The good, the bad and the ugly: quantum tunneling, black holes and NH systems

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Outline

1.1. Motivation (1/3): Black Holes (BHs)

- Thermal systems (T, S).
- Hawking temperature

$$T_{BH} \sim \frac{1}{M_{BH}} \ ,$$

- Bekenstein-Hawking entropy

$$S_{BH} \sim \frac{A_{BH}}{4} .$$

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- BH entropy problem: microstate counting $S_{BH} = \ln \Omega$.
- BH information paradox: pure state $\rightarrow$ mixed thermal state (Hawking radiation) $\rightarrow$ information loss? (would violate QM's unitarity).
- Towards a UV-complete theory of Quantum Gravity?

1. Motivation (2/3): Weyl semimetals (WSMs) & BHs

- WSMs:
  - **CM physics** $\rightarrow$ Electric transport in solids:
    - insulators
    - conductors
    - superconductors
    - semiconductors
    - semimetals $\rightarrow$ **WSMs**
  - Topological materials $\rightarrow$ interesting band structure.

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    - semimetals $\to$ **WSMs**
  - Topological materials $\to$ interesting band structure.

- WSMs & BHs
  - BH analogy:
    - Shape of the band structure $\leftrightarrow$ Spacetime lightcone.
    - Phases of WSMs $\leftrightarrow$ Infalling observer crossing the horizon.
    - Tilt of the Weyl cone $\leftrightarrow$ Tilt of the lightcone of infalling observer (timelike/spacelike) [Kedem, Bergholtz, Wilczek '20].

1. Motivation (3/3): Non-Hermitian (NH) systems

- NH Hamiltonians as **effective descriptions** for dissipative systems (Lindbladian formalism).
- Quantum Gravity in the lab:
  - Using analogue models to mimic or simulate BH physics in a controlled setup.
  - NH systems seem to be a natural candidate for an analogue model to host dissipative processes such as Hawking radiation.
- Aim: Construct analogue NH model displaying Hawking radiation.

2.1. BH Thermodynamics

Similar laws: BH Mechanics (70's) & Thermodynamics (Schwarzschild BH)

Law	Thermodynamics	BH Mechanics
0th	T const. in thermal equilibrium	surface gravity κ const.
1st	$\delta E = T\delta S$	$\delta M = \frac{\kappa}{8\pi G}\delta A$
2nd	$\delta S \geq 0$	$\delta A \geq 0$
3rd	$\nexists$ physical process $T \to 0$	$\nexists$ physical process $\kappa \to 0$

2.2. BH are not that black

- **BH temperature**: Black body radiation at the Hawking temperature:

$$T_{BH} = \frac{\kappa}{2\pi}.$$

- **BH entropy**: Associate an entropy from 1st law $\rightarrow$ Bekenstein-Hawking formula

$$S_{BH} = \frac{A}{4G_D}.$$

  - Entropy not extensive (area not volume) $\rightarrow$ Holography (AdS/CFT).
  - Large # dof $\rightarrow$ requirement of the CFT.

2.3. Hawking effect

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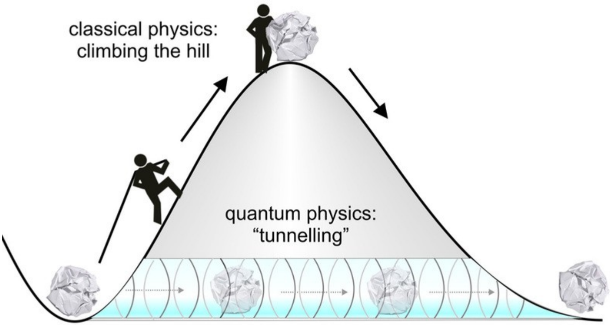
- QFT: The notion of particle is observer-dependent! (e.g. Unruh effect).
- BHs emmit radiation: Particle production, i.e., outgoing flux of quanta.
- Hawking's calculation based on QFT in curved spacetime:
  - Consider scalar field in curved background (also done for spin particels, gauge fields).
  - Bogiliubov transformations relating mode expansions of quantum fields between infalling and asymptotic observer.
  - Finite number of particles w.r.t. the vacuum state of the asymptotic observer [Hawking '75].
  - Result: Thermal Panckian spectrum

$$N \sim \frac{1}{e^{\frac{2\pi\omega}{\kappa}} - 1} \sim \frac{1}{e^{\frac{\beta\omega}{\kappa}} - 1},$$

where $\beta := 1/T$ and hence $T_{BH} = \frac{\kappa}{2\pi} = \frac{1}{8\pi M}$.

