Anyons and lowest Landau level Anyons

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at the very end of Alexios's talk on Monday (last second, slide #99):

1d harmonic Calogero mapped on 2d harmonic anyon

I will discuss how this idea originated also in the context of LLL-anyon and Calogero thermodynamics

in the limit $B \to 0$:

LLL-anyon thermodynamics $\to$ Calogero thermodynamics

a bit strange : project on LLL then take $B \to 0$ limit outline :

as we know N-anyon pb is not solvable

except for some classes of N-anyon eigenstates : the so called ”linear” states

physical interpretation of ”linear” states in terms of LLL states by coupling the anyons to an external magnetic field and projecting on the lowest Landau level

$\Rightarrow$ **LLL-anyon model :**

i) solvable : these ”linear” states form a complete LLL-anyon basis which interpolates from LLL-Bose to LLL-Fermi

ii) a magnifying loop is needed : harmonic well confinement ω

(a magnetic field does not confine anything)

then take the thermodynamic limit $\omega \to 0$

$\Rightarrow$ thermodynamics, LLL-anyon equation of state

$\Rightarrow$ LLL-anyon mean occupation number that interpolates BE to FD on the other hand the 1d Calogero model has the same ”type” of spectrum/thermodynamics as the LLL-anyon model

in fact

$$\textbf{2d LLL-anyon} \stackrel{B\to 0}{\longrightarrow} \textbf{1d Calogero}$$

a dimensional reduction 2d $\to$ 1d has taken place

based on old works from the 90’s and the 2000’s let's start from the beginning

quantum statistics should always be defined for free particles with a free Hamiltonian

$$H'_N = \sum_{i=1}^{N} \frac{\vec{p}_i^{\ 2}}{2m}$$

$\oplus$ particular boundary condition on the N-body wavefunction

$$\psi'(\vec{r}_1, \vec{r}_2, ..., \vec{r}_N)$$

Bose-Fermi : symmetric-antisymmetric boundary conditions

in 2d : anyonic multivalued boundary condition

$\psi'(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N)$ acquires a phase $\exp(i\pi\alpha)$ when two particles are interchanged

$\alpha \in [0,1]$ is the statistical parameter this amounts to write

$$\psi'(\vec{r}_1,\vec{r}_2,...,\vec{r}_N) = \exp(i\alpha\sum_{k<l}\theta_{kl})\psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_N)$$

