Testability of Generalized Multiple-Valued Reed-Muller Circuits

E.V. Dubrova J.C. Muzio

VLSI Design and Test Group
Department of Computer Science
University of Victoria, P.O.Box 3055
Victoria, B.C., Canada, V8W 3P6

e-mail: elena@shannon.uvic.ca
jmuzio@csr.uvic.ca

Abstract

The testability of generalized Reed-Muller circuits realizing m-valued functions in modulo m sum-of-product form, with m being a prime greater than two, is investigated. Two aspects of the problem are considered - the number of tests required for fault detection, and the generation of tests. We prove that just four tests are sufficient to detect all single stuck-at faults on internal lines in the circuit. Furthermore, this set of tests is independent of the function being realized and therefore universal. We give two alternative techniques for testing primary inputs - one by generating a test set of maximum length 2n, where n is the number of primary inputs, and the other by adding to the circuit an extra multiplication mod m gate with an observable output to ensure that the four tests for internal lines also detect all single stuck-at faults on primary inputs.

1 Introduction.

A logic function can be implemented with many different circuit designs. Various realizations of the same function may require different numbers of input vectors as tests. For example, in two-valued logic a two-level sum-of-products realization of the n-variable parity function (which has the value 1 if and only if an odd number of the variables have the value 1) requires all 2^n possible input vectors as tests to detect all single stuck-at faults. However, this function can also be implemented as a multi-level tree of two-input XOR gates, and this realization requires only two tests to detect all single stuck-at faults.

In two-valued systems, testing the multi-level tree of XOR gates is easy because in a fanout free linear circuit any single fault propagates to the output independently of the input vector applied. This property allows the minimization of the number of tests required for fault detection and simplifies the generation of tests for the whole family of circuits, called Reed-Muller (RM) circuits. An RM circuit consists of a multi-level tree of two-input XOR gates, fed by AND gates (fig. 1). Any RM circuit can be tested for all single stuck-at faults with a maximum of $3n+4$ input vectors, where n is the number of primary inputs [5].

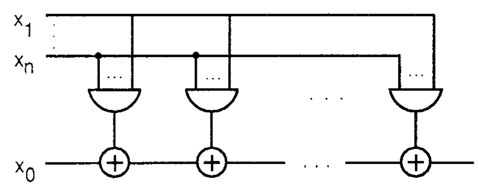

Figure 1: Reed-Muller circuit.

The useful property of a multi-level tree of XOR gates to propagate any single fault to the output remains valid when XOR gates are replaced with sum mod m gates and more than two levels of signals are used in the circuit. Circuits which use more than two levels of signals (voltage or current) are known as multiple-valued (or m-valued) circuits. So, an m-valued circuit based on a multi-level tree of sum mod m gates might possess a useful property of easy testability. One such architecture is a circuit realizing an m-valued function in modulo m sum-of-product form, where m is a prime. This canonical form of m-valued functions, referred to as generalized Reed-Muller (GRM) expansion [9], was proposed by Cohn in 1960 [1]. Modulo m addition and multiplication form a Galois field of order m, or $GF(m)$. An m-valued function $f(x_1, \ldots, x_n)$ has a unique GRM expansion of type [4]:

$$f(x_1, \ldots, x_n) = \sum_{i=0}^{m^n-1} c_i \; x_1^{i_1} \; x_2^{i_2} \ldots \; x_n^{i_n},$$

where $c_i \in \{0, 1, \ldots, m-1\}$ are constants, and $(i_1 i_2 \ldots i_n)$ is the m-ary expansion of i with i_1 being the least significant digit. The term $x_j^{i_j}$ denotes

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the i_jth power of the variable x_j. All operations are in $GF(m)$. Throughout the paper, we assume that m is a prime greater than two.

A GRM expansion can be implemented by the circuit shown on Figure 2. It consists of a linear cascade of two-input sum mod m gates fed by multiplication mod m gates, one corresponding to each product-term of the expansion with non-zero constant c_i, $i \in \{1, \ldots, m^n - 1\}$. The input x_0 has the value of the constant c_0 during normal operation and a value different from c_0 during testing.

