Lecture 7

More on Painlevé II

We continue by analyzing the RHP from the previous lecture, but now we make the reduction

$$\begin{cases} \nu = 0 \\ s_2 = s_5 = 0 \\ s_1 = \varepsilon s \\ s_3 = -\varepsilon s \\ s_4 = -s \\ s_6 = s \end{cases}$$

for some $\varepsilon \in [0, 1]$ and hence consider the RHP:

RH problem 7.0.1. *Let $s \in \mathbb{C}$. And let Γ_ψ and J_ψ as in Figure 7.5. Search for 2×2 analytic function Ψ such that*

- *Ψ is analytic in $\mathbb{C} \setminus \Gamma_\psi$*
- *$\Psi_+(z) = \Psi_-(z) J_\psi$*
- *$\Psi(z) = (I + \mathcal{O}(1/z))\mathrm{e}^{-\sigma_3\theta(z,x)}$, as $z \to \infty$*
- *Ψ is bounded near the origin.*

Here $\theta(z, x) = \mathrm{i}(\frac{4}{3}z^3 + xz)$.

We have shown that there exists a countable set X_s^ε (for which the intersection with any compact set in $\mathbb{C}$ is finite) such the solution to this RHP exists for $x \in \mathbb{C} \setminus X_s^\varepsilon$ and depends analytically on $x \in \mathbb{C} \setminus X_s^\varepsilon$. Moreover,

$$\Psi(z) = \left(I + \Psi_1/z + \mathcal{O}(1/z^2)\right) \mathrm{e}^{-\sigma_3\theta(z,x)}$$

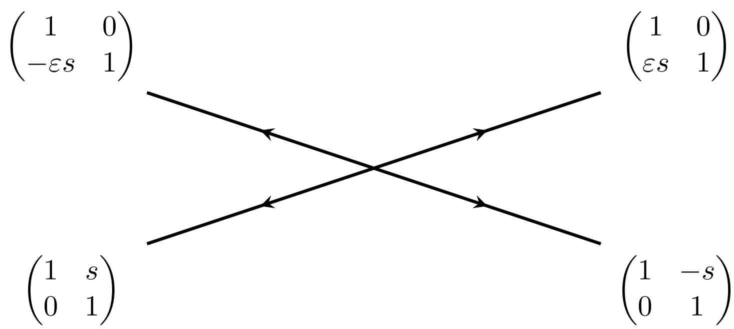

Figure 7.1: The jump contour and matrices for Ψ

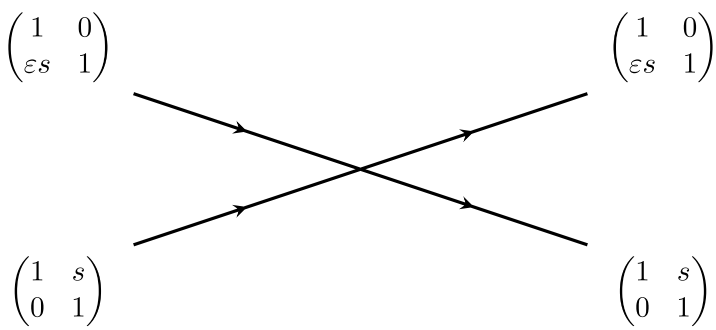

Figure 7.2: The jump contour and matrices for Ψ. Since the orientation in the left rays is reversed, the jump matrices are inverted.

as $z \to \infty$, we have that $u = 2(\Psi_1)_{12}$ solves the equation

$$u_{xx} = xu + 2\varepsilon u^3.$$

Finally, we note that we can change the orientation of the rays at the left half plane and replace the jumps with the inverses. We will assume that J_ψ is then as in Figure 7.

7.1 Continuity in ε

It is clear that when $\varepsilon \downarrow 0$, the equation turns into the the Airy equation. But the question is, whehter the same is true for the solutions. The following theorem shows that we have pointwise convergence for the solutions.