θ_{kl} angle of $\vec{r}_k - \vec{r}_l$ with fixed direction in the plane

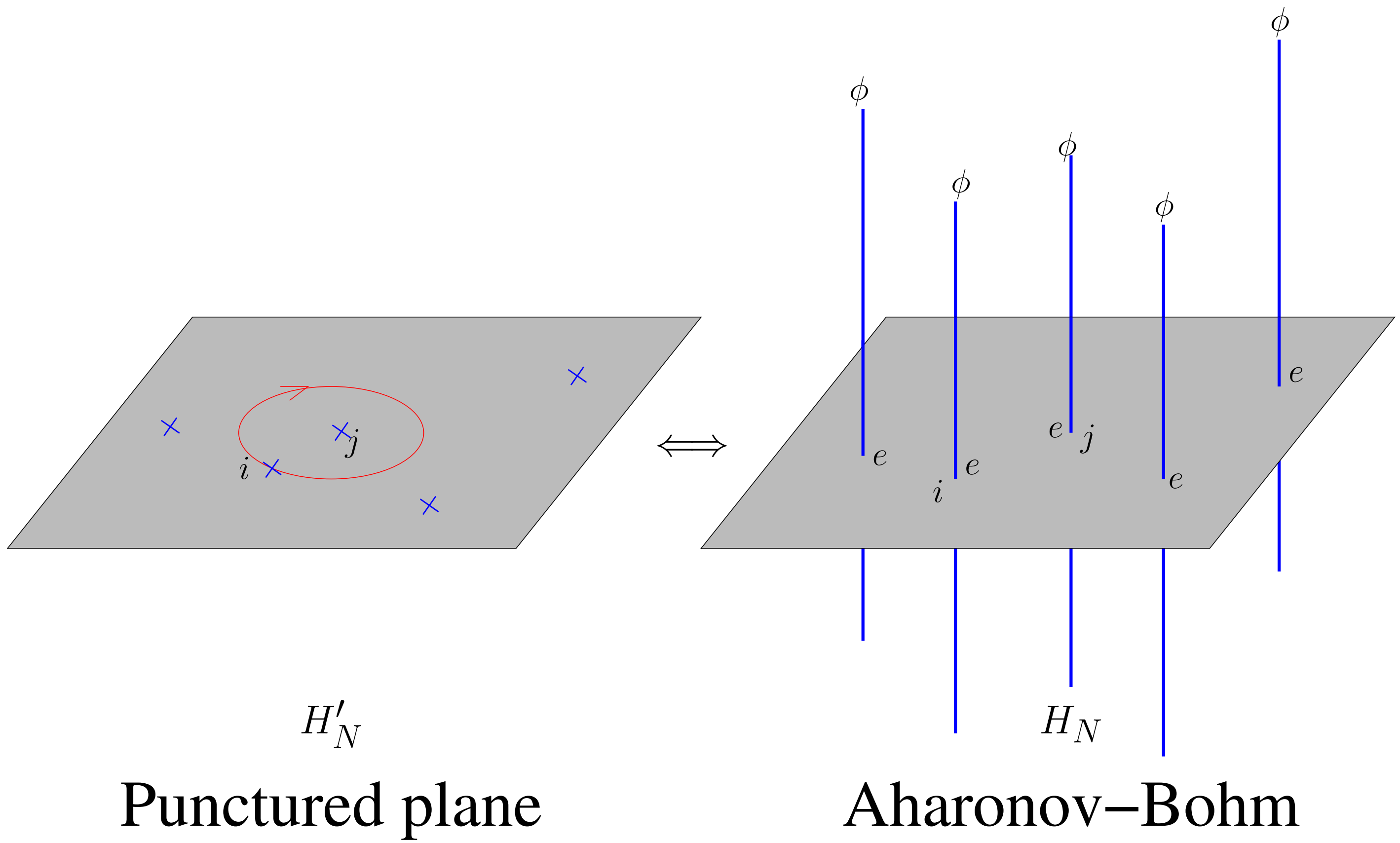
$\psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_N)$ regular wavefunction by convention bosonic

$$H'_N \rightarrow H_N = \sum_{i=1}^{N}\frac{1}{2m}(\vec{p}_i - \vec{A}(\vec{r}_i))^2$$

Aharonov-Bohm vector potential $\vec{A}(\vec{r}_i) = \alpha\sum_{j,j\neq i}\vec{k}\wedge\vec{r}_{ij}/r_{ij}^2$

induces statistical interactions between the bosons

$\alpha = \phi/\phi_0$ where ϕ_0 is flux quantum



i
j
$\Longleftrightarrow$
ϕ
e
H'_N
H_N
Punctured plane
Aharonov–Bohm

H_N is pertubatively divergent : one has to add contact repulsive 2-body interactions

$$\frac{2\pi\alpha}{m}\sum_{k\neq l}\delta(\vec{r}_{kl})$$

i.e. exclusion of the diagonal of configuration space

$$H_N=\sum_{i=1}^{N}\frac{1}{2m}(\vec{p}_i-\alpha\sum_{j\neq i}\frac{\vec{k}\wedge\vec{r}_{ij}}{r_{ij}^2})^2+\frac{2\pi\alpha}{m}\sum_{k\neq l}\delta(\vec{r}_{kl})$$

take advantage of the short distance behavior

$\psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_N)\simeq {r_{kl}}^{\alpha}$ when $\vec{r}_k\to\vec{r}_l$

to write

$$\psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_N) = \prod_{k<l} {r_{kl}}^{\alpha} \tilde{\psi}(\vec{r}_1,\vec{r}_2,...,\vec{r}_N)$$