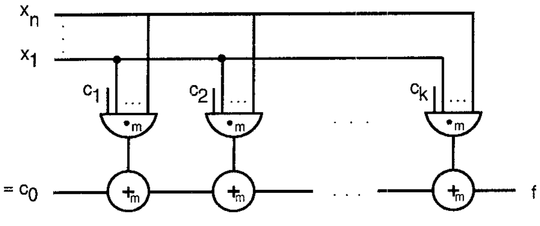

Figure 2: Generalized Reed-Muller circuit.

For example, the 3-variable 3-valued function

$$f(x_1, x_2, x_3) = 1x_1^2x_2 \oplus 2x_1^2x_2^2 \oplus 1x_1x_2x_3 \oplus 2x_1x_2x_3^2$$

can be implemented by the GRM circuit shown in Figure 3. We use the notation " $\oplus$ " for addition mod m and " $\cdot$ " for multiplication mod m. " $\cdot$ " is omitted where obvious.

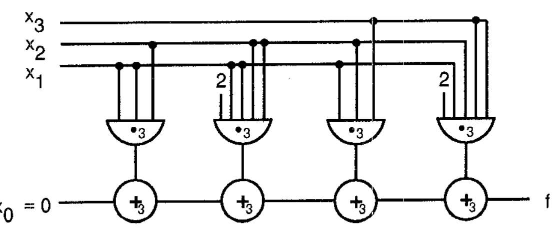

Figure 3: GRM circuit implementing the function from the example.

In this paper, we investigate the testability of GRM circuits. We concentrate on two aspects of the problem:

- The number of tests required for fault detection.
- The generation of tests.

As a fault model, we use a single stuck-at fault model, i.e. we assume that a single line in the circuit can be stuck at some constant logic value from 0 to (m-1). First, we consider detection of internal faults, which occur on the inputs of the individual gates. We prove that only four tests are sufficient to detect all single stuck-at faults on internal lines in a GRM circuit. Furthermore, this set of tests is independent of the function being realized and therefore universal. Second, we give two alternative techniques for testing primary inputs - one by generating a test set of maximum length $2n$, and the other by adding to the circuit an extra multiplication mod m gates with an observable output to ensure that the four tests for internal lines also detect all single stuck-at faults on primary inputs.

The paper is organized as follows. Section 2 estimates the number of tests sufficient to detect all single stuck-at faults on the internal lines of a GRM circuit. In Section 3, the testability of the primary inputs is investigated. Subsection 3.1 describes a procedure for test generation for primary inputs and Subsection 3.2 evaluates its effectiveness. In Section 4 shows that by adding to the GRM circuit an extra multiplication mod m gates with an observable output the number of tests sufficient to detect all single stuck-at faults is reducible to four. In Section 5, some conclusions are drawn and topics for further research are proposed.

2 Testability of internal lines.

It is proved in [2] that in the two-valued RM realization of a Boolean function $f(x_1, \ldots, x_n)$ at most $n + 4$ tests are required to detect all internal single stuck-at faults. The proof is constructive by showing that, independently of the function being realized, a set $T = T_1 \cup T_2$ detects all internal single stuck-at faults. T_1 is defined by the table below

x_0	x_1	x_2	$\ldots$	x_n
0	0	0	$\ldots$	0
0	1	1	$\ldots$	1
1	0	0	$\ldots$	0
1	1	1	$\ldots$	1

It detects all single faults on the inputs of XOR gates and all stuck-at-0 faults on the inputs of AND gates. T_2 is defined by $T_2 := \{t_{21}, t_{22}, \ldots, t_{2n}\}$ with the test t_{2i} having $x_i = 0$ and $x_j = 1$ for all $j \neq i, i, j \in \{1, 2, \ldots, n\}$. It detects all stuck-at-1 faults on the inputs of AND gates. So, the $n + 4$ tests in the test set $T = T_1 \cup T_2$ detect all internal single stuck-at faults in a two-valued RM circuit.

