NORDITA
Nordic Institute for Theoretical Physics

Mathematical Physics of Anyons and Topological States of Matter

March 11 – 16, 2019, Stockholm (Sweden)

Programme

Monday 11/03

10:00 – 10:30 Opening

10:30 – 11:20 J.P. Solovej

11:20 – 11:40 Coffee break

11:40 – 12:30 G. Goldin

12:30 – 14:30 Lunch Break

14:30 – 15:20 E. Yakaboylu

15:20 – 15:40 Coffee break

15:40 – 16:30 S. Larson

16:40 – 17:30 A. Polychronakos

18:00 – 19:00 Reception

Tuesday 12/03

09:30 – 10:20 J. Yngvason

10:20 – 10:40 Coffee break

10:40 – 11:30 M. Correggi

11:40 – 12:30 L. Arkeryd

12:30 – 14:30 Lunch Break

14:30 – 15:20 E. Langmann

15:20 – 15:40 Coffee break

15:40 – 16:30 K. Rajaratnam

16:40 – 17:30 A. Olgiati

Wednesday 13/03

09:30 – 10:20 J. Avron

10:20 – 10:40 Coffee break

10:40 – 11:30 S. Ouvry

11:40 – 12:30 M. Plaschke

12:30 – 19:30 Free afternoon

19:30 – 22:30 Social dinner

Thursday 14/03

09:30 – 10:20 G.M. Graf

10:20 – 10:40 Coffee break

10:40 – 11:30 A.B. Nielsen

11:40 – 12:30 O. Kenneth

12:30 – 14:30 Lunch Break

14:30 – 15:20 L. Oddis

15:20 – 15:40 Coffee break

15:40 – 16:30 V. Qvarfordt

16:40 – 17:30 M. Porta

19:00 – 22:30 Dinner cruise

Friday 15/03

09:30 – 10:20 G. Panati

10:20 – 10:40 Coffee break

10:40 – 11:30 Y. Tournois

11:40 – 12:30 A. Trombettoni

12:30 – 14:30 Lunch Break

14:30 – 18:30 Discussions & closing

Titles and Abstracts

The Boltzmann Haldane Equation for Anyons - Mathematical Results

LEIF ARKERYD
(University of Gothenburg, Gothenburg, Sweden)

The talk will discuss a Boltzmann equation for anyons and kinetic gases with Haldane statistics in general. The focus will be on mathematical results and aspects of their proofs.

Quantized Hall Conductance for Large Interacting Systems

JOSEPH AVRON
(Technion, Haifa, Israel)

I will give an informal geometric description of the work of Hastings and Michalakis—and more precisely, of simplified version of this work due to Bachmann, Bols, De Roeck, and Fraas—on the quantization of the Hall conductance for large interacting systems. In particular, I shall describe a generator parallel transport with good localization properties introduced by Hastings. The talk will not assume prior knowledge of the quantum Hall effect and is partly based on an article in the Bulletin of IAMP.

On the Average-Field Functional for Anyons

MICHELE CORREGGI
("Sapienza" Università di Roma, Rome, Italy)

We discuss the average-field approximation for a trapped gas of non-interacting anyons in the quasi-bosonic regime. In the homogeneous case, i.e., for a confinement to a bounded region, we prove that the energy in the regime of large statistics parameter, i.e., for "less-bosonic" anyons, is independent of boundary conditions and of the shape of the domain. When a non-trivial trapping potential is present, we derive a local density approximation in terms of a Thomas-Fermi-like model. We also discuss some recent numerical simulations and open questions mostly related to the vortex structure of the minimizer. Joint work with R. Duboscq (Toulouse), D. Lundholm (Stockholm) and N. Rougerie (Grenoble).

Group-Theoretical Approach to Anyons and Topological Quantum Configurations

Gerald A. Goldin
(Rutgers University, New Brunswick, US)

Classification of induced unitary representations of the group of diffeomorphisms of physical space and its semidirect products provides a unified description of diverse quantum systems. One of the early discoveries of the possibility of anyon statistics for N particles in two-space stemmed from this approach to quantum theory. The braid group enters naturally, and anyonic creation and annihilation fields satisfying equal-time q-commutation relations arise as the intertwining fields for a hierarchy of such N-anyon diffeomorphism group representations with fixed anyonic phase shift. Higher-dimensional braid group representations induce diffeomorphism group representations describing nonabelian anyons, or plektons. The ideas generalize to configurations of extended objects having nontrivial topology, albeit with some mathematically rigorous results remaining to be demonstrated. This talk reviews briefly the main findings, highlights directions needing investigation, and points toward a possible approach to relativistic theories based on spatial measurements in inertial reference frames.

Disorder and Topology. The Cases of Floquet and of Chiral Systems.

