c_k \) is the scale factor.
- For ideal motion: \( x_1' = x_2' = \cdots = x_n' \)
- Vector of scale factors: \( \mathbf{c} = [c_1, c_2, \ldots, c_n]^T \)

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Ideal Motion

- Let the transform matrix be
  \[ T = \begin{bmatrix} t_1, t_2, \ldots, t_n \end{bmatrix} \]

- Output
  \[ y = T^T x = T^T \begin{bmatrix} c_1 x'_1 \\ c_2 x'_1 \\ \vdots \\ c_n x'_1 \end{bmatrix} = \begin{bmatrix} \sqrt{c^T c} \cdot x'_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \]

  \[ \Rightarrow \sqrt{c^T c} = t_1^T c \]

- Subspace constraint
  \[ 0 = t_k^T c, \ k = 2, \ldots, n \]
Design of Motion-Adaptive Transform

• Classic transform coding
  – Energy compaction
  – Karhunen–Loève Transform (KLT)

• Transform along motion trajectory
  – Invertibility
  – Example: Motion-Compensated Orthogonal Transform (MCOT)[1]

• Our design goal
  – Invertible motion-adaptive transform
  – Optimal energy compaction given the graph

2013-03-21 D. Liu and M. Flierl, "Motion-Adaptive Transforms based on Vertex-Weighted Graphs"
Constrained Energy Compaction

• Let $L$ be the autocorrelation matrix of $x$

$$
\min_{t_k} \quad -t_k^T L t_k, \quad k = 2, \ldots, n,
$$

s.t.  $$
   t_k^T t_k = 1,
$$
   $$
   t_k^T t_j = 0, \quad j = 1, \ldots, k - 1,
$$
   $$
   t_1 = \frac{c}{\|c\|_2}.
$$

• Unconstrained formulation using a Lagrangian cost
Subspace-Constrained Transform (SCT)

Find rotation $A'$ in $(n-1)$-dimensional subspace

$B' = [b_2, b_3, \ldots, b_n]$

$T' = [t_2, t_3, \ldots, t_n]$

$= B'A'$
Subspace-Constrained Transform (SCT)

- Let $M' = B'^T L B'$ be the autocorrelation matrix based on $B'$. $M'$ is an $(n - 1) \times (n - 1)$ matrix.
- We show that the rotation $A'$ is a matrix of eigenvectors for $M'$
- We are free to choose any basis $B'$ to construct our $T$
- $T = [t_1, T'] = [t_1, B'A']$
Discussion

• If $t_1$ is identical to the first basis vector of the KLT, SCT is the same as the KLT.

• In general, SCT approximates the KLT for a given graph.

• The following experiments illustrate
  – $t_1$ approximates the first basis vector of the KLT
  – the energy compaction of SCT is close to that of the KLT
Experimental Setup

• Estimate the autocorrelation matrix $L$

$$L_{ij} = \sum_{r=1}^{N} x_i^{(r)} x_j^{(r)}, \text{ where } N \text{ is the total number of samples, } r \text{ is an instance of a graph}$$

- Consider samples that use the same vertex-weighted graph

2013-03-21  D. Liu and M. Flierl, "Motion-Adaptive Transforms based on Vertex-Weighted Graphs"
Experimental Setup

• A hierarchical decomposition is performed on each GOP

• Results on energy compaction are given for comparison
  – MCOT: basis vectors are dependent on the motion vectors only
two tap, hierarchical Haar
  – SCT: basis vectors $t_2, \ldots, t_n$ are signal dependent
  – KLT: basis vectors $t_1, t_2, \ldots, t_n$ are signal dependent
Experimental Results

- QCIF Foreman, relative energy in the first four temporal subbands, 1st decomposition level

<table>
<thead>
<tr>
<th></th>
<th>Lowband</th>
<th>Highband 1</th>
<th>Highband 2</th>
<th>Highband 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCOT</td>
<td>99.36%</td>
<td>0.12%</td>
<td>0.42%</td>
<td>0.10%</td>
</tr>
<tr>
<td>SCT</td>
<td>99.36%</td>
<td>0.54%</td>
<td>0.08%</td>
<td>0.02%</td>
</tr>
<tr>
<td>KLT</td>
<td>99.42%</td>
<td>0.49%</td>
<td>0.07%</td>
<td>0.02%</td>
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- QCIF City, relative energy in the first four temporal subbands, 1st decomposition level

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</thead>
<tbody>
<tr>
<td>MCOT</td>
<td>93.99%</td>
<td>1.54%</td>
<td>3.13%</td>
<td>1.34%</td>
</tr>
<tr>
<td>SCT</td>
<td>93.99%</td>
<td>4.08%</td>
<td>1.40%</td>
<td>0.53%</td>
</tr>
<tr>
<td>KLT</td>
<td>94.18%</td>
<td>3.95%</td>
<td>1.35%</td>
<td>0.52%</td>
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Experimental Results

• QCIF Foreman, GOP = 16, relative energy in the 2\textsuperscript{nd} decomposition level

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<tr>
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<tr>
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<td>0.30%</td>
<td>0.09%</td>
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<tr>
<td>KLT</td>
<td>98.35%</td>
<td>1.30%</td>
<td>0.28%</td>
<td>0.07%</td>
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• QCIF City, GOP = 16, relative energy in the 2\textsuperscript{nd} decomposition level

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</thead>
<tbody>
<tr>
<td>MCOT</td>
<td>90.12%</td>
<td>2.95%</td>
<td>4.27%</td>
<td>2.66%</td>
</tr>
<tr>
<td>SCT</td>
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<td>7.47%</td>
<td>1.86%</td>
<td>0.55%</td>
</tr>
<tr>
<td>KLT</td>
<td>91.22%</td>
<td>6.50%</td>
<td>1.77%</td>
<td>0.51%</td>
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Conclusion

• We present a class of motion-adaptive transforms that is based on vertex-weighted graphs.

• The vertex-weighted graph determines uniquely the first basis vector of the linear transform.

• This first vector defines a subspace that constrains the energy compaction of our transform.

• SCT achieves optimal energy compaction, given our subspace constraint.