We use a similar approach to prove that, in the GRM realization of an m-valued function, only four tests are required to detect all internal single stuck-at faults. It might appear surprising that the many-valued case requires less tests than the two-valued one. Before giving the result, we explain the intuition behind this phenomenon.

Consider an n-input multiplication mod m gate G with m being a prime. Let $a_i \in M$ be the value of the input variable x_i, for $i \in \{1, 2, \ldots, n\}$, where $M := \{0, 1, ..., m-1\}$. Since the cancellation law of multiplication holds for $GF(m)$ [6], for any $x, y, z \in M$ we have:

$$\text{If } \quad x \neq 0 \quad \text{and} \quad y \neq z \quad \text{then} \quad xy \neq xz$$

It follows from the above that if an input vector $(a_1, \ldots, a_n)$ such that $a_i \neq 0$ for all i, is applied to G, than a change in the value of any single input x_i causes a change in the value on the output. But this implies that $(a_1, \ldots, a_n)$ is a test for all x_i stuck-at-$\overline{a}_i$ faults, where $\overline{a}_i$ denotes any value but a_i, i.e. $\overline{a}_i \in M - \{a_i\}$.

By applying the same reasoning as above, one can see that to detect the remaining stuck-at-a_i faults on each input x_i, another input assignment $(a'_1, \ldots, a'_n)$ such that $a'_i \neq 0$ and $a_i \neq a'_i$ for all i has to be applied.

So any two input assignments $(a_1, \ldots, a_n)$ and $(a'_1, \ldots, a'_n)$ such that none of a_i, a'_i is zero and $a_i \neq a'_i$ for all $i \in \{1, 2, \ldots, n\}$, detect all single stuck-at faults on the inputs of a multiplication mod m gate for m being a prime greater than two. It is easy to see why $m = 2$ is an exception. In the two-valued case there is only one input assignment with all entries different from zero, namely $(11 \ldots 1)$.

Since the cancellation law of addition also holds for $GF(m)$, by applying the similar reasoning as above, we can see that any two input assignments $(a_1, \ldots, a_n)$ and $(a'_1, \ldots, a'_n)$ such that $a_i \neq a'_i$ for all $i \in \{1, 2, \ldots, n\}$, detect all single stuck-at faults on the inputs of an n-input sum mod m gate. Notice, that the requirement $x \neq 0$ is not postulated in the cancellation law of addition, so the entries of the assignments can have value zero as well. Thus, the above statement holds also for the case $m = 2$, i.e. for an XOR gate.

We can now give the main result of the section.

Theorem 1 *There exists a universal set of four tests which detects all single stuck-at faults on internal lines in any GRM circuit.*

Proof: The proof is constructive. Consider the set T consisting of four tests defined by the table below.

x_0	x_1	x_2	$\ldots$	x_n
0	0	0	$\ldots$	0
0	1	1	$\ldots$	1
1	0	0	$\ldots$	0
0	$m-1$	$m-1$	$\ldots$	$m-1$

Let us denote with p_i and r_i the inputs of the ith addition mod m gate in the cascade, as shown on Figure 4.

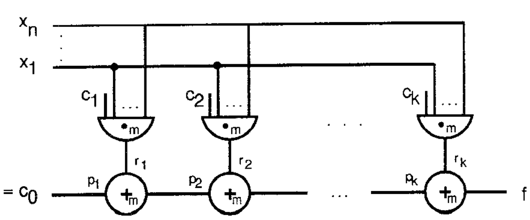

Figure 4: Generalized Reed-Muller circuit.

1. The first test of T results in applying $(0, 0)$ to each pair (p_i, r_i), detecting all stuck-at-$\overline{0}$ faults on p_i and r_i. By stuck-at-$\overline{k}$ faults we mean all stuck-at-l faults for $l \in M - \{k\}$.