$H_N \to \tilde{H}_N$ acting on $\tilde{\psi}(\vec{r}_1,\vec{r}_2,...,\vec{r}_N)$

directly from H'_N

use complex notations

$$H'_N = -2\sum_{i=1}^{N} \partial_i \bar{\partial}_i$$

$$z_{kl} = r_{kl} e^{i\theta_{kl}}$$

$$e^{i\alpha\sum_{k<l}\theta_{kl}} \otimes \prod_{k<l} {r_{kl}}^{\alpha}$$

combine in Jastrow-like factor

$$\prod_{k<l} {z_{kl}}^{\alpha}$$

so write

$$\psi'(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N) = \prod_{k<l} {z_{kl}}^{\alpha} \tilde{\psi}(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N)$$

$$H'_N \to \tilde{H}_N = -2\sum_{i=1}^{N} \partial_i \bar{\partial}_i - 2\alpha \sum_{i<j} \frac{1}{z_i - z_j}(\bar{\partial}_i - \bar{\partial}_j)$$

obvious N-anyon states = analytic functions of the z_i's to better see what is going on add magnifying loop

= confining harmonic trap

i.e. rather start from the free harmonic Hamiltonian

$$H'_N = -2\sum_{i=1}^{N} \partial_i \bar{\partial}_i + \sum_{i=1}^{N} \frac{\omega^2}{2} \bar{z}_i z_i$$

as before free N-anyon wavefunction $\psi'(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N)$

anyon phase/short distance $\prod_{k<l} {z_{kl}}^{\alpha}$

$\oplus$ long distance harmonic $e^{-\omega \sum_{i=1}^{N} z_i \bar{z}_i / 2}$

i.e. write

$$\psi'(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N) = \prod_{k<l} (z_k - z_l)^{\alpha} e^{-\omega \sum_{i=1}^{N} z_i \bar{z}_i / 2} \tilde{\psi}(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N)$$

$$
\begin{aligned}
H'_N \to \tilde{H}_N &= -2\sum_{i=1}^{N}\left[\partial_i\bar{\partial}_i - \frac{\omega}{2}\bar{z}_i\bar{\partial}_i - \frac{\omega}{2}z_i\partial_i\right] \\
&\quad -2\alpha\sum_{i<j}\left[\frac{1}{z_i - z_j}(\bar{\partial}_i - \bar{\partial}_j) - \frac{\omega}{2}\right] + N\omega
\end{aligned}
$$

$\tilde{H}_N$ still acts trivially on symmetrized product $\prod_{i=1}^{N} z_i^{\ell_i}$

$$
0 \leq \ell_1 \leq \ldots \leq \ell_N \Rightarrow \text{ Bose}
$$

$$\Rightarrow \psi'(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N) = \prod_{i<j} (z_i - z_j)^{\alpha} e^{-\omega \sum_{i=1}^{N} z_i \bar{z}_i / 2} \prod_{i=1}^{N} z_i^{\ell_i}$$

$$E_N = \omega \left[\sum_{i=1}^{N} \ell_i + \frac{1}{2} N(N-1)\alpha \right] + N\omega \quad 0 \le \ell_1 \le \ldots \le \ell_N$$

these are ”linear” states interpolating Bose $\alpha = 0 \to$ Fermi $\alpha = 1$

products of 1-body harmonic eidenstate $z_i^{\ell_i}\, e^{-\omega z_i \bar{z}_i / 2}$ energy $\omega(\ell_i + 1)$

$\ell_i = 2$d angular momentum

$\ell_i \ge 0 \Rightarrow \omega(\ell_i + 1) = 1$d harmonic spectrum

$\alpha \to g \Rightarrow N$-anyon ”linear” spectrum $\to N$-body Calogero spectrum

$$E_N = \omega \left[\sum_{i=1}^{N} \ell_i + \frac{N(N-1)}{2} g \right] + \frac{N}{2}\omega \quad 0 \le \ell_1 \le \ell_2 \le \ldots \le \ell_N$$

$$H_N = -\frac{1}{2}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + \sum_{i<j}\frac{g(g-1)}{(x_i-x_j)^2} + \frac{1}{2}\omega^2\sum_{i=1}^{N}x_i^2$$

again 1d confining harmonic well has been added

in the Calogero case ℓ_i is the 1d quantum number for the 1-body harmonic Hermite polynomial

$$H_{\ell_i}(\sqrt{\omega}x_i)e^{-\frac{1}{2}\omega x_i^2} \quad \ell_i \geq 0 \quad \text{energy } \omega(\ell_i + \frac{1}{2})$$