3. Quantum tunneling

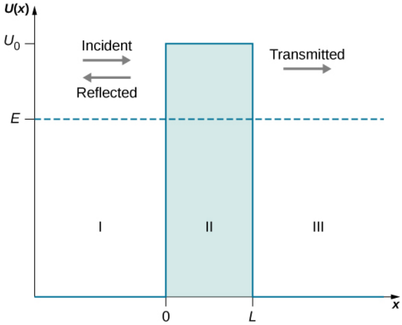


- Classically forbiden phenomenon: particles "roll" on a potential curve.
- In QM: particles can tunnel trhough a potential barrier.
- Can be understood in terms of de Broglie's wave-particle duality $\lambda = h/p$.
- Simple example: particle with finite potential barrier.

3.1. Quantum tunneling through potential barriers (1/2)

- Finite potential barrier of height U_0 and width L

$$U(x) = U_0\,\theta(x) - U_0\,\theta(x-L).$$



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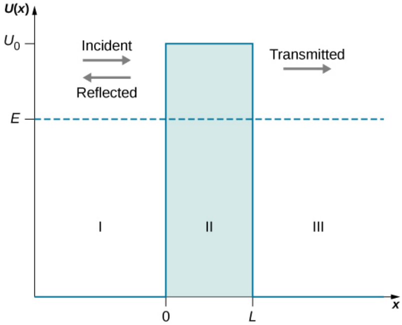
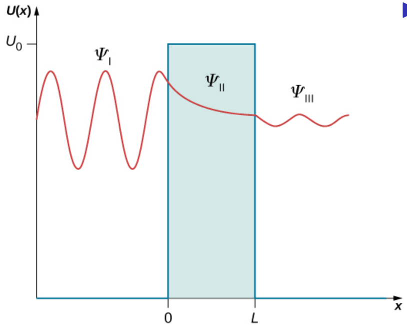


- Solve Schrödinger equation $\frac{-1}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$, for $\psi_I(-\infty < x < 0)$, $\psi_{II}(0 < x < L)$ and $\psi_{III}(L < x < +\infty)$.
- Boundary conditions:
  - continuity $\psi_I(0) = \psi_{II}(0),\ \psi_{II}(L) = \psi_{III}(L)$,
  - smoothness $\psi_I'(0) = \psi_{II}'(0), \psi_{II}'(L) = \psi_{III}'(L)$.

3.1. Quantum tunneling through potential barriers (2/2)



- Oscillatory behavior:
  - region I: Incident and reflected wave
$$\psi_I(x) = Ae^{ikx} + Be^{-ikx},$$
  - region III: Transmitted wave
$$\psi_{III}(x) = Ce^{ikx},$$

where $k := \sqrt{2mE}$ and $A, B, C \in \mathbb{C}$.

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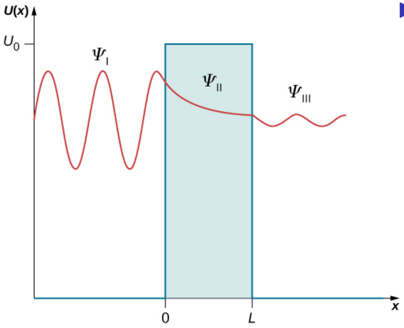

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- Attenuating behavior, region II:

$$\psi_{II}(x) = De^{-2m(U_0-E)x},$$

where $U_0 - E > 0$ and $D \in \mathbb{C}$.

- Transmission coeficient (tunneling probability) $T(U_0, L, E) = |C|^2/|A|^2$.