Gian Michele Graf
(ETH, Zürich, Switzerland)

We will present a new formulation of bulk and edge indices for disordered Floquet systems. A byproduct is a space-time duality stating the equivalence of two settings: two systems may be placed next to one another in space or operate one after the other in time. A different type of systems to be addressed are disordered chiral chains, which may be viewed as Su-Schrieffer-Heeger models with random hopping. There localization occurs at all but possibly one energy, which is enough to endow the model with topological features. Different formulations of the index will be introduced and related to the Lyapunov spectrum of the chain.

A Simple Model Leading to Nonabelian Anyon Behaviour

Oded Kenneth
(Technion, Haifa, Israel)

The zero mode space of the Pauli (or Dirac) hamiltonian in 2d has finite degeneracy determined by the total magnetic flux as observed first by Aharonov and Casher. We consider the adiabatic evolution of the zero modes in the presence of slowly moving

(non integer) magnetic fluxons. Gapless adiabatic theorem guarantees the evolution is governed by an adiabatic connection, which is found to be equal to the Chern connection. The connection is flat (but nontrivial) provided the fluxes satisfy certain inequalities (namely $0 < \phi_a < 1, \sum(1-\phi_a) < 1$). Under the braid group the fluxons than transform as nonabelian anyons belonging to the Gassner representations. (Also known as Burau representation in the special case of identical fluxons.) The inequalities satisfied by the fluxes may be related to unitarity of the representation.

Deformed Calogero-Sutherland Model and Fractional Quantum Hall Effect

EDWIN LANGMANN
(KTH, Stockholm, Sweden)

I discuss a quantum field theory of anyons on a circle that provides an effective description of certain fractional quantum Hall effect systems. This model has a rich mathematical structure related to quantum integrable systems of Calogero-Moser-Sutherland type, special functions, and soliton theory. In particular, I describe a natural generalization of the Calogero-Sutherland model describing two kinds of particles and its relation to a quantum version of the Benjamin-Ono equation.

Exclusion Bounds for Extended Anyons

SIMON LARSON
(KTH, Stockholm, Sweden)

In this talk we discuss energy bounds for the homogeneous gas of extended anyons. The main results are many-body magnetic Hardy inequalities and local exclusion principles, leading to estimates for the ground-state energy of the anyon gas in the full parameter range. The methods also yield improvements for ideal (non-extended) anyons.

Based on joint work with D. Lundholm and P. T. Nam.

Anyons in Fractional Quantum Hall Models

ANNE E.B. NIELSEN
(Max Planck Institute, Dresden, Germany)

There is currently much interest in investigating the possibilities for obtaining fractional quantum Hall physics in lattice systems, both because it may lead to new ways to realize the effect, e.g. in cold atoms, and because the lattice gives rise to new features and opportunities. Here, we show that a variant of matrix product states can be used to construct lattice wavefunctions and parent Hamiltonians with fractional quantum Hall properties. We show how to introduce quasiholes and quasielectrons into the models, and we determine the size, shape, charge, and braiding statistics of the anyons for both Laughlin and Moore-Read states. In the continuum, it is known to be difficult to construct wavefunctions for quasielectrons. We show that the same is not true in lattice systems, since the quasielectrons can be constructed directly as inverse quasiholes. We also discuss how one can interpolate between lattice and continuum fractional quantum Hall models.

Quadratic Forms for Anyons

LUCA ODDIS
("Sapienza" Università di Roma, Rome, Italy)

We review the main issues concerning the well-posedness (as suitable self-adjoint operators) of the Hamiltonians of two non interacting spinless anyons. We show that such operators can be identified with a one-parameter family of self-adjoint extensions of a suitable symmetric operator with Aharonov-Bohm-like magnetic potential. We also derive the explicit expressions of the corresponding quadratic forms and prove their closedness and boundedness from below. We present a decomposition of the domains of these self-adjoint operators different from the one coming from the Von-Neumann theorem in terms of the behaviour of the wave functions near the coincidence set. The proof can be extended to the case with interaction under suitable assumptions on the potential. Then we examine the N-anyon case and exploit the informations given by the 2-particle case to generalise the construction of the quadratic forms. In particular, we consider wave functions split in a regular part, belonging to the Friedrichs extension and a singular part, which reproduces the asymptotic behaviour near the coincidence hyperplanes seen in the previous case. Joint work with M. Correggi ("Sapienza" University of Rome).

