

Advanced Logic Design 2012

Assignment 2, 8 points, due 08/11

(1) **1 point** Consider the Boolean function $f : \{0, 1\}^3 \rightarrow \{0, 1, -\}$ which is defined by the following truth table:

a	b	c	f(a,b,c)
0	0	0	—
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

- (1a) List on-set, don't care set and off-set of f as sets of cubes.
- (1b) List on-set, don't care set and off-set of f in parallel encoding ((min,max) pairs).
- (1c) List on-set, don't care set and off-set of f in sequential encoding.
- (2) **2 points** Let c_1 be the cube $(1 \text{ } - \text{ } -)$ and c_2 be the cube $(- \text{ } 1 \text{ } 1)$. Using the parallel encoding ((min,max) pairs):
- (2a) Compute the intersection of c_1 and c_2 .
- (2b) Compute the supercube of c_1 and c_2 .
- (2c) Check whether c_2 is contained in c_1 .
- (2d) Compute the complement of a union of c_1 and c_2 .

For each case, explain which formula you have used and show all steps of the computation.

- (3) **2 points** Repeat (2) for the sequential encoding.

(4) **1 point** Prove that $a + 1 = 1, \forall a \in B$, for the Boolean algebra $(B; +, \cdot, ' ; 0, 1)$, where $B = \{0, 1\}$.

(5) **1 point** Prove that $a + a \cdot b = a, \forall a, b \in B$, for the Boolean algebra $(B; +, \cdot, ' ; 0, 1)$, where $B = \{0, 1\}$.

(6) **1 point** Check whether the following algebra $(B; +, \cdot, ' ; \mathbf{0}, \mathbf{1})$ is a Boolean algebra:

$$B = \{0, 1, 2, 3\}$$

”+” is the maxim (the largest of two values)

”.” is the minimum (the smallest of two values)

”/'” is the complement dened by: $x' = (x + 1) \bmod 4$, i.e. $0' = 1, 1' = 2, 2' = 3, 3' = 0$.

$$\mathbf{0} = 0$$

$$\mathbf{1} = 3$$