3.2. Hawking radiation as quantum tunneling (1/4)

[Parikh, Wilczek '99]

- Alternative derivation of the Hawking effect as a tunneling process.
- Relates ImS (clasically forbidden processes) to $e^{-\beta\omega}$ (Boltzmann factor characteristic of thermal systems at eq.).

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- Limitations: neglecting backreaction (semiclassical) and thus considering an eq. process (slow BH evaporation, large M).
- Massless Hawking radiation: Hawking radiation can also be thought in terms of arbitrary spin massive particles and gauge fields.
- WKB approximation (semicalssical treatment): radiation modelled as s-wave (wavefunction as planar wave solution, slow-varying potential)

$$\psi(x) = e^{iS(x)} \simeq \frac{A}{\sqrt{p(x)}} e^{\pm i \int_{x_0}^{x} p(x')dx'},$$

where $S \in \mathbb{C}$, $p(x)$ is the momentum and $A \in \mathbb{R}$ is const.

3.2. Hawking radiation as quantum tunneling (2/4)

- Method:
  - 1) Start from 4-D Schwarszchild metric in Gustrand-Painlevé coordinates

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dT^2 + 2\sqrt{\frac{2M}{r}} dT \; dr + dr^2 + r^2 d\Omega^2.$$

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  - 3) 2 types of contributions:
    - Outgoing shell of energy ω (massless particles) $M \to M - \omega$.
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  - 3) 2 types of contributions:
    - Outgoing shell of energy ω (massless particles) $M \to M - \omega$.
    - Ingoing shell of energy $-\omega$ (massless antiparticles) $M \to M + \omega$.
  - 4) Compute $\mathrm{Im}S$ (for particles in the following)

$$\begin{aligned}\mathrm{Im}S = \mathrm{Im} \int_{r_{in}}^{r_{out}} p_r dr &= \mathrm{Im} \int_{M}^{M-\omega} \int_{r_{in}}^{r_{out}} \frac{dr}{\dot{r}} dH \\ &= \mathrm{Im} \int_0^{\omega} \int_{r_{in}}^{r_{out}} \frac{dr}{1 - \sqrt{\frac{2(M-\omega')}{r}}} (-d\omega'),\end{aligned}$$

using Hamilton's canonical eq. $\dot{r} = \frac{dH}{dp_r}$, $H = M - \omega$ and the modified radial geodesics.

3.2. Hawking radiation as quantum tunneling (3/4)

- 5) Complex contour integral: $\omega' \to \omega' - i\epsilon$ to ensure positive energy solutions decay in time.
  Result:
  $$\mathrm{Im}S = 4\pi\omega\left(M - \frac{\omega}{2}\right).$$

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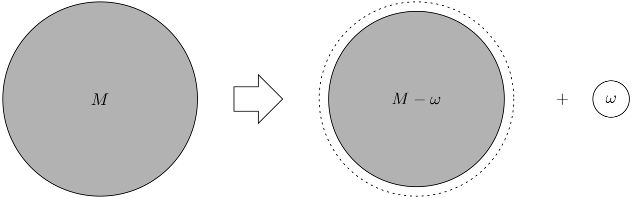
  Result:
$$\mathrm{Im}S = 4\pi\omega \left(M - \frac{\omega}{2} \right).$$

- 6) It can be shown that same contribution appears from antiparticles
$$\mathrm{Im}S = \mathrm{Im} \int_{r_{out}}^{r_{in}} p_r dr = \mathrm{Im} \int_0^{-\omega} \int_{r_{out}}^{r_{in}} \frac{dr}{-1 + \sqrt{\frac{2(M+\omega')}{r}}} d\omega'$$
$$= 4\pi\omega \left(M - \frac{\omega}{2} \right).$$



3.2. Hawking radiation as quantum tunneling (4/4)

- 7) Semiclassical emission rate: adding the two channels

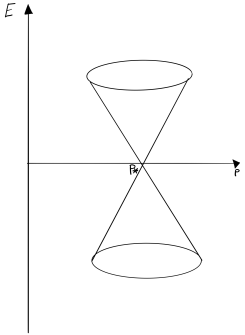
$$\Gamma \sim e^{2\mathrm{Im}S} = e^{-8\pi\omega(M-\omega/2)}.$$

  - From the first term, by identifying with the Boltzman factor $e^{-8\pi\omega M} \sim e^{-\beta\omega}$ we obtain the Hawking temperature

$$T_{BH} = \frac{1}{8\pi M}.$$

  - The correction term with ω^2 raises the effective temperature of the BH as it radiates (accelerated evaporation).

