Zero-Energy States in Supersymmetric Matrix Models

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Abstract

We summarize some of our recent work on the problem of understanding existence, uniqueness, and structure of zero-energy states in Sp(N) × SU(N) invariant supersymmetric matrix models.

Introduction

Supersymmetric matrix models arise from reduced Super Yang-Mills theory and regularized superpermutation. They have been proposed to describe aspects of a so-called M-theory, providing a unified description of the fundamental physical interactions, and, especially in that context, the question of existence, uniqueness, and structure of zero-energy ground states of these models is extremely relevant. See e.g. [1] for a review.

The models, depending on the parameters \( d = 2, 3, 5 \), or 9 and \( N = 2, 3, \ldots \), are supersymmetric quantum mechanical systems defined by the Hilbert space

\[
\mathcal{H} = L^2(\mathbb{R}^d \otimes \mathbb{R}^{N\times N}) \otimes \mathcal{F}, \quad \mathcal{F} = C^\infty(\mathbb{R}^d + \mathbb{R}^d),
\]

and the Hamiltonian operator

\[
H = \frac{1}{2} \sum_{A,B} \frac{f_{A,B}}{C^A_B} \partial_A \theta_A \theta_B - \frac{1}{2} + V \gamma \mathcal{H}_F.
\]

(1)

where the \( f_{A,B} \) are totally antisymmetric SU(\( N \)) structure constants, \( \gamma \) generate a matrix representation of the Clifford algebra over \( \mathbb{R}^d \), acting irreducibly on \( \mathbb{R}^{N\times N} \), while \( \mathcal{H}_F \) generate the Clifford algebra over \( \mathbb{R}^d \otimes \mathbb{R}^{N\times N} \), acting irreducibly on \( \mathcal{F} \).

The Hamiltonian is invariant under SU(\( N \)) transformations generated by

\[
J_A = \frac{1}{2} \sum_{B,C} \frac{f_{A,B,C}}{C^B_C} \theta_B \theta_C \mathcal{F}^*_A,
\]

as well as under Spin(\( d \)) generated by

\[
J_{\alpha} = \frac{1}{2} \sum_{A,B} \frac{f_{A,B}}{C^A_B} \mathcal{F}^*_A \theta_{\alpha},
\]

where the SU(\( N \)) invariant operator

\[
H_{\gamma}(x) = -\Delta x + \sigma \gamma(\gamma + 2W(x)\lambda^3),
\]

comprising a set of supersymmetric harmonic oscillators in the \( \gamma \)-coordinates, was introduced and studied in [8]. Using the knowledge of how its spectrum and eigenfunctions depend on the parameter \( x \), we prove the following

Theorem. For any \( \lambda \geq 0 \) there exists a sequence \( (\theta_k) \) of rapidly decaying smooth SU(\( N \))-invariant functions such that \( (\theta_k) \to 1 \) and

\[
\| H - \lambda \theta_k \| \rightarrow 0 \quad \text{as} \quad \lambda \to + \infty.
\]

It follows that the essential spectrum of \( H \) on \( \mathcal{H}_{\gamma}^{\text{phys}} \), just as for \( \mathcal{H}_{\gamma}^{\text{tyo}} \), is equal to \( \mathbb{R}_+ \).

Spin(9) average of a truncated model

Motivated by the fact that a zero-energy state must be invariant under \( \text{Spin}(d) \), we average the truncated Hamiltonian \( H_{\gamma}(9) \), and its eigenstates, with respect to the action (2) of Spin(9).

The average of the operator \( H_{\gamma}(9) \) w.r.t. Spin(9) is equal to \( H \). Furthermore, if we restrict these operators to the subspace of Spin(9) \( \times \) SU(\( N \)) invariant states, then \( H_{\gamma}^{\text{tyo}} = H \). The first statement follows by a direct computation, relating the terms of (4) to those of (1), while the second statement follows by considering the corresponding quadratic form on the space of invariants states of spin(9) = U(\( 9 \)).

Construction by recursive methods

An alternative, more direct, approach to determining the structure of zero-energy states is via recursive methods. Using the grading of the Fock space \( F \) in terms of fermion number, we derived recursive relations for the ground state in [10]. The corresponding structure was analyzed in the asymptotic description of the models, where we also revealed a dynamical SU(2) symmetry. An alternative recursive formulation was introduced in [11], where we instead of fermion number used the graded structure of the Clifford algebra of \( \mathbb{H} \) as a reducible representation.

In [12] we use representation theory of Spin(9) to explicitly construct the form of the zero-energy eigenfunction for \( d = 9 \), \( N = 2 \) at the origin, and give recursive relations relating it to higher order terms.

References


This work is supported by the Swedish Research Council and the Marie Curie Training Network ENIGMA (contract MRTN-CT-2004-5652).