Alexios : N-body Calogero states mapped on N-anyon "linear" states

$$H_{\ell_i}(\sqrt{\omega}x_i) \leftrightarrow z_i^{\ell_i}$$

explicit construction of the mapping N-body kernel

”linear” states share the same quantum numbers as LLL states

give a physical interpretation : physics in the LLL

i.e. couple to a magnetic field and start from the free Hamiltonian

$$H'_N = -2\sum_{i=1}^{N}(\partial_i - \frac{\omega_c}{2}\bar{z}_i)(\bar{\partial}_i + \frac{\omega_c}{2}z_i) + \frac{1}{2}\omega^2\sum_{i=1}^{N}\bar{z}_i z_i$$

ω_c half cyclotron frequency

$$\omega^2 \to \omega^2 + \omega_c^2 = \omega_{\mathrm{t}}^2$$

write

$$\psi'(\vec{r}_1,\vec{r}_2,...,\vec{r}_N) = \prod_{k<l}(z_k - z_l)^{\alpha} e^{-\omega_t \sum_{i=1}^{N} z_i\bar{z}_i/2}\,\tilde{\psi}(\vec{r}_1,\vec{r}_2,...,\vec{r}_N)$$

$$
\begin{aligned}
H'_N \to \tilde{H}_N \quad &= \quad -2\sum_{i=1}^{N}\left[\partial_i\bar{\partial}_i - \frac{\omega_t+\omega_c}{2}\bar{z}_i\bar{\partial}_i - \frac{\omega_t-\omega_c}{2}z_i\partial_i\right] \\
&\quad -2\alpha\sum_{i<j}\left[\frac{1}{z_i-z_j}(\bar{\partial}_i-\bar{\partial}_j) - \frac{\omega_t-\omega_c}{2}\right] + N\omega_t
\end{aligned}
$$

again $\tilde{H}_N$ acts trivially on $\prod_{i=1}^{N} z_i^{\ell_i}$

same eigenstates where ω replaced by ω_t

product of 1-body harmonic-LLL states

$$
z_i^{\ell_i}\, e^{-\omega_t z_i \bar{z}_i/2} \quad \ell_i \geq 0 \quad \text{energy } (\omega_t-\omega_c)\ell_i + \omega_t
$$

spectrum

$$
E_N = (\omega_t - \omega_c)\left[\sum_{i=1}^{N}\ell_i + \frac{1}{2}N(N-1)\alpha\right] + N\omega_t
$$

when $\omega \to 0 \Rightarrow \omega_t = \sqrt{\omega^2 + \omega_c^2} \to \omega_c$

one gets LLL-anyon eigenstates

$$\psi' = \prod_{i<j} (z_i - z_j)^\alpha e^{-\omega_c \sum_{i=1}^{N} z_i \bar{z}_i / 2} \prod_{i=1}^{N} z_i^{\ell_i}$$

$$E_N = N\omega_c$$

degenerate spectrum N anyons in the LLL

complete interpolation in the LLL : LLL-Bose $\to$ LLL-Fermi

$\Rightarrow$ the "solvable" LLL-anyon model

LLL 1-body eigenstates

$$z_i^{\ell_i} e^{-\frac{1}{2}\omega_c z_i \bar{z}_i} \quad \ell_i \geq 0 \quad \text{energy } \omega_c$$

energy ℓ_i-independent $\Rightarrow$ infinite Landau degeneracy which scales as surface S of the plane

$$N_L = BS/\phi_0$$

magnetic field does not confine (translation invariant, continuous spectrum)

with the harmonic well the degeneracy is lifted

$$z_i{}^{\ell_i} e^{-\frac{1}{2}\omega_c z_i \bar{z}_i} \xrightarrow{\omega \neq 0} z_i{}^{\ell_i} e^{-\frac{1}{2}\omega_t z_i \bar{z}_i}$$