Theorem 7.1.1. *Let* $x \in \cap X_{\varepsilon\in[0,1]}$. *For* $\varepsilon \in [0,1]$ *denote the solution to the RHP by* Ψ^{ε} *and* $u_{\varepsilon} = 2(\Psi_1^{\varepsilon})_{12}$. *Then* $\varepsilon \mapsto u_{\varepsilon}(x)$ *is a continuous function on* $[0,1]$.

Proof. Fix $\varepsilon \in [0,1]$ and $\delta \in [0,1]$ close to ε. Then we define

$$\Phi_{\delta} = \Psi_{\delta}\Psi_{\varepsilon}^{-1}.$$

Then Φ_{δ} satisfies the RHP

1. Φ_{δ} is analytin in $\mathbb{C} \setminus \Gamma_{\Phi_{\delta}}$
2. $\Phi_{\delta,+} = \Phi_{\delta,-}J_{\Phi_{\delta}}$
3. $\Phi_{\delta}(z) = I + \mathcal{O}(1/z)$ as $z \to \infty$.

where $\Gamma_{\Phi_{\delta}} = \Gamma_{\Psi^{\varepsilon}}$ and

$$J_{\Phi_{\delta}} = \Phi_{\delta,-}^{-1}\Phi_{\delta,+} = \Psi_{\varepsilon,-}\Phi_{\delta,-}^{-1}\Psi_{\delta,+}\Phi_{\varepsilon,+}^{-1}\Psi_{\varepsilon,-}J_{\Psi_{\delta}}J_{\Psi_{\varepsilon}}^{-1}\Phi_{\varepsilon,+}^{-1}.$$

Now $J_{\Psi_{\delta}}J_{\Psi_{\varepsilon}}^{-1} = I$ for the jumps on the two rays in the lower half plane. In the upper half plane we find

$$J_{\Phi_{\delta}} = \Psi_{\varepsilon,-}\begin{pmatrix} 1 & 0 \\ (\delta-\varepsilon) & 0 \end{pmatrix}\Phi_{\varepsilon,+}^{-1}.$$

Hence (why?)

$$\|J_{\Phi_{\delta}} - I\|_{\infty,2} = \mathcal{O}(\delta-\varepsilon), \qquad \delta \to \varepsilon.$$

Hence we see that $\Psi_{\delta}(z) \to \Psi_{\varepsilon}(z)$ uniformly for compact subsets of $\mathbb{C} \setminus \Gamma_{\Phi_{\delta}}$ by Proposition **??**. Hence also $\Psi_{\delta}^{-1}(z) \to \Psi_{\varepsilon}^{-1}(z)$ and $\frac{\partial}{\partial z}\Psi_{\delta}(z) \to \frac{\partial}{\partial z}\Psi_{\varepsilon}(z)$ uniformly for compact subsets of $\mathbb{C}\setminus\Gamma_{\Phi_{\delta}}$. Hence $A_{\delta} \to A_{\varepsilon}$ and $u_{\delta}(x) \to u_{\varepsilon}(x)$ as $\delta \to \varepsilon$. □

7.2 Pole-free solutions

Now take $\varepsilon = 1$, so that we are studying the Painlevé II equation[1]

$$u_{xx} = xu + 2u^3.$$

Solutions to this equation typically have poles and it is often important to know where they are. The follwoing theorem states that for certain values of s there are no poles on the real line.

[1]with $\alpha = 0$

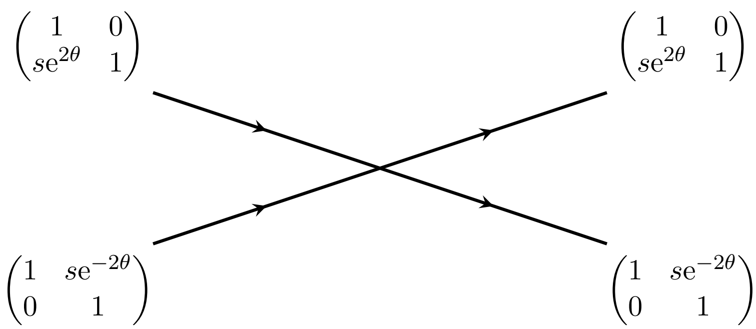

Figure 7.3: The jump contour and matrices for Φ.