Stability of the Laughlin Phase in Presence of Interactions

ALESSANDRO OLGIATI
(CNRS, Grenoble, France)

The Laughlin wave function is at the basis of the current understanding of the fractional quantum Hall effect, nevertheless, many of its fundamental properties are yet to be understood. I will present a model for its response, within Laughlin's ansatz, to variations of the external potential and of the interaction among particles. Our main result is that the energy is asymptotically captured by the minimum of an effective functional with variational constraints fixed by the incompressibility of the liquid. Moreover, as was already known for the Laughlin wave-function, the one-body density converges to the characteristic function of a set. This is based on an ongoing project with Nicolas Rougerie.

Anyons and Lowest Landau Level Anyons

STEPHANE OUVRY
(LPTMS, Université Paris-Sud, Paris, France)

I present a review on the LLL-anyon model, its themodynamics and its relation to Calogero thermodynamics.

The Localization-Topology Correspondence: Periodic Systems and Beyond

GIANLUCA PANATI
("Sapienza" Università di Roma, Rome, Italy)

As realized in a breakthrough paper by Thouless et al., for gapped periodic 2D systems a relevant Transport-Topology Correspondence holds true, in the sense that a non-vanishing Hall conductivity corresponds to a non-trivial topology of the space of occupied states, decomposed with respect to the crystal momentum – a space which is called Bloch bundle in the recent literature.

More recently, a related Localization-Topology Correspondence has been noticed and mathema-tically proved for 2D and 3D gapped periodic quantum system. The result states that the Bloch bundle is trivial if and only if there exists a system of composite Wannier functions on which the expectation value of the squared position operator is finite. In other words, whenever the system is in a Chern-non-trivial phase, the composite Wannier functions are very delocalized, while in the Chern trivial phase they can be chosen exponentially localized (joint work with D. Monaco, A. Pisante and S. Teufel).

During my talk, I will report on this result and the essential ideas of its proof, as well as on the ongoing attempt to generalize this correspondence to non-periodic gapped quantum systems (work in progress with G. Marcelli and M. Moscolari).

Local Anyons on the Universal Covering of the Circle leading to Cone-local Anyons on $\mathbb{R}^2$

MATTHIAS PLASCHKE
(Universität Wien, Vienna, Austria)

Basic considerations in algebraic quantum field theory in lower dimensions show the possibility of quantum field nets with Anyon statistics. It is also possible to derive some of their properties in a mathematically rigorous manner like their commutation relations, the non-triviality of 2π-rotations, localization in a kind of covering space and especially a No-Go theorem for free fields. This also shows that explicit constructions of such fields are relatively hard to find.

In 1+1 dimensions it is possible to explicitly define a local covariant net of quantum fields with anyonic statistics using the method of implementable Bogoliubov transformations. A similar construction can also be used on the circle S_1, leading eventually to a local net on the universal covering space of S_1. These Anyon fields transform under a representation of the covering group of $U(1)$ for arbitrary real-valued spin and their commutation relations depend on the relative winding number of localization regions. By taking the tensor product with a local covariant field theory on $\mathbb{R}^2$ one can obtain a non-relativistic field net for Anyons in two dimensions. These fields are then localized in so-called paths of cones, which constitute the best possible localization for Anyons in two spatial dimensions.

The Calogero Model: Physics, Mathematics and Recent Results

ALEXIOS POLYCHRONAKOS
(CUNY, New York, US)

A review of the Calogero integrable model and its various generalizations will be given, with emphasis on mathematical properties and recent results. Topics will include integrability, the connection with fractional statistics, their matrix model and operator formulations, their various reductions and extensions, and their hydrodynamic description, properties and solitons.

Edge Universality in Interacting Topological Insulators

MARCELLO PORTA
(Universität Tübingen, Tübingen, Germany)

In the last few years there has been important progress on the rigorous understanding of the stability of gapped topological phases for interacting condensed matter systems. Most of the available results deal with bulk transport, for systems with no boundaries. In this talk, I will consider interacting 2d topological insulators on the cylinder. According to the bulk-edge duality, one expects robust gapless edge modes to appear. By now, this has been rigorously understood for a wide class of noninteracting topological insulators; the main limitation of all existing proofs is that they do not extend to interacting systems. In this talk I will discuss the bulk-edge duality for a class of interacting 2d topological insulators, including the Haldane-Hubbard model and the Kane-Mele-Hubbard model. Our theorems give a precise characterization of edge transport: besides the bulk-edge duality, the interacting edge modes satisfy the Haldane relations, connecting the velocities of the edge currents, the edge Drude weights and the edge susceptibilities. The proofs are based on rigorous renormalization group, with key nonperturbative inputs coming from the combination of lattice and emergent Ward identities. Based on joint works with G. Antinucci (Zurich) and V. Mastropietro (Milan).

