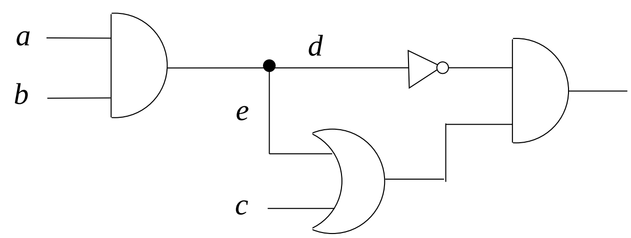
Solutions to FTC'2006 Final Exam

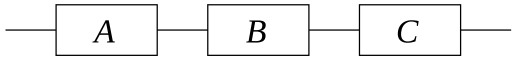
1) **(6 points)** (a) Check whether all single stuck-at faults in the circuit shown below are detectable. Recall that, in order to check this, it is sufficient to find tests for all primary inputs and all fanout branches (marked by d and e). If faults of these lines are detectable, all faults are detectable. Fill the table below. Show only one test per fault. Do not use don't care symbol, always specify a, b and c to 0 or 1. If no test exists for a given fault, write "–".



The table below shows all possible correct answers. You were required to show only one test per fault.

stuck-at-fault on line	test (a,b,c)
a: stuck-at-0	(1,1,1)
a: stuck-at-1	(0,1,1)
b: stuck-at-0	(1,1,1)
b: stuck-at-1	(1,0,1)
c: stuck-at-0	(0,0,1), (0,1,1), (1,0,1)
c: stuck-at-1	(0,0,0), (0,1,0), (1,0,0)
d: stuck-at-0	(1,1,0), (1,1,1)
d: stuck-at-1	(0,0,1), (0,1,1), (1,0,1)
e: stuck-at-0	-
e: stuck-at-1	(0,0,0), (0,1,0), (1,0,0)

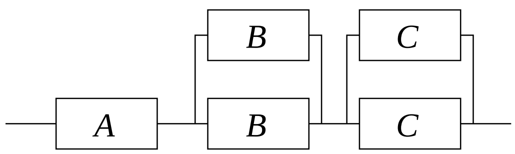
2) **(7 points)** You company produces a system which consists of three non-redundant components: A, B and C:



The failure rates of components are $\lambda_a = 0.01$, $\lambda_b = 0.11$ and $\lambda_c = 0.12$ per year, respectively, and the costs are 100, 50 and 80 SEK, respectively.

The warranty for this system is 1 year. Your boss decides that too many items are returned for repair during the warranty period. He gives you a task of improving the reliability of the system so that no more than 4% of items are returned for repair during the first year of operation. This should be done by adding no more than two redundant components to the system (i.e. no more than two blocks to the Reliability Block Diagram above). Find a Reliability Block Diagram for the system which meets your boss's requirement and is the cheapest.

The Reliability Block Diagram for the system which has at least 96% reliability during the first year of operation and is the cheapest is shown below:



The reliability is given by

$$R_{system} = R_A(1-(1-R_B)^2)(1-(1-R_C)^2) = 0.9668.$$

3) **(12 points)** You need to buy chains for your car for a 36-hour mountain trip in the winter. Local police requires that you have chains at all 4 wheels to drive safely. The shop offers you two choices of sets of chains (each set consists of 4 identical chains):

1. with the failure rate of a chain $\lambda_1 = 0.1$ per 24 hours, cost 200 SEK
2. with the failure rate of a chain $\lambda_2 = 0.4$ per 24 hours, cost 100 SEK

(a) Compute the reliability of your car during the trip, R_1, for the case when you buy only one set of more expensive chains. Assume that the failures of chains are independent events.

$$R_1 = (e^{-\lambda_1 t})^4 = e^{-0.4 \cdot 36/24} = e^{-0.6} = 0.5488.$$

(b) You cannot compute directly the reliability for the case when you buy two sets of cheaper chains, R_2, because one set is spare and, therefore, Markov chains need to be used for the reliability evaluation. However, you can find an lower bound R_{lb} for the value of the reliability by assuming that a spare chain can fail with the same failure rate λ_2 and the failures of all 8 chains are independent events. Although such an assumption is over-pessimistic, if the computed value $R_{lb} > R_1$, then $R_2 > R_1$, so you can draw a conclusion that it is better to buy two sets of cheaper chains. However, if $R_{lb} < R_1$, then no conclusion can be drawn.

