Advanced Logic Design 2012
Assignment 4, 14 points, due 22/11

(1) **6 points** Draw the composition tree for each Boolean function f given below. Write the expression for the resulting decomposition of f.

(a)

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	1	0	1	0
01	1	0	0	0
11	1	1	1	1
10	1	1	0	0

(b)

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	0	0	0
10	0	0	0	0

(c)

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	1	0	1	0
01	1	0	0	1
11	1	0	0	1
10	1	0	0	1

(2) **3 points** Consider the Boolean functions:

(a) $f = abce + bcde + ch + khe$
(b) $f = abd + acdh + acdek + kdh$
(c) $f = abeh + acdh + aceh + abc + bd$

Answer the following two questions about each function:

1. Find all kernels of f
2. Using all or some kernels from (2), write f in a factorized form trying to minimize the number of literals. Count the number of literals.

(3) **3 points** Let $f(X,Y)$ be a Boolean function of type $\{0,1\}^n \rightarrow \{0,1\}$, where X and Y are disjoint sets of variables. Suppose we have a ROBDD representing f which has variables of X on the top and variables of Y on the bottom of the ROBDD. We know that the following property is true:

The set of variables X above a cut line of a ROBDD representing $f(X,Y)$ is a bound set for $f(X,Y)$ if and only if the paths crossing the cut line from above point to at most 2 distinct nodes below the cut line.

However, it is also possible to check whether the set below the cut line, Y, is a bound set directly on a ROBDD. Your task is to formulate which conditions should be satisfied in this case, i.e. your answer should be of type:

The set of variables Y below a cut line of a ROBDD representing $f(X,Y)$ is a bound set for $f(X,Y)$ if and only if ... (your conditions) ...

Note, that you are not allowed to re-order the ROBDD.

(4) **2 points** A 4-LUT (look-up table) FPGA technology mapping problem consists in finding a decomposition of an n-variable function $\{0,1\}^n \rightarrow \{0,1\}$ into a network of 4-variable functions $\{0,1\}^4 \rightarrow \{0,1\}$. Each of 4-variable functions is implemented by a LUT. The LUTs are connected in correspondence with the decomposition.

What is the maximum number of LUTs needed to implement a 5-variable function (any 5-variable function, not just some specific one)? Draw a diagram showing how LUTs should be connected.