Theorem 7.2.1. *Let u be the solution of the PII equation with Stokes parameters $s_2 = 0$ and $s_6 = -s_4 = s$. Then for $s \in \mathrm{i}[-1,1]$ the function u has no poles in $\mathbb{R}$.*

Proof. It is sufficient to prove that the RHP 7.3.4 has a solution for every $x \in \mathbb{R}$. We can turn this into an equivalent RHP by considering $\Phi = \Psi e^{\sigma_3 \theta(z,x)}$. Then we obtain the following RHP for Φ

- Φ is analytic in $\mathbb{C} \setminus \Gamma_\psi$
- $\Phi_+(z) = \Phi_-(z) J_\psi$
- $\Phi(z) = (I + \mathcal{O}(1/z))$, as $z \to \infty$
- Φ is bounded near the origin.

The jump structure is represented in Figure 7.2 We now use the Vanishing lemma for Φ (see **??**)
Vanishing Lemma: There exists a unique solution for Φ^h iff the homogenous RHP

- Φ^h is analytic in $\mathbb{C} \setminus \Gamma_\psi$
- $\Phi^h_+(z) = \Phi^h_-(z) J_\psi$
- $\Phi^h(z) = \mathcal{O}(1/z)$, as $z \to \infty$
- Φ^h is bounded near the origin.

Suppose now that Φ^h is a solution to the homogoneous RHP. THen it remains to show that $\Phi^h = 0$. We first define a new function Ξ as indicated in Figure 7.2 Then we obtain the following RHP for Ξ (prove!)

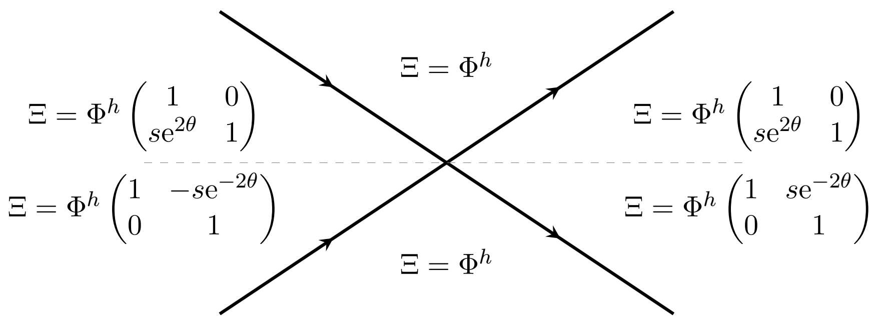

Figure 7.4: The definition of Ξ.

- Ξ is analytic in $\mathbb{C} \setminus \mathbb{R}$
- $\Xi_+(y) = \Xi_-(y) \begin{pmatrix} 1+s^2 & s\mathrm{e}^{-2\theta} \\ s\mathrm{e}^{-2\theta} & 1 \end{pmatrix}$ for $y \in \mathbb{R}$.
- $\Xi(z) = \mathcal{O}(1/z)$, as $z \to \infty$

Now define $G(z) = \Xi(z)\Xi(\bar{z})^*$. Then G is analytic in the upper half plane. Morevover $G(z) = \mathcal{O}(1/z^2)$ and hence

$$\int_{\mathbb{R}} G_+(y)\mathrm{d}y = 0.$$

By taking the adjoint we also get

$$\int_{\mathbb{R}} (G_+(y) + G_+(y)^*)\mathrm{d}y = 0. \tag{7.2.1}$$

Now for $y \in \mathbb{R}$ we obtain

$$G(y) = \Xi_+(y)\Xi_-(y)^* = \Xi_-(y) \begin{pmatrix} 1+s^2 & s\mathrm{e}^{-2\theta} \\ s\mathrm{e}^{-2\theta} & 1 \end{pmatrix} \Xi_-(y)^*$$

Hence since $x \in \mathbb{R}$, $\bar{s} = -s$ and $\overline{\theta}(y,x) = -\theta(y,x)$ we have

$$G_+(y) + G_+(y)^* = 2\Xi_-(y) \begin{pmatrix} 1-|s|^2 & 0 \\ 0 & 1 \end{pmatrix} \Xi_-(y)^*, \tag{7.2.2}$$

which is positive definite.