$$\omega_c \xrightarrow{\omega \neq 0} (\omega_t - \omega_c)\ell_i + \omega_t$$

$$N\omega_c \xrightarrow{\omega \neq 0} (\omega_t - \omega_c)\left[\sum_{i=1}^{N} \ell_i + \alpha \frac{N(N-1)}{2}\right] + N\omega_t$$

non degenerate , α dependence

LLL-anyon thermodynamics

take advantage of ω as a long distance regulator

i.e. start from non degenerate spectrum

inverse temperature $\beta = \frac{1}{kT}$

N-anyon partition function $Z_N = \sum_{0 \le \ell_1 \le \ell_2 \le \ldots \le \ell_N} e^{-\beta E_N}$

fugacity $z = e^{\beta\mu}$

grand partition function $Z = \sum_{N=0}^{\infty} z^N Z_N$

thermodynamical potential $\ln Z \rightarrow$ cluster expansion $\ln Z = \sum_{n=1}^{\infty} z^n b_n$

b_n is the n-th cluster coefficient : $b_2 = Z_2 - Z_1^2/2$, $b_3 = Z_3 - Z_2 Z_1 + Z_1^3/3, \ldots$

when ω small in fact $\beta\omega$ small (dimensionless) one obtains the n-th cluster coefficient

$$b_n \simeq \frac{1}{\beta(\omega_t - \omega_c)} \frac{e^{-n\beta\omega_c}}{n^2} \prod_{k=1}^{n-1} \frac{k - n\alpha}{k}$$

remember $\omega_t^2 = \omega^2 + \omega_c^2$: in the thermodynamic limit $\beta\omega \to 0$

$$\frac{1}{\beta(\omega_t - \omega_c)} \simeq \frac{1}{\beta \frac{\omega^2}{2\omega_c}} = \frac{2\beta\omega_c}{(\beta\omega)^2} \to ??$$

one has to give a meaning to $\frac{1}{(\beta\omega)^2}$ when $\beta\omega \to 0$

to do so consider free harmonic oscillators :

in the thermodynamic limit $\beta\omega \to 0$ one should have

harmonic n-th cluster $\to$ infinite box n-th cluster this is the case if

$$\frac{1}{n(\beta\omega)^2} \longrightarrow \frac{S}{2\pi\beta}$$

S infinite surface of the plane

using this thermodynamic limit prescription

$$b_n = \frac{1}{\beta(\omega_t - \omega_c)} \frac{e^{-n\beta\omega_c}}{n^2} \prod_{k=1}^{n-1} \frac{k - n\alpha}{k} \underset{\beta\omega\to 0}{\longrightarrow} b_n = N_L \frac{e^{-n\beta\omega_c}}{n} \prod_{k=1}^{n-1} \frac{k - n\alpha}{k}$$

again $N_L = BS/\phi_0 =$ Landau degeneracy one wants

$$\ln Z = \sum_{n=1}^{\infty} b_n z^n$$

one gets

$$\ln Z = N_L \ln y$$

where the function y satisfies to

$$y - z e^{-\beta\omega_c} y^{1-\alpha} = 1$$

$\Rightarrow$ LLL-anyon thermodynamics is

$$\ln Z = N_L \ln y \oplus y - z e^{-\beta\omega_c} y^{1-\alpha} = 1$$

mean energy $\bar{E} = -\frac{\partial \ln Z}{\partial \beta}$ and mean particle number $\bar{N} = z\frac{\partial \ln Z}{\partial z}$

y function of $ze^{-\beta\omega_c} \Rightarrow \bar{E} = \bar{N}\omega_c$ i.e. LLL

$$\Rightarrow \text{ filling factor } \nu = \frac{\bar{N}}{N_L} = z\frac{\partial \ln y}{\partial z}$$

$$y - ze^{-\beta\omega_c} y^{1-\alpha} = 1 \Rightarrow y = \frac{1+(1-\alpha)\,\nu}{1-\alpha\,\nu} \Rightarrow ze^{-\beta\omega_c} = \frac{\nu}{(1+(1-\alpha)\nu)^{1-\alpha}(1-\alpha\,\nu)^{\alpha}}$$