Non-Abelian Anyons: Statistical Repulsion and Topological Quantum Computation

VIKTOR QVARFORDT
(Stockholms Universitet, Stockholm, Sweden)

Abelian anyons are described by abelian representations of the exchange (braid) group, that is, an exchange phase which continuously interpolates between bosons and fermions. The Pauli principle for fermions can be generalized to an effective pairwise ”statistical repulsion” for such anyons using a relative Poincaré inequality. Non-abelian anyons are much more complex and their statistical repulsion is yet largely unexplored. The framework of modular tensor categories is used to show how the statistical repulsion of non-abelian anyons depends on the exchange symmetry. The Fibonacci anyon model as well as the Ising anyon model is studied, for which explicit results are obtained. If there is time, we also discuss how Fibonacci anyons can be used to implement topological quantum computation.

Vortex Lattice Solutions of the ZHK Chern-Simons Equations

KRISHAN RAJARATNAM
(University of Toronto, Toronto, Canada)

We study the ZHK Chern-Simons equations which occur in the study of the fractional quantum hall effect of condensed matter physics. We first give a background of these equations and briefly describe their relation to physics. Then we state and sketch ideas behind our first result on the existence of vortex lattice solutions of these equations. Then we shall describe the physically interesting solution whose lattice shape minimizes the average energy per lattice cell. Time permitting, we describe the generalization of these results to find solutions of the ZHK Chern-Simons equations on Riemann surfaces of higher genus g. Finally, we shall present a result on the orbital stability of the vortex lattice solutions under perturbations which preserve the lattice.

Non-abelian Anyons in Paired Spin-singlet States

YORAN TOURNOIS
(Stockholms Universitet, Stockholm, Sweden)

The fractional quantum Hall effect is the paradigmatic example of topological order, and is believed to host quasiparticles which carry a fractional charge and obey fractional statistics of abelian or non-abelian type. An important tool to study the physics of fractional quantum Hall phases has been model wave functions, which are in turn analyzed using conformal field theory (CFT). In this talk, I will introduce the connection between CFT and the fractional quantum Hall effect, and use it to discuss braiding properties of non-abelian anyons in so-called paired spin-singlet states.

Off-Diagonal Long-Range Order in Low-Dimensional Quantum Systems

ANDREA TROMBETTONI
(CNR & SISSA, Trieste, Italy)

A quantum system exhibits off-diagonal long-range order (ODLRO) when the largest eigenvalue of the one-body-density matrix scales as N, where N is the total number of particles. More generally, if the largest eigenvalue scales as N^C to define the scaling exponent C, then $C = 1$ corresponds to ODLRO and $C = 0$ to the single-particle occupation of the density matrix orbitals. When $0 < C < 1$, C can be used to quantify deviations from ODLRO. In this talk I start by discussing the behaviour of the exponent C in a variety of one-dimensional bosonic and anyonic quantum systems. For the 1D Lieb-Liniger Bose gas we find that for small interactions C is close to 1. 1D anyons provide the possibility to fully interpolate between $C = 1$ and 0 and to show the singularity of the non-interacting limit for 1D bosons. I will then focus on the Tonks-Girardeau limit, in which C approaches the value $1/2$, discussing the case in which the gas is trapped in a general confining potential and showing the universality of the scaling of the largest eigenvalue of the one-body-density matrix with the number of particles. Finally, I will comment about the case of two-dimensional quantum systems.

Emergent Gauge Fields and Anyons in Quantum Impurity Problems

ENDERALP YAKABOYLU
(IST Austria, Klosterneuburg, Austria)

We have demonstrated that in any impurity problem a many-body environment manifests itself as an external gauge field with respect to the impurities interacting with it. Hence, an impurity problem can be viewed as interaction of charged particles with this gauge field. In this talk, I will give two examples that show non-trivial topological behavior. In the first example, I will show that a rotating impurity dressed by many-body field excitations, which is so-called angulon, can be seen as a point charge on a two-sphere interacting with a non-Abelian magnetic monopole. This problem exhibits a topological transition associated with making the monopole Abelian. In the second example, I will demonstrate that identical impurities interacting with a two-dimensional many-particle environment obey anyonic statistics. In particular, the bath manifests itself as an external magnetic flux tube with respect to the impurities, and acts as a statistical gauge field after a certain critical coupling. In the final part, I will consider an N-impurity problem and discuss how to realize Abelian anyons using quantum impurities.

Anyons in Algebraic Quantum Field Theory

JAKOB YNGVASON
(Universität Wien, Vienna, Austria)

Algebraic quantum field theory is a general, mathematically precise framework for (relativistic) quantum fields that is particularly well suited for structural analysis. The talk will give an account of the basic concepts of this approach and some results pertaining to anyons, in particular theorems of Jens Mund and Jacques Bros concerning non-existence of ‚free‘ relativistic anyons in $2+1$ space-time dimensions.