First assume that $s \neq \pm i$. Then (7.2.1) and (7.2.2) together imply that $\Xi_-(y) = 0$ for every y. By also using the jump structure we see that also

$X_+(y) = 0$. Hence $\Xi = 0$ by analyticity. Therefore $\Phi^h = 0$ and we have proved the vanishing lemma and therewith the statement for $s \neq \pm i$.

Now suppose that $s = \pm i$. Then (7.2.2) turns into

$$G_+(y) + G_+(y)^* = 2\Xi_-(y)\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\Xi_-(y)^*, \tag{7.2.3}$$

By combining this with (7.2.1) we thus obtain the the elements in the second column $\Xi_-2j = 0$. Moreover by the jump condition

$$\Xi_+ = \Xi_-\begin{pmatrix} 0 & \mathrm{e}^{-2\theta} \\ s\mathrm{e}^{-2\theta} & 1 \end{pmatrix} \tag{7.2.4}$$

we have that hte first column of X_+ is zero. It remains to prove that the first column of Ξ_- and the second column of Ξ_+ are zero. For this we use Carlson's Theorem [].

Define the function

$$h(z) = \begin{cases} \Xi_{j2}(z) & \operatorname{Im} z > 0 \\ \Xi_{j1}(z)\mathrm{e}^{-2\theta} & \\ s\mathrm{e}^{-2\theta} & -1 < \operatorname{Im} z < 0 \end{cases}$$

Then by (7.2.4) we have that h extends to an analytic function on $\{\operatorname{Im} z > -1\}$ and a continuous function on $\{\operatorname{Im} z \geq -1\}$. Moreover, it can checked by the form of θ that h is uniformly bounded and that

$$h(z) = \mathcal{O}(\mathrm{e}^{-c(\operatorname{Re} z)^2})$$

as $z \to \infty$ with $\operatorname{Im} z = -1$, for some positive constant c. Carlson's theorem tells us that there is no other such funcion with these properties than the trivial function. Hence $h = 0$ and this proves that also the first column of X_- and the second column of Ξ_+ are zero. Hence $\Xi = 0$ and $\Phi^h = 0$ and this proves the anishing lemma and therewith the statement for $s = \pm i$.

□

7.3 Asymptotic analysis of special Painlevé II solutions

We recall the following RHP from last lecture.

RH problem 7.3.1. *Let $s \in \mathbb{C}$. And let Γ_ψ and J_ψ as in Figure 7.5. Search for 2×2 analytic function Ψ such that*

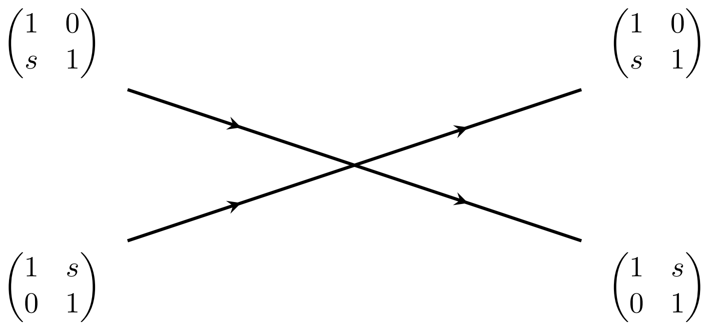

Figure 7.5: The jump contour and matrices for Ψ

- Ψ *is analytic in* $\mathbb{C} \setminus \Gamma_\psi$
- $\Psi_+(z) = \Psi_-(z) J_\psi$
- $\Psi(z) = (I + \mathcal{O}(1/z))\mathrm{e}^{-\sigma_3 \theta(z,x)}$, *as* $z \to \infty$
- Ψ *is bounded near the origin.*

Here $\theta(z,x) = \mathrm{i}(\frac{4}{3}z^3 + xz)$.