$$\alpha = 0 \Rightarrow \nu = \frac{ze^{-\beta\omega_c}}{1+ze^{-\beta\omega_c}} \quad \text{BE}$$

$$\alpha = 1 \Rightarrow \nu = \frac{ze^{-\beta\omega_c}}{1-ze^{-\beta\omega_c}} \quad \text{FD}$$

equation of state $\ln Z = N_L \ln y \Rightarrow \ln Z = N_L \ln(\frac{1+(1-\alpha)\,\nu}{1-\alpha\,\nu})$

$$\Rightarrow \ln Z = \beta P S = N_L \ln(\frac{1+(1-\alpha)\,\nu}{1-\alpha\,\nu})$$

$\Rightarrow$ critical filling $\nu = 1/\alpha$ where the pressure P is infinite

$\Rightarrow$ at critical filling as many anyons as possible per LLL quantum state

$$\text{entropy}\ \ \mathbf{S} = \ln Z + \beta \bar{E} - (\ln z)\bar{N} = N_L \ln \frac{(1+\nu(1-\alpha))^{1+\nu(1-\alpha)}}{(1-\nu\alpha)^{1-\nu\alpha}\ \ \nu^{\nu}}$$

at critical filling entropy vanishes

$\Rightarrow 0 = \ell_1 = \ell_2 = \ldots = \ell_N \Rightarrow$ non degenerate N-anyon groundstate so from

$$\psi' = e^{-\frac{1}{2}\omega_c \sum_{i=1}^{N} z_i \bar{z}_i} \prod_{i<j} {z_{ij}}^{\alpha} \prod_{i=1}^{N} {z_i}^{\ell_i} \qquad 0 \leq \ell_1 \leq \ell_2 \leq ... \leq \ell_N = 0$$

at critical filling $0 = \ell_1 = \ell_2 = \ldots = \ell_N$

$$\psi' = e^{-\frac{1}{2}\omega_c \sum_{i=1}^{N} z_i \bar{z}_i} \prod_{i<j} {z_{ij}}^{\alpha} \qquad \nu = \frac{1}{\alpha}$$

Laughlin wavefunction at FQHE filling $\nu = \frac{1}{2m+1}$

$$\psi_{\text{Laughlin}} = e^{-\frac{1}{2}\omega_c \sum_{i=1}^{N} z_i \bar{z}_i} \prod_{i<j} {z_{ij}}^{2m+1} \qquad \nu = \frac{1}{2m+1}$$

$$\nu \leq 1/\alpha$$

$$\nu = N/N_L \Rightarrow 0 \leq N_L - N\alpha$$

$\Rightarrow$ Haldane exclusion statistics

at critical filling $N_L = N\alpha$

$\alpha = \phi/\phi_o \Rightarrow N\phi = N_L\phi_o = BS$ total screening

$$Z = \sum z^N Z_N = y^{N_L} \Rightarrow Z_N = e^{-\beta N \omega_c} N_L \prod_{k=2}^{N} \frac{k + N_L - N\alpha - 1}{k}$$

N anyons in the LLL energy $N\omega_c$ degeneracy

$$N_L \prod_{k=2}^{N} \frac{k + N_L - N\alpha - 1}{k} = \frac{N_L}{N!} \frac{(N + N_L - N\alpha - 1)!}{(N_L - N\alpha)!}$$

$\alpha = 1, 2, 3, \ldots$ counts number of ways to put on a circle N particles in N_L quantum states such that there are at least $\alpha - 1$ empty states in between two occupied states

= exclusion statistics

a last remark : $\alpha \in [0,2]$

but because of B field

$$\alpha \in [0,1] \neq \alpha \in [2,1]$$

when $\alpha \to 2^-$ $\nu \to 1/2 =$ Bose gaz at critical filling $1/2$

LLL-anyon thermodynamics is

$$\ln Z = N_L \ln y \quad \oplus \quad y - z e^{-\beta\omega_c} y^{1-\alpha} = 1$$

