Archipelagic perspectives on mathematics, physics

and perceptible spectra of reality

Djurönäset, 30 August - 3 September, 2021

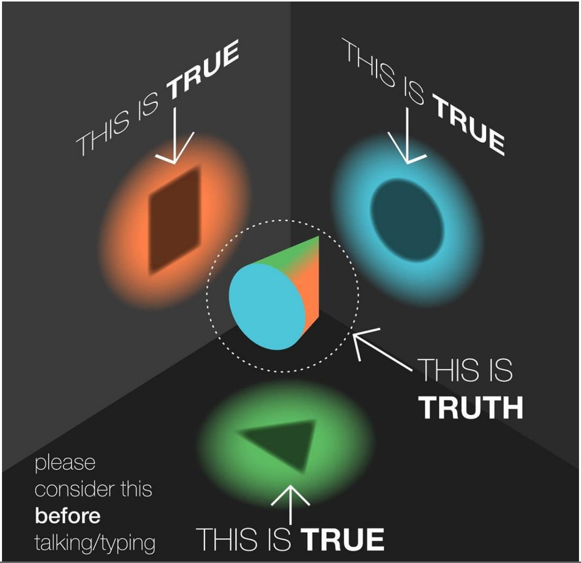
Göran Gustafsson Foundation

UPPSALA UNIVERSITET

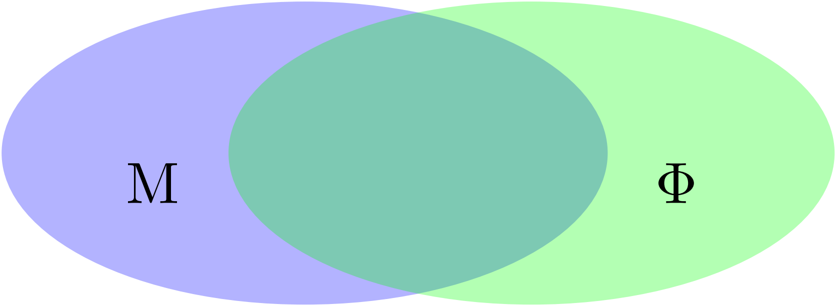
Nordita, March 2019

Archipelago

Perspectives



Math $\leftrightarrow$ Phys



More?

PERCEIVABLE vs. PERCEPTIBLE [Quora]

Perceptible and *perceivable* both relate to *perceiving* (to become aware of or become conscious of by some means). The difference is that one of them has a mental component whereas the other doesn't.

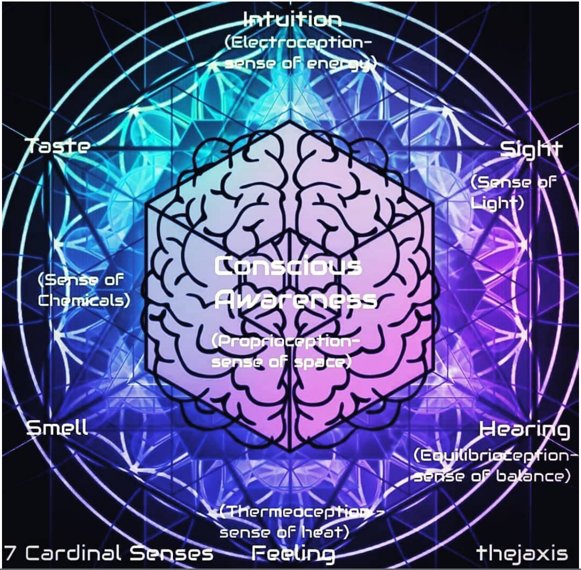
Perceivable is being detectable chiefly **by sight or hearing**. There is no mental or intellectual component involving the 'mind' — just the senses.

- *a silhouette perceivable through the mist*

Perceptible is the one that additionally includes a 'mind' component — capable of or easily **detectable by the senses**, or easily **grasped by the mind**. Since this one covers both sensory and mental means, this is always the safer bet.

- *perceptible changes in behaviour — because you can 'sense' it in some way*
- *perceptible differences in the two concepts — you can mentally grasp them*

Spectra of reality? Subjective vs. Objective



What is real? How do you define Real?