We have shown that there exists a countable set X_s (for which the intersection with any compact set in $\mathbb{C}$ is finite) such the solution to this RHP exists for $x \in \mathbb{C} \setminus X_s$ and depends analytically on $x \in \mathbb{C} \setminus X_s$. Moreover,

$$\Psi(z) = \left(I + \Psi_1/z + \mathcal{O}(1/z^2)\right) \mathrm{e}^{-\sigma_3 \theta(z,x)}$$

as $z \to \infty$, we have that $u = 2(\Psi_1)_{12}$ solves the equation

$$u_{xx} = xu + 2u^3.$$

In this lecture we will analyze the asymptotic behavior of $u(x)$ as $x \to +\infty$. As we will see, the solutions u with this choice of Stokes parameters tends to zero rapidly as $x \to +\infty$. Therefore, the term u^3 is negligible in the Painlevé II equation. When we neglect this term, the Painlevé II equation turns into the Airy equation. The latter equation has only one solution that converges to zero as $x \to \infty$ and this is $\mathrm{Ai}(x)$. As we will see, we indeed have the following asymptotic behavior: $u(x) = \mathrm{i}s\,\mathrm{Ai}(x)(1 + o(1))$ as $x \to \infty$.

Overview of the analysis

We will find the asymptotic behavior for $u(x)$ as $x \to \infty$, by first finding the asymptotic behavior of the solution $\Psi(z) = \Psi(z; x)$ as $x \to \infty$ and then extracting the information for $u(x)$. The procedure goes as follows. We will define a sequence fo transforms

$$Y \mapsto T \mapsto S \mapsto R.$$

Each transformation is easy and, in particular, invertible. After each transformation, we obtain a new function that solves a new RH problem. In the end, the goal of the sequence is to end up with a RH problem for R of the form

$$\begin{cases} R_+ = R_- J_R \\ R(z) = I + \mathcal{O}(1/z), \quad z \to \infty, \end{cases}$$

with $J_R \to I$ as $x \to +\infty$. Then we can use the theory of Lectures 2 and 3 to deduce that $R \to I$ as $x \to \infty$. Moreover, we obtain a Neuman series for the solution $R(z) = I + \sum_{j=1}^{\infty} R^{(j)}(z)$. We then trace back the transformations and obtain the asymptotic behavior of Ψ and hence u.

First transformation $\Psi \mapsto T$

We define the function

$$T = \Psi \mathrm{e}^{\sigma_3 \theta}$$

Then T is the unique solution to the following RHP.

RH problem 7.3.2. *Let $s \in \mathbb{C}$. And let Γ_T and J_T as in Figure 7.6. Search for 2×2 analytic function S such that*

- *T is analytic in $\mathbb{C} \setminus \Gamma_\psi$*
- *$T_+(z) = T_-(z) J_T(z)$*
- *$T(z) = (I + \mathcal{O}(1/z))$, as $z \to \infty$*
- *T is bounded near the origin.*

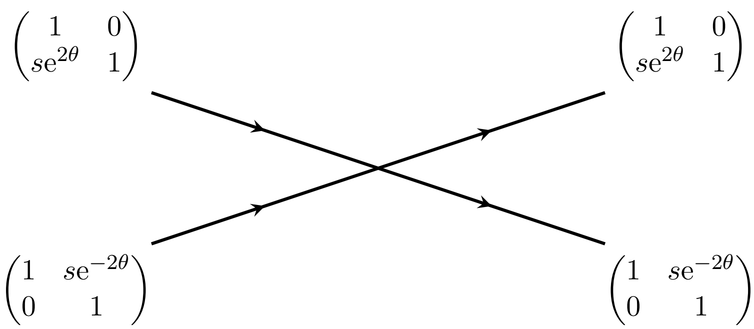

Figure 7.6: The jump contour and matrices for T

The second transformation $T \mapsto S$

Since we are interested in the behavior at $x \to +\infty$, we will assume $x > 0$ and define

$$S(z) = T(\sqrt{x}z).$$

Then S is the unique solution to the following RHP, where mention that we define ϕ via the relation $x^{3/2}\phi(z) = 2\theta(x^{1/2}z, x)$, hence

$$\phi(z) = 2\mathrm{i}(\frac{4}{3}z^3 + z). \tag{7.3.1}$$

RH problem 7.3.3. *Let $s \in \mathbb{C}$. And let Γ_S and J_S as in Figure 7.7. Search for 2×2 analytic function S such that*