2. The second test of T results in applying $(*, c_i)$ to each pair (p_i, r_i), where c_i is the constant (non-zero) which is fed into the ith multiplication mod m gate and $*$ denotes any value from M. It detects all r_i stuck-at-0 faults.

   This test also detects all stuck-at-$\overline{1}$ faults on the inputs of the multiplication mod m gates.

3. The third test of T results in applying $(1, 0)$ to each pair (p_i, r_i), detecting all p_i stuck-at-0 faults.

4. The fourth test of T applies the value $(m - 1)$ to the inputs of multiplication mod m gates, detecting all stuck-at-1 faults on them.

Hence the four tests completely test the internal lines for all single stuck-at faults.

□

The above theorem gives the number of tests sufficient to detect all internal single stuck-at faults in a GRM circuit. Since the proof is constructive, it shows how to generate the test set itself. This test set is independent of the function being realized, and therefore universal. In the next section we investigate the testability of the primary inputs of a GRM circuit.

3 Testability of primary inputs.

We show that the number of tests sufficient to detect all stuck-at faults on primary inputs in a GRM circuit, as well as in an arbitrary m-valued combinational logic circuit realizing a function $f(x_1, \ldots, x_n)$, is at most $2n$. For any variable x_i, provided $f(x_1, \ldots, x_n)$ is not degenerate in x_i, there exist values $a_1, \ldots, a_n$ and $a'_i \neq a_i$ such that

$$f(a_1, \ldots, a_{i-1}, a_i, a_{i+1}, \ldots, a_n) \neq f(a_1, \ldots, a_{i-1}, a'_i, a_{i+1}, \ldots, a_n) \quad (1)$$

Since the change in the value of x_i from a_i to a'_i causes a change in the value of f, the input vector $t_1 = (a_1, \ldots, a_{i-1}, a_i, a_{i+1}, \ldots, a_n)$ is a test for all x_i stuck-at faults which set the output of the circuit to a logic value different from $f(a_1, \ldots, a_n)$. On the other hand, the input vector $t_2 = (a_1, \ldots, a_{i-1}, a'_i, a_{i+1}, \ldots, a_n)$ is a test for all x_i stuck-at faults which set the output of the circuit to $f(a_1, \ldots, a_n)$. Thus $T = \{t_1, t_2\}$ is a test set for all single stuck-at faults on x_i. Considering all inputs, a set of $2n$ tests for all single stuck-at faults on primary inputs can be obtained.

While the fact that a test set of length $2n$ exists is straightforward, the problem of generating this test set is not trivial. The next section presents a procedure for finding $2n$ tests for detecting all single stuck-at faults on primary inputs of a GRM circuit.

3.1 A procedure for test generation.

It is shown in [2] that in the two-valued case, the number of tests required to detect all single stuck-at faults on primary inputs in an RM circuit is $2n_e$, where n_e is the number of primary inputs appearing in an even number of product-terms in the RM expansion of the n-variable function being realized. The following procedure is applied to find the test set. For a primary input x_i, all AND gates having x_i as input are considered. From these, a gate G_i with the minimal number of other inputs is selected. Further, two tests, t_{i1} and t_{i2} are defined in the following way:

t_{i1} := specifies $x_i = 0$, all other inputs of G_i to 1, and all other primary inputs to 0.

t_{i2} := specifies $x_i = 1$, all other inputs of G_i to 1, and all other primary inputs to 0.

The test t_{i1} detects x_i stuck-at-1, and the test t_{i2} detects x_i stuck-at-0. The procedure is repeated for all n inputs.