What is real? How do you define Real?

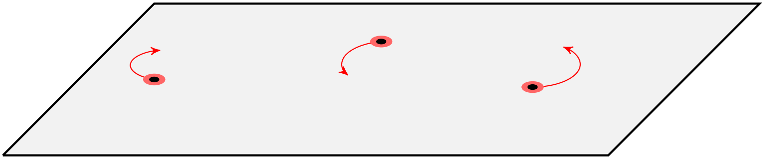
Story time...

Outline

1. Some preliminaries: identity, QM, observable, contextuality ...

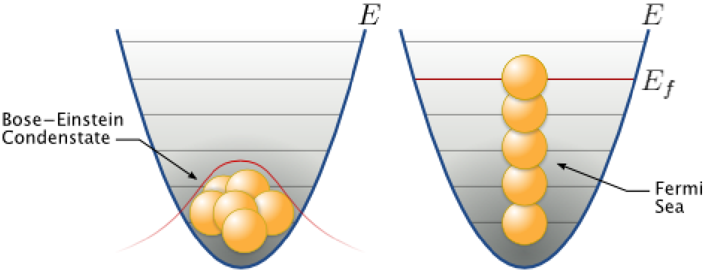
2. Workshop info

Perspective boils down to identity (logical or experienced)



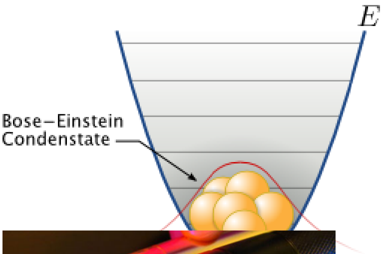
Locally identifiable $\rightarrow$ individualized/localized
Globally unidentifiable $\rightarrow$ unified/omnipresent

Quantum statistics: bosons – fermions – anyons

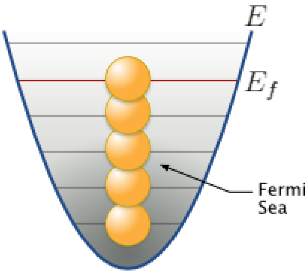


Cold **bosons** Cold **fermions**

Quantum statistics: bosons – fermions – anyons

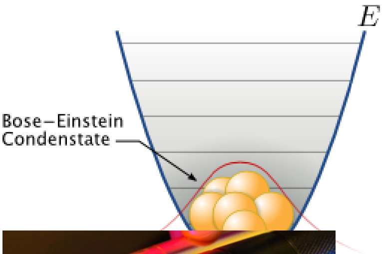


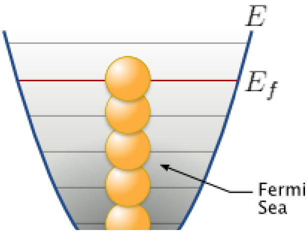
coherence $\rightarrow$ optics



Cold **fermions**

Quantum statistics: bosons – fermions – anyons





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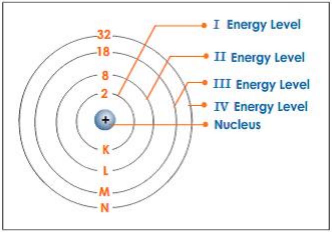
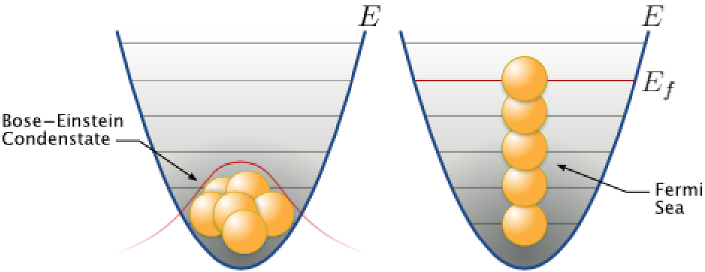