1-body LLL density of states $\rho_{LLL}(\varepsilon) = N_L \delta(\varepsilon - \omega_c)$

$$\Rightarrow \ln Z = \int_0^\infty \rho_{LLL}(\varepsilon) \ln y(\varepsilon)\, d\varepsilon \quad \oplus \quad y(\varepsilon) - z e^{-\beta\varepsilon} y(\varepsilon)^{1-\alpha} = 1$$

factorization can be generalized to

$$\ln Z = \int_0^\infty \rho(\varepsilon) \ln y(\varepsilon)\, d\varepsilon \quad \oplus \quad y(\varepsilon) - z e^{-\beta\varepsilon} y(\varepsilon)^{1-\alpha} = 1$$

mean occupation number $n(\varepsilon)$ per quantum state at energy ε

$$\bar{N} = z\frac{\partial \ln Z}{\partial z} = \int_0^\infty \rho(\varepsilon) n(\varepsilon)\, d\varepsilon$$

LLL-anyons :

ν filling factor = mean occupation number per LLL state energy ω_c

$$\nu \to n(\varepsilon)$$

so

$$ze^{-\beta\omega_c} = \frac{\nu}{(1+(1-\alpha)\nu)^{1-\alpha}(1-\alpha\,\nu)^{\alpha}} \to ze^{-\beta\varepsilon} = \frac{n(\varepsilon)}{(1+(1-\alpha)n(\varepsilon))^{1-\alpha}(1-\alpha\, n(\varepsilon))^{\alpha}}$$

is there another microscopic N-body model which would have this type of thermodynamics ?

yes : the Calogero model

Calogero thermodynamics : again add 1d confining harmonic well to lift the degeneracy

$\Rightarrow$ N-body harmonic Calogero spectrum

$$E_N = \omega \left[\sum_{i=1}^{N} \ell_i + \frac{N(N-1)}{2} g \right] + \frac{N}{2}\omega \quad 0 \le \ell_1 \le \ell_2 \le \ldots \le \ell_N$$

in the thermodynamic limit $\beta\omega \to 0$ use again thermodynamic limit prescription in 2d one had

$$\frac{1}{n(\beta\omega)^2} \rightarrow \frac{S}{2\pi\beta}$$

S infinite surface of 2d plane

in 1d

$$\frac{1}{\sqrt{n}\beta\omega} \rightarrow \frac{L}{\sqrt{2\pi\beta}}$$

L infinite length of 1d line

$\Rightarrow$ one obtains the Calogero n-th cluster coefficient

$$b_n = \frac{L}{\sqrt{2\pi\beta n}} \frac{1}{n} \prod_{k=1}^{n-1} \frac{k-ng}{k}$$

from which one wants to compute $\ln Z = \sum_{n=1}^{\infty} b_n z^n$

remember LLL-anyon n-th cluster coefficient

$$b_n = N_L \frac{e^{-n\beta\omega_c}}{n} \prod_{k=1}^{n-1} \frac{k-n\alpha}{k} \Rightarrow \ln Z = N_L \ln y \oplus y - ze^{-\beta\omega_c} y^{1-\alpha} = 1$$

the Calogero n-th cluster coefficient has an extra

$$\frac{1}{\sqrt{2\pi\beta n}} = \frac{1}{2\pi} \int_{-\infty}^{\infty} dp \;\; e^{-n\beta \frac{p^2}{2}}$$

$$b_n = \frac{L}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-n\beta \frac{p^2}{2}}}{n} \prod_{k=1}^{n-1} \frac{k-ng}{k} \, dp \Rightarrow \ln Z = \frac{L}{2\pi} \int_{-\infty}^{\infty} \ln y \, dp \oplus y - ze^{-\beta \frac{p^2}{2}} y^{1-g} = 1$$

$p =$ 1d plane wave momentum energy $\varepsilon = p^2/2$

$$\Rightarrow \ln Z = \int_0^\infty \rho_0(\varepsilon) \ln y(\varepsilon)\, d\varepsilon \oplus y(\varepsilon) - z e^{-\beta\varepsilon} y(\varepsilon)^{1-g} = 1$$

with

$$\rho_0(\varepsilon) = \frac{L}{\pi\sqrt{2\varepsilon}}$$

the free 1-body density of states in 1d

nothing but in the thermodynamic limit Hermite polynomial

$$H_{\ell_i}(\sqrt{\omega} x_i) e^{-\frac{1}{2}\omega x_i^2} \quad \text{energy } \omega(\ell_i + \frac{1}{2})$$