- *S is analytic in $\mathbb{C} \setminus \Gamma_\psi$*
- *$S_+(z) = S_-(z)J_S(z)$*
- *$S(z) = (I + \mathcal{O}(1/z))$, as $z \to \infty$*
- *S is bounded near the origin.*

The transformation $S \mapsto R$

In the next transformation we will deform the contours. We denote

$$\gamma_\pm = \{t \pm \mathrm{i}\sqrt{t^2 + 1/4}\}.$$

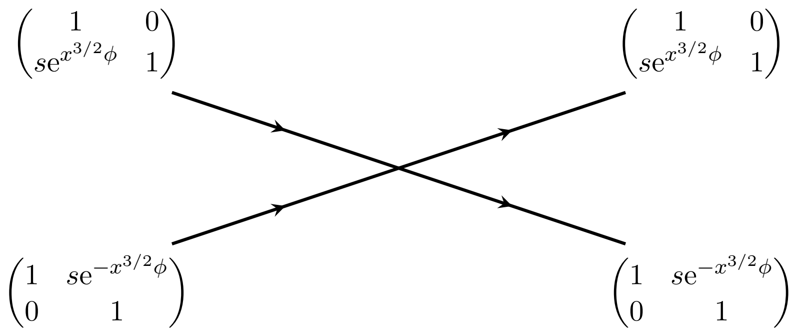

Figure 7.7: The jump contour and matrices for S

The relevance of the contours $\gamma_\pm$ is in the following. A straightforward calculation shows that

$$\frac{\mathrm{d}}{\mathrm{d}t} \operatorname{Re} \phi(t + \mathrm{i}\sqrt{t^2 + 1/4}) = -\frac{8(t + 4t^3)}{\sqrt{1 + 4t^2}}.$$

Hence the maximum of $\operatorname{Re}\phi$ on γ_+ is at $t = 0$ which is at $z = \mathrm{i}/2$. Similarly,

$$-\frac{\mathrm{d}}{\mathrm{d}t} \operatorname{Re} \phi(t - \mathrm{i}\sqrt{t^2 + 1/4}) = -\frac{8(t + 4t^3)}{\sqrt{1 + 4t^2}}.$$

Hence the maximum of $-\operatorname{Re}\phi$ on γ_- is at $t = 0$ which is at $z = -\mathrm{i}/2$. Moreover,

$$\phi(\mathrm{i}/2) = -\phi(-\mathrm{i}/2) = -2/3.$$

Hence we have in particular that $\operatorname{Re}\phi < 0$ on γ_+ and $-\operatorname{Re}\phi < 0$ on γ_-.

Now we define R as follows. The rays Γ_S and $\gamma_\pm$ partition the plane in a number of regions. The region enclosed by γ_+ and the two rays of Γ_s in the upper half plane we denote by Ω_+. The region enclosed by γ_- and the two rays of Γ_s in the lower half plane we denote by Ω_+. See also Figure 7.8 We then define

$$R(z) = \begin{cases} S(z) \begin{pmatrix} 1 & 0 \\ -\mathrm{e}^{x^{3/2}\phi(z)} & 0 \end{pmatrix}, & z \in \Omega_+ \\ S(z) \begin{pmatrix} 1 & \mathrm{e}^{-x^{3/2}\phi(z)} \\ 0 & 0 \end{pmatrix}, & z \in \Omega_- \\ S(z), & \text{otherwise.} \end{cases}$$

Then R solves the following RHP (exercise!)