Pauli's exclusion $\rightarrow$ chemistry

Quantum statistics: bosons – fermions – anyons

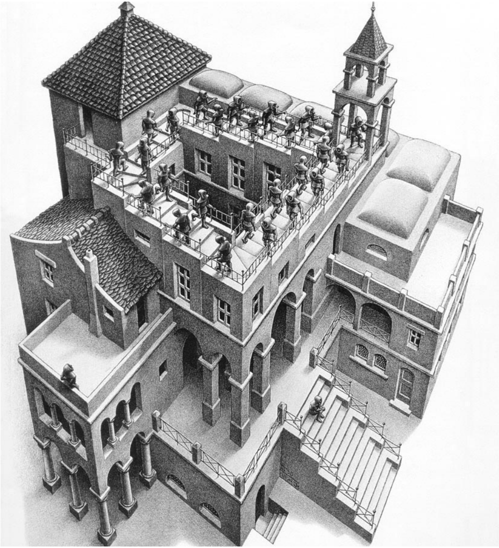


Cold **bosons** $\uparrow$ Cold **fermions**

"anyons"?

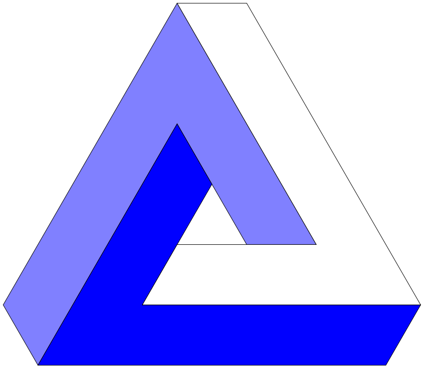
Math-phys problem since 1977...

What is QM?



Escher, Ascending and descending

What is QM?



Penrose-Reutersvärd tribar, 1934
"Impossibility in its purest form."

Observables

- information that subsystems have about each other
- observable aspect $\rightsquigarrow$ definite value/data/outcomes (log. excl.)

$$\begin{array}{lll} A \;\; B \;\; C \in \hat{\mathcal{O}} \subseteq \hat{\mathcal{A}} & & \text{(partial) algebra of obss} \\ \wr\!\downarrow \;\; \wr\!\downarrow \;\; \wr\!\downarrow \qquad \Psi \in \mathcal{S} & & \text{states} \\ a \;\;\; b \;\;\; c \;\in \mathcal{O} \subseteq \mathcal{A} & & \text{comm. algebra of outcomes} \end{array}$$

- **obss** describe possible info/knowledge obtainable from the sys.
- **states** describe the actual info/knowledge, i.e. current 'reality'
- "**measurement**" projects possibilities to actualities

$$\mathbb{P}(A \overset{\Psi}{\rightsquigarrow} a) = \langle\Psi|a\rangle\langle a|\Psi\rangle / \left\|\Psi\right\|^2 , \qquad \Psi \mapsto |a\rangle\langle a|\Psi$$

- obss A, B **commensurable** if $[A, B] = 0$ ($\Rightarrow$ sim. knowledge) (uncertainty principle)

Observables

Ex:

$$X = (+1)|\uparrow\rangle\langle\uparrow| + (-1)|\downarrow\rangle\langle\downarrow| = \begin{bmatrix} +1 & 0 \\ 0 & -1 \end{bmatrix} \rightsquigarrow x \in \{+1, -1\}$$

$$P = |\downarrow\rangle\langle\uparrow| + |\uparrow\rangle\langle\downarrow| = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \rightsquigarrow p \in \{+1, -1\} = \sigma(P)$$

observables $\hat{\mathcal{O}} = \mathrm{Span}_{\mathbb{R}}\{1, X, P\} \subset \hat{\mathcal{A}} = \mathbb{C}^{2\times 2}$

states $\Psi = \alpha|\uparrow\rangle + \beta|\downarrow\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \in \mathcal{S} = \mathbb{C}^2$

non-commensurable: $[X, P] = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \notin \hat{\mathcal{O}}$

Ex: position $\hat{x} = \int_{\mathbb{R}} x|x\rangle\langle x|dx \;\rightsquigarrow\; x \in \mathbb{R}$, $\Psi \in \mathcal{S} = L^2(\mathbb{R})$
momentum $\hat{p} = -i\partial_x \;\rightsquigarrow\; p \in \mathbb{R}$, $[\hat{x}, \hat{p}] = i1$ (Heisenberg u. p.)