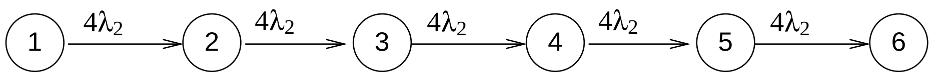
Write an expression for R_{lb} during the trip, compute it, and compare it to R_1. Can you draw a conclusion about which chains are better to buy?

Let R be the reliability of one cheap chain. $R = e^{-\lambda_2 t} = e^{-0.4 \cdot 36/24} = e^{-0.6} = 0.5488$. Then, R_{lb} is given by:

$$R_{lb} = R^8 + 8R^7(1-R) + 28R^6(1-R)^2 + 56R^5(1-R)^3 + 70R^4(1-R)^4 = 0.7373.$$

Since $R_{lb} > R_1$, it is better to buy two sets of cheaper chains.

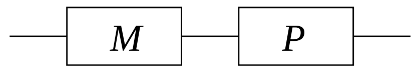
(c) Draw a Markov chain for reliability evaluation for the case when you buy two sets of cheaper chains. Explain the meaning of all states. Do not write the transition matrix.



The states are numbered according to the following table:

State	Description
1	all chains are fine
2	1st chain failed and replaced by a spare, 3 spares left
3	2nd chain failed and replaced by a spare, 2 spares left
4	3rd chain failed and replaced by a spare, 1 spares left
5	4th chain failed and replaced by a spare, no spares left
6	5th chain failed, failed state

4) **(7 points)** A non-redundant system consists of two components - a memory M and a processor P:

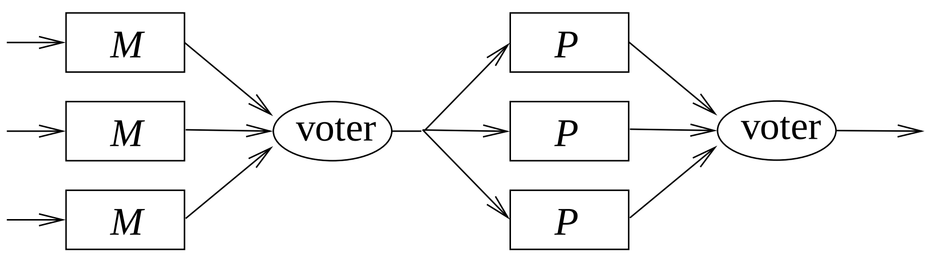


The reliabilities of the memory and the processor during the first year of operation are $R_M = 0.947$ and $R_P = 0.95$, respectively.

You are asked to use the passive N-modular redundancy to increase 1-year reliability of the system to at least 0.98 while keeping the hardware overhead to minimum. You can allocate redundancy either at component level, or at system level. Each component M or P contributes 1 unit of overhead.

Assuming that the voter is perfect (i.e. it cannot fail) and that its size is negligible compared to the size of components (i.e. it contributes 0 units of overhead), what is the smallest value of N which meets the above reliability requirement? Draw the structure of the resulting system. Write an expression which you used for computing its reliability.

The reliability of at least 98% during the first year of operation is achieved for $N = 3$ if component-level redundancy is used:



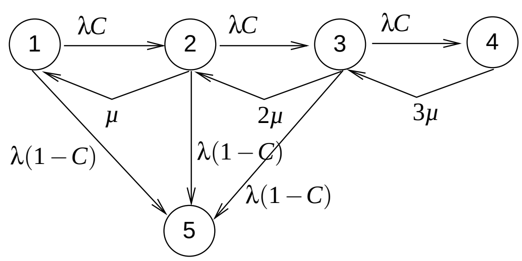
The reliability of the system above is given by

$$R_{system} = (R_M^3 + 3R_M^2(1 - R_M))(R_P^3 + 3R_P^2(1 - R_P)) = 0.9847.$$

5) **(10 points)** Draw a Markov chain for the cold standby redundancy system with three modules for the cases listed below. For all cases, assume that: (1) the failure rate of each module is λ, (2) the fault coverage is C, i.e. the probability that a fault is detected by the fault detection unit is C, (3) the switch is perfect, i.e. if a fault is detected, the system will reconfigure as expected, (4) each module has its own repair crew, (5) the repair rate of each module is μ.