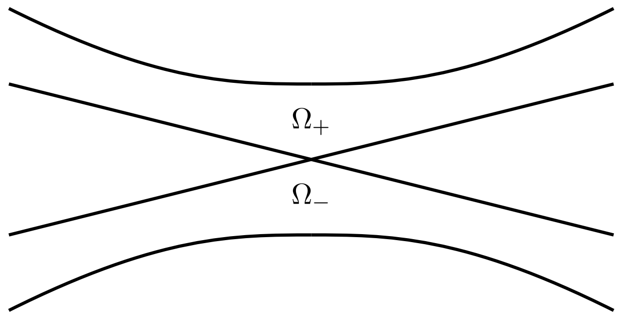

Figure 7.8: Partitioning of the plane in the definition of R

RH problem 7.3.4. *Let $s \in \mathbb{C}$. And let Γ_R and J_R as in Figure 7.9. Search for 2×2 analytic function R such that*

- *R is analytic in $\mathbb{C} \setminus \gamma_\pm$*
- $R_+(z) = R_-(z) \begin{pmatrix} 1 & 0 \\ \mathrm{e}^{x^{3/2}\phi(z)} & 0 \end{pmatrix}$ *for $z \in \gamma_+$.*
- $R_+(z) = R_-(z) \begin{pmatrix} 1 & \mathrm{e}^{-x^{3/2}\phi(z)} \\ 0 & 0 \end{pmatrix}$ *for $z \in \gamma_+$.*
- $R(z) = (I + \mathcal{O}(1/z))$*, as $z \to \infty$*

Note that since ϕ is analytic, also R is analytic away from the contours. Also, because of the behavior of $\phi(z)$ as $z \to \infty$ one can easily check (exercise) that $R(z) = I + \mathcal{O}(1/z)$ as $z \to \infty$. Strictly speaking, R has still jumps on the rays of Γ_S. However, since $R_+ = R_-$ on Γ_s we can extend R analytically on Γ_S.

Note that this transformation did nothing else then just deforming the contour. In particular, we see that for RHP's we can deform the contours in the same way we can deform contours for contour integrals, as long as the jump matrices involve analytic functions.

Since we have that $\pm \operatorname{Re} \phi < 0$ on $\gamma_\pm$, we see that the jumps J_R for R converge exponentially fast to 0 as $x \to +\infty$ (in fact, the dominant part fot he jump is near $\pm i/2$. In particular, one can show easily that

$$\|J_R - I\|_{2,\infty} = \mathcal{O}(\exp(-Cx^{3/2}), \qquad x \to +\infty,$$

for some constant $C > 0$.

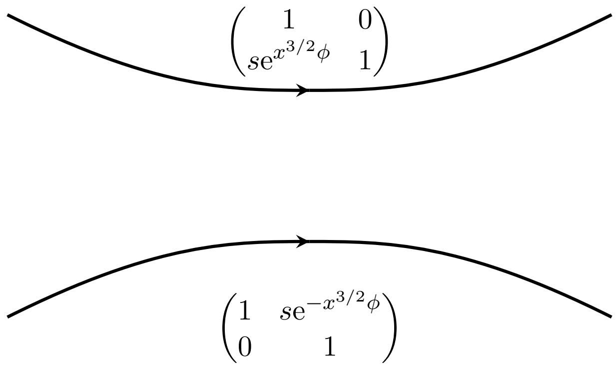

Figure 7.9: The jump contour and matrices for R

But then we know by the general theory for RHP in Lectures 2 and 3 that, for sufficiently large x, there is a solution to R that can be represented using a Neumann series giving

$$R(z) = I + \sum_{j=1}^{\infty} R^{(j)}(z).$$

In principle, each term in the series can be computed using the principles from Lectures 2 and 3. However, we will show here that there is a more direct approach that gives the terms.

Solving R heuristically

Since we know that there is Neumann series solving the RHP for R, we can justify the following heuristic procedure. Instead of a series let us try a solution of the form

$$R(z) = I + \Delta R(z).$$

We know that $\Delta R(z)$ should be small as $x \to +\infty$.