Observables: N-particle systems

Ex: (distinguishable) $\hat{x}_{jk}$ commensurable but not with $\hat{H}$

$$\hat{\mathrm{x}} \rightsquigarrow \mathrm{x} = (x_{jk})_{j=1}^{N}{}_{k=1}^{d} \in \mathbb{R}^{Nd} = \sigma(\hat{\mathrm{x}}), \quad \hat{H} = \sum_{j,k} \hat{p}_{jk}^2 = -\Delta \rightsquigarrow \mathrm{p}^2$$

$$\Psi \in L^2(\mathbb{R}^{Nd}), \qquad \mathbb{P}(\hat{\mathrm{x}} \overset{\Psi}{\rightsquigarrow} \mathrm{x}) = |\Psi(\mathrm{x})|^2$$

Ex: (identical)

$$\hat{\mathrm{x}} \rightsquigarrow \mathrm{x} \in \left(\mathbb{R}^{Nd} \setminus \triangle\right) /_{S_N} = \sigma(\hat{\mathrm{x}}), \qquad \hat{H} = -\Delta^{\mathcal{A}}$$

$$\Psi \in \Gamma\left(E \to \sigma(\hat{\mathrm{x}}); \mathbb{C}^D\right)$$

$\mathcal{A}$ locally flat connection on $U(D) \Rightarrow$ bosons, fermions or anyons

Observables: angular momentum

$$L_1 := \hat{x}_2\hat{p}_3 - \hat{x}_3\hat{p}_2 \qquad \tfrac{1}{i}[L_1, L_2] = L_3 \text{ cycl.} \rightsquigarrow \{-\ell, -\ell+1, \dots, \ell\}$$

$$L_2 := \hat{x}_3\hat{p}_1 - \hat{x}_1\hat{p}_3 \qquad \mathbf{L}^2 := L_1^2 + L_2^2 + L_3^2 \rightsquigarrow C_{\text{rep}} = \ell(\ell+1)$$

$$L_3 := \hat{x}_1\hat{p}_2 - \hat{x}_2\hat{p}_1 \qquad \text{spin } \ell \in \mathbb{Z}/2$$

$$\ell = \tfrac{1}{2}: \quad L_1 = \tfrac{1}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad L_2 = \tfrac{1}{2}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad L_3 = \tfrac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

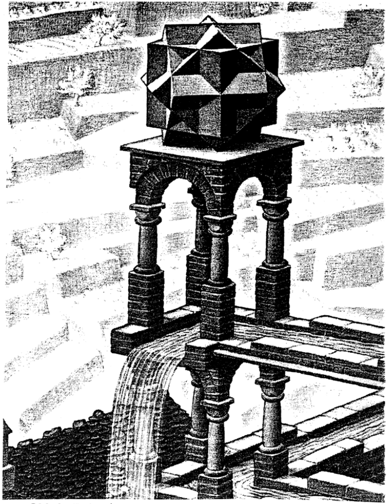
$$\ell = 1: L_1 = i\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \; L_2 = i\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \; L_3 = i\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$L_1^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad L_2^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad L_3^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$[L_j^2, L_k^2] = 0 \; \forall j, k$ only for $\ell \in \{0, \frac{1}{2}, 1\}$

Bell-Kochen-Specker paradox

N. David Mermin: Hidden variables and the two theorems of John Bell 809



©M. C. Escher / Cordon Art – Baarn – Holland.

FIG. 2. The tower on the left of M. C. Escher's engraving "Waterfall." © M. C. Escher/ Cordon Art, Baarn, Holland. The ornament atop the tower consists of three superimposed cubes. One of the cubes has all its edges horizontal or vertical. The other two are given by rotating this one through 90 degrees about each of the two perpendicular horizontal lines that connect the midpoints of opposite vertical edges. The 33 uncolorable directions used in the proof of the Bell-KS theorem in Peres, 1991, lie along the lines connecting the common center of the cubes to their vertices and the centers of their edges and faces.