$\omega \to 0$ and $\ell_i \to \infty$ with $\ell_i \omega \longrightarrow p_i^2/2$

$H_{\ell_i}(\sqrt{\omega} x_i) \underset{\omega\to 0}{\longrightarrow}$ plane wave $\cos(p_i x_i)$

so same factorization for LLL-anyon and Calogero thermodynamics ($g = \alpha$)

$$\ln Z = \int_0^\infty \rho(\varepsilon) \ln y(\varepsilon)\, d\varepsilon \oplus y(\varepsilon) - z e^{-\beta\varepsilon} y(\varepsilon)^{1-\alpha} = 1$$

LLL-anyon : $\rho(\varepsilon) \to \rho_{\mathrm{LLL}}(\varepsilon) = N_L \delta(\varepsilon - \omega_c)$

Calogero : $\rho(\varepsilon) \to \rho_0(\varepsilon) = \frac{L}{\pi\sqrt{2\varepsilon}}$

not a surprise : as we know in an harmonic well LLL-anyon and Calogero have similar spectrum

$$E_N = (\omega_t - \omega_c)\left[\sum_{i=1}^{N} \ell_i + \frac{N(N-1)}{2}\alpha\right] + N\omega_t \qquad 0 \le \ell_1 \le \ell_2 \le \ldots \le \ell_N$$

$$E_N = \omega\left[\sum_{i=1}^{N} \ell_i + \frac{N(N-1)}{2}\alpha\right] + \frac{N}{2} \qquad 0 \le \ell_1 \le \ell_2 \le \ldots \le \ell_N$$

identical when $\omega_c = 0$

first project on LLL then $B \to 0$

once in the LLL the $B \to 0$ limit induces dimensional reduction 2d $\to$ 1d :

anyons are 2d

LLL-anyons are still 2d :

eventhough in the LLL there is only one quantum number per particle = angular momentum $\ell_i \geq 0$ the LLL-anyon thermodynamical potential

$$\ln Z = N_L \ln y$$

scales like the infinite 2d surface S of the plane

$$N_L = BS/\phi_0 \Rightarrow 2\text{d}$$

$$\text{when } B \to 0 : 2\text{d} \to 1\text{d}$$

for the thermodynamical potential this means

$$\int_0^\infty \rho_{LLL}(\varepsilon) \ln y(\varepsilon)\, d\varepsilon \xrightarrow[B\to 0]{} \int_0^\infty \rho_0(\varepsilon) \ln y(\varepsilon)\, d\varepsilon$$

so for the density of states

$$\rho_{LLL}(\varepsilon) \xrightarrow[B\to 0]{} \rho_0(\varepsilon)$$

i.e.

$$N_L \delta(\varepsilon - \omega_c) \xrightarrow[B\to 0]{} \frac{L}{\pi\sqrt{2\varepsilon}}$$

which is

$$\frac{eBS}{2\pi} \delta(\varepsilon - \frac{eB}{2}) \xrightarrow[B\to 0]{} \frac{L}{\pi\sqrt{2\varepsilon}}$$

seems per se meaningless

becomes meaningful thanks to harmonic well regularization

nothing but

$$2d - \text{LLL} \;\; \omega_c \xrightarrow{\omega \neq 0} (\omega_t - \omega_c)\ell_i + \omega_t \xrightarrow{B \to 0} \omega(\ell_i + 1) \xrightarrow{\omega \to 0} 1d - \text{plane wave} \;\; p_i^2/2$$

$$\text{gap} \; = \; \frac{\omega^2}{2\omega_c} \xrightarrow{B \to 0} \text{gap} \; = \; \omega$$

question : can we do the same thing for the Calogero-Sutherland model ?

= Calogero on a circle

N-body eigenstates = Jack polynomials

2d anyons $\to$ 1d Calogero Sutherland ?

long distance regulator ω replaced by R radius of the circle

thermodynamic limit $\omega \to 0$ is $R \to \infty$

anyons on a sphere radius R : difficult

ongoing work