By inserting this into the RHP for R and reorganizig terms we obtain

$$\Delta R_+(z) = \Delta R_-(z) + (J_R - I) + \Delta R_-(J_R - I)$$

Since we know that ΔR and $(J_R - I)$ are small as $x \to +\infty$, we ignore this term in the equation and solve ΔR from

$$\Delta R_+(z) \approx \Delta R_-(z) + (J_R - I)$$

which, together with $\Delta R(z) = \mathcal{O}(1/z)$ as $z \to \infty$ gives,

$$\Delta R(z) \approx \begin{pmatrix} 0 & \frac{s}{2\pi \mathrm{i}} \int_{\gamma_-} \frac{\mathrm{e}^{-x^{3/2}\phi(y)}}{y-z} \mathrm{d}y \\ \frac{s}{2\pi \mathrm{i}} \int_{\gamma_+} \frac{\mathrm{e}^{x^{3/2}\phi(y)}}{y-z} \mathrm{d}y & 0 \end{pmatrix}$$

Of course, this is only an approximate solution for ΔR.

By iterating this procedure one can correct ΔR in the same way and so on. In this way, step by step, we will end up with the full series for $R = I + \sum_{j=1}^{\infty} R^{(j)}$. Indeed, by inserting the series in the RH problem for R and collecting terms that are of the same size we obtain the system of equations

$$\begin{cases} R_+^{(j+1)} = R_-^{(j+1)} + R_-^{(j)}(I - J_R) \\ R^{(j)}(z) = \mathcal{O}(1/z), \quad z \to \infty \end{cases} \tag{7.3.2}$$

which can be solved iteratively.

Asymptotics for u

Note that

$$\begin{aligned} u = 2(\Psi_1)_{12} &= \lim_{z\to\infty} 2z \left(\Psi(z)\mathrm{e}^{\sigma_3\theta}\right)_{12} \\ &= \lim_{z\to\infty} 2z \left(T(z)\right)_{12} \\ &= \lim_{z\to\infty} 2z \left(S(z/\sqrt{x})\right)_{12} \\ &= \lim_{z\to\infty} 2z \left(R(z/\sqrt{x})\right)_{12}. \end{aligned}$$

Hence

$$u \approx -\frac{s\sqrt{x}}{\pi \mathrm{i}} \int_{\gamma_-} \mathrm{e}^{-x^{3/2}\phi(y)} \mathrm{d}y,$$

as $x \to \infty$. Now note that by a simple change of variables and using the integral expression for the Airy function from Lecture 1 we obtain (exercise!)

$$-\frac{\sqrt{x}}{\pi \mathrm{i}} \int_{\gamma_-} \mathrm{e}^{-x^{3/2}\phi(y)} \mathrm{d}y = \mathrm{i}s \operatorname{Ai}(x),$$

and hence $u_1 \approx \mathrm{i} \operatorname{Ai}(x)$, for large x.

After rigorous justification (exercise!) of the heuristic argument one obtains

$$u(x) = \mathrm{i}s \operatorname{Ai}(x)(1 + o(1)), \qquad x \to \infty.$$

Clearly $s = -\mathrm{i}$ is special. In fact this solution to the Painlevé II equation has a name in the literature and is referred to as the *Hastings-McLeod solution*.

Finally, we remark that a more detailed treatment of R will give an full asymptotic series for $u(x)$ as $x \to \infty$.

Bibliography

[1] M.J. Ablowitz and A.S. Fokas, Complex variables: introduction and applications. Second edition. Cambridge Texts in Applied Mathematics. Cambridge University Press, Cambridge, 2003.

[2] K. F. Clancey and I. Gohberg, Factorization of matrix functions and singular integral operators. Operator Theory: Advances and Applications, 3. Birkhuser Verlag, Basel-Boston, Mass., 1981

[3] P.A. Deift,Orthogonal polynomials and random matrices: a Riemann-Hilbert approach. Courant Lecture Notes in Mathematics, 3. New York University, Courant Institute of Mathematical Sciences, New York; American Mathematical Society, Providence, RI, 1999. viii+273 pp.

[4] A.S. Fokas, A. Its, A.A. Kapaev and V.Y. Novokshenov, Painlev Transcendents. The Riemann-Hilbert approach. Mathematical Surveys and Monographs, 128. American Mathematical Society, Providence, RI, 2006. xii+553 pp.

[5] I. Gohberg and N. Krupnik, One-dimensional linear singular integral equations. I. Introduction. Operator Theory: Advances and Applications, 53. Birkhuser Verlag, Basel, 1992.