Bell-Kochen-Specker paradox

Family of obss: $S^2 \ni \mathbf{e} \mapsto L_{\mathbf{e}} = e_1 L_1 + e_2 L_2 + e_3 L_3,\ L_{\mathbf{e}}^2$

A function $f \colon E \subseteq S^2 \to \{0,1\}$ has the **101 property** if

- $f(\mathbf{e}) = f(-\mathbf{e})\ \forall \mathbf{e} \in E$,
- for any orthonormal frame $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\} \subseteq E$ in $\mathbb{R}^3$,

$$\big(f(\mathbf{e}_1), f(\mathbf{e}_2), f(\mathbf{e}_3)\big) \in \big\{(0,1,1), (1,0,1), (1,1,0)\big\}$$

Theorem (BKS paradox): Let $E \subset S^2$, $|E/_\sim| = 33$, be the Peres subset of directions. There exists no function $f \colon E \to \{0,1\}$ with the 101 property.

$\Rightarrow$ no hidden variables! / no objective reality!

hidden variables :$\Leftrightarrow$ $\exists$ imbedding of p.a. $\hat{A} \hookrightarrow$ comm.a. $\mathcal{A}$

Contextuality [Abramsky,Barbosa,Mansfield,2017]

$(X, \mathcal{M}, O)$:
measurements $X = \{a, \alpha, b, \beta\}$, outcomes $O = \{0, 1\}$,
measurement contexts $C \in \mathcal{M} = \big\{\{a, b\}, \{a, \beta\}, \{\alpha, b\}, \{\alpha, \beta\}\big\}$
(commensurable measurements, i.e. can be performed together)

empirical model $e\colon C \mapsto e_C$:
probability dist. on meas. outcomes: $O^C \ni t \mapsto e_C(t) \in [0, 1]$
marginalization: for $U \subseteq C$, $t \in O^U$,

$$e_C|_U(t) := \sum_{s \in O^C,\, s|_U = t} e_C(s)$$

demand compatibility of all marginals: (cf. sheaf)

$$\forall C, C' \in \mathcal{M} \qquad e_C|_{C \cap C'} = e_{C'}|_{C \cap C'}$$

Contextuality [Abramsky,Barbosa,Mansfield,2017]

noncontextual: $\exists$ global assignment of outcomes to all meass (h.v.)

$$\exists d \in O^X \text{ s.t. } \forall C \in \mathcal{M} \;\; d|_C = e_C$$

strongly contextual: $\nexists g \in O^X$ s.t. $\forall C \in \mathcal{M} \;\; e_C(g|_C) > 0$
(non)**contextual fraction** can be computed via linear prog.:

$$e = (1-\lambda)e^{NC} + \lambda e^{SC}, \qquad \lambda \in [0,1]$$

Bell inequality: $\displaystyle\sum_{C\in\mathcal{M},\, t\in O^C} a_{(C,t)} e_C(t) \;\leq\; R$

Theorem: Contextuality must be present in an empirical model whenever it admits a nonlinear function to be computed with a sufficiently large probability of success. The higher the desired success probability, the larger the contextual fraction must be.

Reality is

Locally	**Globally**
flat	curved
linear / chronological	nonlinear / kairological
unentangled	entangled
individualized	unified
noncontextual	contextual
logically consistent	logically inconsistent
subjective	approximately objective?

Consciousness as a flow towards objectivity / logical consistency?

Satire on false perspective

"Whoever makes a Design without the Knowledge of Perspective will be liable to such Absurdities as are shewn in this Frontispiece."
William Hogarth, 1754

Strange loop / Self-entanglement

$$0 \rightarrow \infty \rightarrow$$

Strange loop / Self-entanglement

This workshop

A forum for exchanging and discussing the most interesting ideas we have come across — with great setting & minimal constraints!

Important this week

- Staff: Lena (here), Johanna Danielsson (there)
- Operator 08-571 490 60
- Access
- Breakfasts
- Lunches
- Dinners
- Facilities
- Work

Schedule updates

- Monday:
- Tuesday:
- Wednesday:
- Thursday:
- Friday:

Enjoy!