Topics in Applied Algebraic Geometry
Homework 1 (Sandra)

due date: Thursday 2017.04.11

Problem 1. Notation as in Lecture 1 (Sandra)

(1) Show that the map $\phi : \mathbb{P}^3_{\mathbb{R}} \to SO_3(\mathbb{R})$ defined as $\phi(a)x = axa^*$ is an isomorphism.

(2) Show that the map

$$\phi : \mathcal{Q}'_{\mathbb{R}} \to SO_3(\mathbb{R}) \times \mathbb{R}^3, (p, q) \mapsto (\phi(q), \frac{pq^*}{qq^*})$$

is an isomorphism.

Problem 2. The Plücker embedding allows us to consider the Grassmannian of lines in $\mathbb{P}^3$ as a quadric hypersurface Q in $\mathbb{P}^5$.

(1) Fix a point $p \in \mathbb{P}^3$ and let X be the set of lines passing through p. Show that X is a plane lying on Q. (Hint: up to a linear change of coordinates, you may assume that p = (0, 0, 0, 1).)

(2) Fix a line $L \subset \mathbb{P}^3$ and let Y be the set of lines meeting L. Show that Y is the intersection of Q with a hyperplane.