Unfortunately, this simple procedure cannot be used directly for the case $m > 2$ because of the following reason. A GRM expansion of an m-valued function $f(x_1, \ldots, x_n)$ can have $(m-1)$ different powers of each variable x_i involved in the product-terms. If $m = 2$, only one power of an x_i is employed, and thus, a *single* gate G_i with the minimal number of other inputs can always be selected. By assigning all but x_i inputs of G_i to value 1, and all other primary inputs to 0, a *single* path from x_i to the output is sensitized. So, the effect of a fault on x_i is always propagated to the output. In a GRM circuit, there may be more than one multiplication mod m gate depending on x_i and k other primary inputs. If these k primary inputs are assigned to 1 and the rest of the primary inputs to 0, then the effect of a fault on x_i is propagated along multiple paths, and thus may be canceled out by the sum mod m cascade. Therefore, in case of GRM circuits, all m^k possible combinations of values for k primary inputs, not assigned to 0, should be examined to find out which one makes the output sensitive to x_i. Such an assignment always exists, provided the circuit doesn't have redundant multiplication mod m gates.

As an illustration, consider the GRM circuit on Figure 3 and suppose we generate tests for primary input x_1. All four multiplication mod m gates have x_1 as input, but the first and the second gates depend on the minimal number of other primary inputs (x_2 only). If we set $x_2 = 1$ and $x_3 = 0$, then the circuit implements the function $f(x_1, 1, 0) = 1x_1^2 \oplus 2x_1^2 = 0$, i.e. the output is not sensitive to x_1. However, for the input assignment $x_2 = 2$ and $x_3 = 0$, the circuit implements the function $f(x_1, 2, 0) = 2x_1^2 \oplus 2x_1^2 = 1x_1^2$, and thus the output is sensitive to x_1.

Summarizing, the modified procedure for finding the test set of size $2n$ for detecting all single stuck-at faults on primary inputs of a GRM circuit is:

Procedure for test generation for primary inputs of a GRM circuit:

1. Consider all multiplication mod m gates having x_i as input;
2. From these, select the gates depending on the minimal number of other primary inputs. Define $H := \{x_j \mid x_j$ is a primary input on which all selected gates depend$\}$;
3. With $x_j = 0$ for all $x_j \notin H$, find an assignment A for the primary inputs in $H - \{x_i\}$ under which the output is sensitive to the input x_i, i.e. for some values $a_i \neq a_i'$ the transition in the value of x_i from a_i to a_i' causes a change in the value of the output. The simplest way to find such an assignment depends on the mechanism used to specify the circuit;
4. Define two tests t_{i1} and t_{i2} in the following way:

   t_{i1} := specifies $x_i = a_i$, primary inputs in $H - \{x_i\}$ in correspondence with the assignment A, and $x_j = 0$ for all $x_j \notin H$.

   t_{i2} := specifies $x_i = a_i'$, primary inputs in $H - \{x_i\}$ in correspondence with the assignment A, and $x_j = 0$ for all $x_j \notin H$.
5. Repeat the procedure for all primary inputs.

The complexity of the procedure depends on the size of H. The smaller the size of H, the easier it is to find the assignment A. In the simplest case when $|H| = 1$, there is a multiplication mod m gate(s) in the circuit depending on primary input x_i only (i.e. realizing some power of x_i). Then, to generate tests for x_i stuck-at faults, only m values of x_i should be examined to find a_i and a_i' satisfying:

$$f(0, \ldots, 0, a_i, 0, \ldots, 0) \neq f(0, \ldots, 0, a_i', 0, \ldots, 0)$$

In the next section we show that for a random GRM circuit implementing an m-valued n-variable function, the probability that $|H| \leq 2$ is greater than 99,99% for any x_i, provided $n \geq 3$ and $m \geq 3$.

3.2 Evaluation of the effectiveness of the procedure.

As shown in the previous section, it is easy to find a test for a primary input x_i of a GRM circuit if the circuit has a multiplication mod m gate depending on x_i and a small number k of other primary inputs. In this section we estimate how often this is the case. Lemmas 1 and 2 give the mathematical foundation of the result.

Lemma 1 *In the GRM expansion of an m-valued n-variable function, the number $N_{k,n}$ of k-variable product-terms which include a given variable x_i is at most:*

$$N_{k,n} = (m-1)^k \frac{(n-1)!}{(n-k)!(k-1)!}$$

where $1 \leq k \leq n$ and $i \in \{1, 2, \ldots, n\}$.