(a) Do availability evaluation. Write the resulting transition matrix (do not write the system of equations).

This task has several possible correct solutions, depending on the assumptions one makes. Assuming that undetected faults are not repaired (because the repair crew does not know that they occur), the Markov chain looks as follows:



The states are numbered according to the following table:

State	Description
1	all components work
2	1st component failed, detected and replaced, 1 spare left
3	2nd component failed, detected and is replaced, no spares left
4	3rd component failed and detected, failed state
5	the fault in the components is not detected, failed state

The availability of the system is given by the sum of probabilities of being in the states 1, 2 and 3.

(b) Do safety evaluation. Assume that the system cannot be repaired if it failed non-safe. Otherwise, it can be repaired. Write the resulting transition matrix (do not write the system of equations).

The Markov chain looks identical in the task 5a. The state 4 is failed-safe and the state 5 is failed non-safe states. The safety of the system is given by the sum of probabilities of being in the states 1, 2, 3 and 4.

6) **(5 points)** (a) Construct the parity check matrix H and the generator matrix G for a linear code for 6-bit data, which can correct 1 error and detect one more error.

Every 3 columns of H should be linearly independent to encode code distance 4. The least codeword length which satisfies this is $n = 11$. One possible H is:

$$H = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Other solutions are possible. For example, the first 6 columns of H can be any permutation of 5-bit binary vectors with three 1's in each.

The corresponding to H generator matrix is:

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}.$$

7) **(5 points)** Construct the parity check matrix H and the generator matrix G for a $(7,4)$ separable Hamming code.

$$H = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}, \quad G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

b) Use the generator matrix from (a) to encode the data $[1110]$.

$$c = d \cdot G = [1110] \cdot \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} = [1110000].$$

c) Suppose that the first bit of the codeword from (b) is flipped to the opposite value. Can you correct this error? If your answer is "yes", explain how would you do error-correction. If your answer is "no", explain why not.

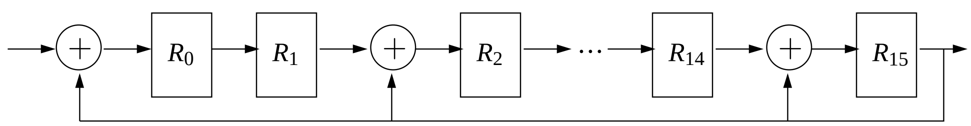
Yes, the error can be corrected by comparing the non-zero syndrome to the columns of H. The syndrome will match the 1st column, so the error is in the 1st bit. This bit has to be complemented to correct the error.

8) **(5 points)** (a) You are to select a cyclic code for error detection in 256-bit data. You know that burst errors of length 16 or less bits are most common for your application. Which of the following generator polynomials will you select in order to guarantee 16-bit burst error detection with a minimum information overhead?

(1) $g(x) = 1 + x + x^5 + x^9 + x^{12} + x^{17}$
(2) $g(x) = 1 + x^2 + x^{15} + x^{16}$
(3) $g(x) = 1 + x + x^2 + x^4 + x^7 + x^8 + x^{10} + x^{11} + x^{12} + x^{16} + x^{22} + x^{23} + x^{26} + x^{32}$
(4) $g(x) = 1 + x^3 + x^7 + x^9 + x^{14} + x^{26} + x^{78} + x^{99} + x^{127} + x^{240}$

The generator polynomial (2) guarantees 16-bit burst error detection with a minimum information overhead.

(b) Draw an LFSR decoding circuit for the chosen generator polynomial.



9) **(3 points)** Your communication system is using the alternating logic time redundancy in combination with an additional parity check bit to correct permanent single-bit errors. Suppose that you change the system as follows: Instead of alternating data with its complement, you submit the same data twice. As before, an additional parity check bit is attached to the data.

(a) Will such a system be able to correct or detect permanent single-bit errors?

Permanent single-bit errors can be detected but not corrected, because the faulty bit cannot be located.

(a) Will such a system be able to correct or detect transient single-bit errors?

Transient single-bit errors can be both, detected and corrected, assuming that the duration of the error is small and only one of the submitted words will be affected.