4.1. Weyl systems (1/2)



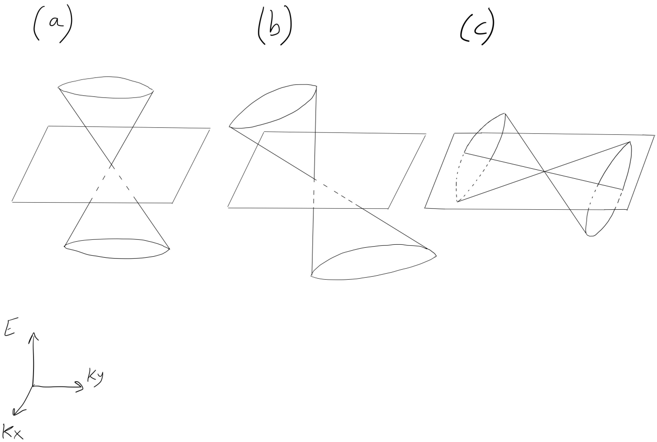
- Special electronic band structure $\rightarrow$ Weyl cone.
- Semimetal (valence and conduction bands intersect at the Weyl point p_*).
- Around the Weyl point: effective low-energy description as free Weyl fermions (HEP/CM) $\rightarrow$ Weyl Hamiltonian (linear dispersion relation)

$$H = a(p)\sigma_0 + \vec{b}(p) \cdot \vec{\sigma},$$

with $\vec{b}(p = p_*) = 0 \rightarrow 3$ eqs. (degeneracy condition).

4.1. Weyl systems (2/2)

- Tilting: $\nu = \left.\frac{da(p)}{dp}\right|_{p=p_*}$
  - (a) Untilted ($\nu = 0$)
  - (b) Type I ($|\nu| < 1$)
  - (c) Type II ($|\nu| > 1$)



4.2. From Weyl sytems to BHs

- Recall tilting of the cone dispersion in WSMs: at the critical tilt the Weyl cone resembles the (tilted) lightcone of an infalling observer in the vicinity of a BH horizon.
- Mapping WSMs to BHs and WHs [Volovik '16]. Mathematical mapping between the Weyl Hamiltonian and spacetime metric via the vielbein formalism.

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- Mapping WSMs to BHs and WHs [Volovik '16]. Mathematical mapping between the Weyl Hamiltonian and spacetime metric via the vielbein formalism.
- In GR: relation between a metric describing curved spacetime g_{ij} and Minkowski spacetime $\eta_{\mu\nu}$ in terms of vielbeins (frame fields) e^{α}_{μ}

$$g_{ij} = \eta_{\mu\nu} e^{\mu}{}_{i} e^{\nu}{}_{j} \quad .$$

- The vielbeins result in the same ones of the Gullstrand-Painlevé metric describing a Schwarzschild BH.
- Aim: extend this to NH systems.

5. Hawking radiation in analogue NH system

"Artificial Hawking radiation in Non-Hermitian Parity-Time symmetric systems"

[Stålhammar, JLA, Rødland, Kunst, arXiv:2106.05030].

- Question: are NH systems better suited to display analogue Hawking radiation?
- NH Weyl-like Hamiltonian representing a two-band system linear in momentum.
- Recall analogy between band structures and artificial event horizons in Hermitian systems (Weyl cone/lightcone).

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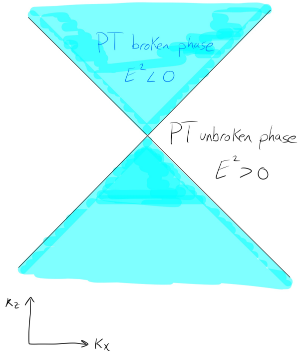
- NH Weyl-like Hamiltonian representing a two-band system linear in momentum.
- Recall analogy between band structures and artificial event horizons in Hermitian systems (Weyl cone/lightcone).
- Weyl point $\rightarrow$ Exceptional points (forming a cone).
- Exceptional points identified as BH horizon.
- Inspired by the computation of Hawking radiation from quantum tunneling à la Parikh-Wilczek, we interpret dissipative behavior as the creation and emission of light-like particle-antiparticle pairs.