Proof: In a GRM expansion, each variable can have $(m-1)$ different powers. Thus, there can be constructed $(m-1)^k$ different product-terms consisting of k fixed variables. Since the number of choices of $(k-1)$ variable from $(n-1)$ is $\begin{pmatrix} n-1 \\ k-1 \end{pmatrix}$, the maximum number $N_{k,n}$ of k-variable product-terms which include a given variable x_i is:

$$N_{k,n} = (m-1)^k \begin{pmatrix} n-1 \\ k-1 \end{pmatrix} = (m-1)^k \frac{(n-1)!}{(n-k)!(k-1)!}.$$

□

For example, for $m = 3$, $n = 3$ and a variable x_1 we have:

$k = 1: \quad N_{1,3} = 2^1 \begin{pmatrix} 2 \\ 0 \end{pmatrix} = 2$

product-terms: x_1, x_1^2

$k = 2: \quad N_{2,3} = 2^2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 8$

product-terms: $x_1x_2, x_1x_2^2, x_1^2x_2, x_1^2x_2^2,$ $x_1x_3, x_1x_3^2, x_1^2x_3, x_1^2x_3^2$

$k = 3: \quad N_{3,3} = 2^3 \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 8$

product-terms:$x_1x_2x_3, x_1x_2x_3^2, x_1x_2^2x_3,$ $x_1x_2^2x_3^2, x_1^2x_2x_3, x_1^2x_2x_3^2, x_1^2x_2^2x_3, x_1^2x_2^2x_3^2$

Lemma 2 *The fraction $X_{k,n}$ of GRM expansions of m-valued n-variable functions not having a product-term of k or less variables which include a given variable x_i is:*

$$X_{k,n} = \frac{1}{m^{\left(\sum_{i=1}^{k} N_{i,n}\right)}}$$

where $1 \le k \le n$ and $i \in \{1, 2, \ldots, n\}$.

Proof: Since m^n is the maximum number of different product-terms in a GRM expansion of an m-valued n-variable function, and $N_{k,n}$ is the maximum number of k-variable product-terms which include a given variable x_i, the number of the GRM expansions not having a product-term of k or less variables which include a given variable x_i is

$$m^{\left(m^n - \sum_{i=1}^{k} N_{i,n}\right)}$$

So, the fraction of the GRM expansions of m-valued n-variable functions not having a product-term of the latter type is

$$\frac{m^{\left(m^n - \sum_{i=1}^{k} N_{i,n}\right)}}{m^{m^n}} = \frac{1}{m^{\left(\sum_{i=1}^{k} N_{i,n}\right)}}$$

□

For example, for $m = 3, n = 3$ and a variable x_i, $i \in \{1, 2, 3\}$, we have:

$$X_{1,3} = 1/3^2 \approx 0.11$$
$$X_{2,3} = 1/3^{10} \approx 1.69 \times 10^{-5}$$
$$X_{3,3} = 1/3^{18} \approx 2.89 \times 10^{-9}$$

The result $X_{2,3} \approx 1.69 \times 10^{-5}$ implies that for $m = 3, n = 3$ and a given primary input x_i, the percentage of GRM circuits not having a multiplication mod m gate realizing x_i^k or $x_i^k x_j^p$, where x_j is some other primary input and k and p are some powers of x_i and x_j, is extremely small. Since the fraction $\frac{1}{m^{\left(\sum_{i=1}^{k} N_{i,n}\right)}}$ decreases as m and n increase, for larger values of m and n the value of $X_{2,n}$ becomes even smaller. So, for a random GRM circuit implementing an m-valued n-variable function, the probability that $|H| \le 2$ is greater than 99,99 % for any x_i, provided $n \ge 3$ and $m \ge 3$.

In the next section we show that by adding to the GRM circuit an extra multiplication mod m gate with an observable output the number of tests sufficient to detect all single stuck-at faults is reducible to four.

