Position and Orientation Based Formation Control of Multiple Rigid Bodies with Collision Avoidance and Connectivity Maintenance

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Abstract— This paper addresses the problem of position- and orientation-based formation control of a class of second-order nonlinear multi-agent systems in a 3D workspace with obstacles. More specifically, we design a decentralized control protocol such that each agent achieves a predefined geometric formation with its initial neighbors, while using local information based on a limited sensing radius. The latter implies that the proposed scheme guarantees that the initially connected agents remain always connected. In addition, by introducing certain distance constraints, we guarantee inter-agent collision avoidance as well as collision avoidance with the obstacles and the boundary of the workspace. Finally, simulation results verify the validity of the proposed framework.

I. INTRODUCTION

During the last decades, decentralized control of multi-agent systems has gained a significant amount of attention due to the great variety of its applications, including multi-robot systems, transportation, multi-point surveillance and biological systems. The main focus of multi-agent systems is the design of distributed control protocols in order to achieve global tasks, such as consensus, and at the same time fulfill certain properties, e.g., network connectivity.

A particular multi-agent problem that has been considered in the literature is the formation control problem, where the agents represent robots that aim to form a prescribed geometrical shape, specified by a certain set of desired relative configurations. The main categories of formation control that have been studied in the related literature are ([1]) position-based control, displacement-based control, distance-based control and orientation-based control.

In position-based formation control, the agents control their positions to achieve the desired formation, as prescribed by some desired position offsets with respect to a global coordinate system. When orientation alignment is considered as a control design goal, the problem is known as orientation-based (or bearing-based) formation control. The desired formation is then defined by relative inter-agent orientations. The orientation-based control steers the agents to configurations that achieve desired relative orientation angles. In this work, we aim to design decentralized control protocols

such that both position- and orientation-based formation are achieved.

The literature in position-based formation control is rich, and is traditionally categorized in single or double integrator agent dynamics and directed or undirected communication topologies (see e.g. [2]–[15]). Orientation-based formation control has been addressed in [16]–[19], whereas the authors in [19]–[21] have considered the combination of position- and orientation-based formation.

The dominant case in the related literature of formation control is the two-dimensional one with simple dynamics and point-mass agents. In real applications, however, the engineering systems may have nonlinear 2nd order dynamics, for which due to imperfect modeling the exact model is not a priori known. Other objectives concern connectivity maintenance, collision avoidance between the agents as well as collision avoidance between the agents and potential obstacles of the workspace, which renders the formation control problem a particularly challenging task. According to the authors' best knowledge, the combination of the aforementioned specifications has not been addressed in the related literature.

Motivated by this, we aim to address here the position-based formation control problem with orientation alignment for a team of rigid bodies operating in 3D space, with 2nd order nonlinear dynamics. We propose a decentralized control protocol that guarantees a geometric prescribed position- and orientation-based formation between initially connected agents. The proposed methodology guarantees inter-agent collision avoidance and collision avoidance with the obstacles and the boundary of the workspace. In parallel, connectivity maintenance of the initially connected agents as well as representation singularity avoidance are ensured. In order to deal with the aforementioned specifications, we employ a novel class of potential functions. A special case of correct-by-construction potential functions, namely *navigation functions*, has been introduced in [22] for the single-robot navigation, and has been employed in multi-agent formation control in [23]–[28]. A more general potential function framework has been employed in [5]. The aforementioned works, however, have only addressed the single integrator case, with no straightforward extension to higher-order systems. The authors in [29] deal with the double integrator case, but the goal was only navigation of the agents to specific points.

In our previous work [30], we treated a similar problem by utilizing a Prescribed Performance Control (PPC) scheme instead (for PPC controller design we refer to [31]), while

only guaranteeing collision avoidance between neighboring agents forming a tree, with no obstacles or representation singularity avoidance. The main contribution of this paper is a novel decentralized control protocol scheme that generalizes [30] and solves a wider class of problems of multiple rigid bodies under Lagrangian dynamics with guaranteed collision avoidance among the agents, collision avoidance between agents and obstacles as well as singularity avoidance.

The remainder of the paper is structured as follows. Section II gives the necessary notation. Section III provides the system dynamics and the formal problem statement. Section IV discusses the technical details of the solution and Section V is devoted to a simulation example. Finally, conclusions and future work are discussed in Section VI.