5.1. NH model (1/2)

- Our NH (tilted) Weyl-like PT symmetric 2-band model

$$H = k_x\sigma_x + k_y\sigma_y + i(k_z - \kappa k_x)\sigma_z.$$

- Energy levels given by $E_\pm = \pm\sqrt{k_x^2 + k_y^2 - (k_z - \kappa k_x)^2} := \pm\sqrt{D}$.
- Exceptional cone (EC): $D = 0 \to k_z = \kappa k_x \pm \sqrt{k_x^2 + k_y^2}$.



5.1. NH model (2/2)

- Problem! The vielbein formalism to obtain the analogue spacetime metric can only be done for Hermitian systems.
- Trick: We can think of k_z as the eigenvalue of a (Hermitian) Dirac-like operator

$$\hat{k}_z = \kappa k_x \sigma_0 + k_x \sigma_x + k_y \sigma_y .$$

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- The EC gives rise to an analogue metric

$$ds^2 = -(1-\kappa^2)dt^2 + dx^2 + dy^2 + 2\kappa dx dt,$$

resembling the previous metric of a Schwarzschild BH in Gullstrand-Painlevé coordinates if $dy = d\Omega = 0$ and making the identifications

$$x \sim r, \; t \sim T, \; \kappa \sim \sqrt{\frac{2M}{r}}.$$

- We have extended the relation between Hermitian Weyl cones and artificial lightcones in the vicinity of a BH to ECs in NH PT-symmetric systems.

5.2. Spontaneous particle emission

Following the Parikh-Wilczek method:

- Evaluate the Im part of the action on the EC

$$\mathrm{Im}S = 4\pi M|k_z|.$$

- Semi-classical emission rate

$$\Gamma \sim e^{-8\pi M|k_z|},$$

  which is highly suppressed for high $|k_z|$ since $M \gg 1$ (semiclass. limit), meaning that the tunneling process is more likely for $|k_z|$ small.

6. Conclusions & outlook

- Particle emission as dissipation: we relate the spontaneous particle emission with the presence of bulk Fermi states (BFSs) which are dissipative states characteristic of NH systems. In our model they correspond to $D \leq 0$ (interior of the EC).
- Dissipative behavior at $k_z = 0$ can be regarded as spontaneous tunneling of light-like quasi-particles giving the leading order contribution to Hawking radition.
- There is also dissipation for finite $k_z \neq 0$ and perhaps it could be related to massive modes contributing to the total Hawking radiation process.

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- There is also dissipation for finite $k_z \neq 0$ and perhaps it could be related to massive modes contributing to the total Hawking radiation process.
- The role of PT symmetry: broken and unbroken symmetry phases $\rightarrow$ Hermitian/non-Hermitian sectors.
- Investigate the possible connection between the entropy of the analogue black hole and the entanglement entropy of the model.
- Experimental relevance: towards modelling black hole evaporation in an analogue physical system.

THE END

Thank you for your attention!

Questions?

7. Neglecting commutator contributions*

When $\kappa \to \kappa(r)$ we neglected commutator contributions:

$$[H, \kappa(r)] \propto [k_r, \kappa(r)] \,,$$

and

$$[H, k_r] \propto [k_r, \kappa(r)] \,,$$

and evaluates to

$$[k_r, \kappa(r)] \propto \frac{\partial \kappa(r)}{\partial r} \propto \frac{M_0}{r^2 \sqrt{\frac{2M_0}{r}}} .$$

- When moving away from the horizon this goes as $r^{-3/2}$, and thus decays for increasing r.
- At the horizon, the commutator is

$$[k_r, \kappa(r)]|_{r=2M_0} \propto M_0^{-1} . \tag{1}$$

As we assumed $M_0 \gg 1$, this contribution indeed disappears.