On Dependable Criteria for Dynamic Reordering Algorithms

E. V. Dubrova
Electronic System Design Lab
Royal Institute of Technology
KTH-Electrum, Electrum 229
S-164 40 Kista, Sweden

D.M. Miller
VLSI Design and Test Group
University of Victoria
P.O.Box 3055, Victoria, B.C.
Canada, V8W 3P6

Extended Abstract

In this paper, we analyze two criteria which can be used for successive improvement of an existing Reduced Ordered Binary Decision Diagrams (ROBDD) variable order in dynamic reordering algorithms. Dependability of such criteria is usually the key to efficiency of a reordering algorithm.

ROBDDs are a graphical data structure for the efficient representation of Boolean functions [1]. Functions are represented by directed, acyclic graphs, which are built for some chosen ordering of the function variables. Normally, the size of the ROBDD varies for different variable orderings and, for some functions, it is highly sensitive to the ordering. Searching for the variable order which minimizes the size of the graph requires, in the worst case, time exponential in the number of variables [4]. Finding such a variable ordering efficiently remains a major open problem related to ROBDD. Exact algorithms for its solution are feasible only for functions of a small number of variables.

A number of heuristic procedures have been developed, using various strategies to produce a "good" ordering within a reasonable time. Methods for automatic *dynamic reordering* of variables have been shown to be appealing for handing large problems and are currently the subject of much investigation.

Reordering algorithms are based on successive improvement of existing orders according to some local search strategy. Usually, a key to developing an efficient search strategy lies in formulating a dependable criterion for grouping the variables. Three candidates for such a criterion suggested so far are

1. Keeping symmetric variables adjacent
2. Minimizing width of the ROBDD
3. Keeping bound-set variables adjacent

The study of criterion (1) was started in [5]. It was empirically observed, that symmetric variables tend to be adjacent in the best ordering for ROBDDs without complemented edges. Since then, keeping symmetric variables together has been considered a good criterion and a number of heuristic procedures for computing a variable orderings based on this criterion have been developed, including whose in [7] and [8]. However, a counterexample has been found [8], showing a function for which no order with the symmetric variables adjacent is optimal.

Criterion (2) was considered in [6]. The author observed that minimizing width of the ROBDD often leads to a reduction in the number of nodes. This search strategy was used in the heuristic algorithm for finding a best variable ordering reported in [6]. However, in this paper we give a counterexample, showing a function for which no minimal-weight ordering is the best. Let $BO(f)$ be a set of all best orderings of a functions f, and let $MW(f)$ be a set of all its minimal-width orderings. The following theorem is proved in the paper:

Theorem 1 *There exists a function for which none of the minimal-width orderings is the best:*

$$\exists f : \quad BO(f) \cap MW(f) = \emptyset$$

Theorem 1 shows that minimizing the total width of the ROBDD is not always a dependable criterion. Such cases, however, tend to be extremely rare in practice. Their existence doesn't diminish the practical value of the algorithm reported in [6], but should be noted as a possibility.

Keeping variables from a bound set of a function f adjacent is another intuitive criterion for grouping variables. The study of (3) was started in [3] and continued in [2]. The existence of a homomorphism was investigated, between two structures $(S_1, \circ)$ and $(S_2, \bullet)$, where S_1 is the set of all non-degenerate functions of n variables or less, S_2 is the set of all sets, which are best orderings of the functions from S_1, $\circ$ is the functional substitution operation, and $\bullet$ is the ordering substitution operation. These two structures are homomorphic if, and only if, there exists a mapping $\alpha : S_1 \to S_2$ assigning to any function $f \in S_1$ the set of best orderings for $f(X) = g(h(Y), Z)$ from S_2, such that $\alpha(g \circ h) = \alpha(g) \bullet \alpha(h)$ for all $g, h \in S_1$ for which the operation $\circ$ is defined. A theorem was proved showing that no such mapping exists for the case $n \geq 5$. This implies that sometimes none of the orderings generated from the best orderings of g and h are best ones for $f(X) = g(h(Y), Z)$.

In this paper we investigate further the relation between the best orderings and bound-set-preserving orderings. Let $BO(f)$ be a set of all best orderings of a functions f, and let $BST(f)$ be a set of all its bound-set-preserving orderings. The following two theorems are proved in the paper:

Theorem 2 *There exists a function with a best ordering which is not bound-sets-preserving:*

$$\exists f : \quad BO(f) - BST(f) \neq \emptyset$$

Theorem 3 *There exists a function with a bound-sets-preserving ordering which is not best:*

$$\exists f : \quad BST(f) - BO(f) \neq \emptyset$$

Presently we are studying how the functions with $BO(f) - BST(f) \neq \emptyset$ can be identified and enumerated. It would also be interesting to see how the percentage of these cases changes as n increase. But we still don't know whether there is a function for which the intersection of these two sets is empty. We believe that no such function exists, and work on proving the conjecture that no function has $BO(f) \cap BST(f) = \emptyset$.

References

[1] R.E. Bryant, Graph-Based Algorithm for Boolean Function Manipulation, *IEEE Transactions on Computers* **C-35** No. 8 (1986), 677-691.

[2] E.V. Dubrova, D.M. Miller, J.C. Muzio, On the Relation Between Disjunctive Decomposition and ROBDD Variable Ordering, in *Proc. of 1997 IEEE Pacific Rim Conference on Communications, Computer and Signal Processing* Victoria, B.C., Canada, August 20-22, 1997.

[3] E.V. Dubrova, D.M. Miller, J.C. Muzio, On the Best ROBDD Variable Ordering for Functions with Disjunctive Decompositions, *IEE Journal of Electronic Letters*, **33** (1997), 1198-1200.

[4] S.J. Friedman, K.J. Supowit, Finding the Optimal Variable Ordering for Binary Decision Diagrams, in *Proc. 24th ACM/IEEE Design Automation Conf.* (1987), 348-355.

[5] S.-W. Jeong, B. Plessier, G.D. Hatchel, F. Somenzi, Variable Ordering and Selection for SSM traversal, in *Proceedings of the IEEE Int. Conf. on Computer Aided Design* (1991), 476-479.

[6] S. Minato, Minimum-width method of variable ordering for binary decision diagrams, *IEICE Trans. Fundamentals* **E-75-A** No. 3 (1992), 392-399.

[7] D. Möller, P. Molitor, R. Drechsler, Symmetry based variable ordering for ROBDDs, in *IFIP Workshop on Logic and Architecture Synthesis* (1994).

[8] S. Panda, F. Somenzi, Who are the Variables in Your Neighborhood, in *Proceedings of IEE/ACM Workshop on Logic Synthesis* (1995), 1-10.