4 Testability by hardware redundancy.

It is proved in [2] that, by providing a two-valued RM circuit with an extra AND gate having an observable output, $n + 4$ tests for internal lines also detect all single stuck-at faults on primary inputs. We show that a similar technique can be used to ensure that the four tests for internal lines given by Theorem 1 also detect all single stuck-at faults on primary inputs of a GRM circuit.

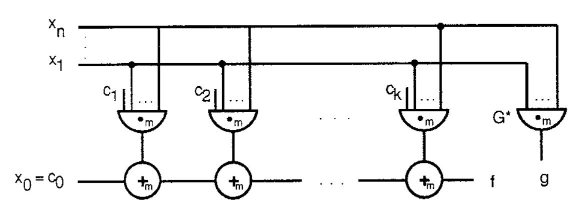

Figure 5: GRM circuit with an extra multiplication mod m gate G^*.

Consider a GRM circuit realization of an m-valued function $f(x_1, \ldots, x_n)$ having an extra multiplication mod m gate G^* depending on all input variables $x_1, \ldots, x_n$ and with an output g (fig. 5). If g is also observable, then two input assignments $(a_1, \ldots, a_n)$ and $(a_1', \ldots, a_n')$, such that none of a_i, a_i' is zero and $a_i \neq a_i'$ for all i, detect all single stuck-at faults on the inputs of G^*. These two tests also detect single stuck-at faults on primary inputs $x_1, \ldots, x_n$ since a single path is sensitized from each x_i to the output g. Observing the second and the fourth tests from the test set T from Theorem 1:

x_0	x_1	x_2	$\ldots$	x_n
0	0	0	$\ldots$	0
0	1	1	$\ldots$	1
1	0	0	$\ldots$	0
0	$m-1$	$m-1$	$\ldots$	$m-1$

we see that the assignments for $x_1, \ldots, x_n$ satisfy the requirements $a_i, a'_i \neq 0$ and $a_i \neq a'_i$ for all $i \in \{1, 2, \ldots, n\}$. Thus, the test set T detects all single stuck-at faults on primary inputs as well as on the inputs of G^*.

So, by adding to the GRM circuit an extra multiplication mod m gate with an observable output the number of tests sufficient to detect all single stuck-at faults is reducible to four.

5 Conclusion.

In this paper, we investigate the testability of generalized Reed-Muller circuits realizing m-valued functions in modulo m sum-of-product form, with m being a prime greater than two. We consider two aspects of the problem - the number of tests required for fault detection, and the generation of tests.

We prove that there exists a set of four tests detecting all single stuck-at faults on internal lines in the circuit. Furthermore, this set of tests is independent of the function being realized and therefore universal.

We propose two alternative techniques for testing primary inputs. The first one is to generate a test set of maximum length $2n$. We give a procedure for finding this test set and analyze its effectiveness. It is shown that the procedure effectively generates the tests for a primary input x_i when the circuit has a multiplication mod m gate depending on x_i and a small number k of other primary inputs. The smaller the k, the easier it is to find the test for x_i. We prove that for a random GRM circuit implementing an m-valued n-variable function, the probability that $k \leq 1$ is greater than 99,99 % for any x_i, provided $n \geq 3$ and $m \geq 3$. Since this probability is very high, it is most likely that tests for primary inputs can be generated effectively using the proposed procedure.

The second technique for testing of primary inputs we propose is to modify the circuit in such a way that the four tests for internal lines also detect all single stuck-at faults on primary inputs. We show that this can be accomplished by adding to the circuit an extra multiplication mod m gate with an observable output. The big advantage of this approach as compared to the first one is that the set of tests detecting all single stuck-at faults in the circuit is reduced to just four tests and, moreover, this set is universal and therefore no test generation procedure is required.

As a topic for further research we suggest the investigation of the testability of other types of m-valued circuits based on a multi-level tree of sum mod m gates. Two such architectures have been proposed so far: one based on the operations of addition and multiplication modulo m and the literal operators ([3]), and one based on addition modulo m, minimum and the literal operators ([10]). These m-valued circuits might also possess the useful property of easy testability.

Acknowledgment

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