II. NOTATION

The set of positive integers is denoted by $\mathbb{N}$. The real n-coordinate space, with $n \in \mathbb{N}$, is denoted by $\mathbb{R}^n$; $\mathbb{R}^n_{\geq 0}$ and $\mathbb{R}^n_{>0}$ are the sets of real n-vectors with all elements nonnegative and positive, respectively. Given a set S, we denote by $|S|$ its cardinality and by $S^N = S \times \cdots \times S$ its N-fold Cartesian product. The notation $\|x\|$ is used for the Euclidean norm of a vector $x \in \mathbb{R}^n$. Define by $I_n \in \mathbb{R}^{n \times n}, 0_{m \times n} \in \mathbb{R}^{m \times n}$ the identity matrix and the $m \times n$ matrix with all entries zeros, respectively. A matrix $S \in \mathbb{R}^{n \times n}$ is called skew-symmetric if and only if $S^\top = -S$; $\mathcal{B}(c, r) = \{x \in \mathbb{R}^3 : \|x - c\| \leq r\}$ is the 3D sphere of center $c \in \mathbb{R}^3$ and radius $r \in \mathbb{R}_{>0}$ and $\mathring{\mathcal{B}}(c, r)$ its interior. Given a scalar function $y : \mathbb{R}^n \to \mathbb{R}$ and a vector $x \in \mathbb{R}^n$, denote by $\nabla_x y(x) = \frac{\partial y(x)}{\partial x} = [\frac{\partial y(x)}{\partial x_1}, \ldots, \frac{\partial y(x)}{\partial x_n}]^\top \in \mathbb{R}^n$ the gradient of y. The vector connecting the origins of coordinate frames $\{A\}$ and $\{B\}$ expressed in frame $\{C\}$ coordinates in 3D space is denoted by $p^C_{B/A} \in \mathbb{R}^3$. We further denote by $q_{B/A} = [\phi_{B/A}, \theta_{B/A}, \psi_{B/A}]^\tau \in \mathbb{T}$ the Euler angles representing the orientation of frame $\{B\}$ with respect to frame $\{A\}$, with $\mathbb{T} = [-\pi, \pi] \times [-\frac{\pi}{2}, \frac{\pi}{2}] \times [-\pi, \pi]$. The angular velocity of frame $\{B\}$ with respect to $\{A\}$, expressed in frame $\{C\}$ coordinates, is denoted by $\omega^C_{B/A} \in \mathbb{R}^3$. We also use the notation $\mathbb{M} = \mathbb{R}^3 \times \mathbb{T}$. For notational brevity, when a coordinate frame corresponds to an inertial frame of reference $\{0\}$, we will omit its explicit notation (e.g., $p_B = p^0_{B/0}, \omega_B = \omega^0_{B/0}$). All vector and matrix differentiations are derived with respect to the inertial frame $\{0\}$, unless otherwise stated.

III. PROBLEM FORMULATION

A. System Model

Consider a set of N rigid bodies, with $\mathcal{V} = \{1, 2, \ldots, N\}$, $N \geq 2$, operating in a workspace $W \subseteq \mathbb{M}$, with coordinate frames $\{i\}, i \in \mathcal{V}$, attached to their centers of mass. The workspace is assumed to be modeled as a bounded sphere $W = \mathring{\mathcal{B}}(p_w, r_w)$ with center p_w and radius r_w. Without loss of generality, we assume that $p_w = 0_{3 \times 1}$, representing an inertial reference frame $\{0\}$. The subscript w stands for the workspace W. We consider that each agent occupies a sphere $\mathcal{B}(p_i, r_i)$, where $p_i \in \mathbb{R}^3$ is the position of the agent's center of mass and $r_i < r_w$ is the agent's radius. We also denote

by $q_i \in \mathbb{T}, i \in \mathcal{V}$, the Euler angles representing the agents' orientation with respect to $\{0\}$, with $q_i = [\phi_i, \theta_i, \psi_i]^\top$. By defining $x_i \in W, v_i \in \mathbb{R}^6$, with $x_i = [p_i^\top, q_i^\top]^\top, v_i = [\dot{p}_i^\top, \omega_i^\top]^\top$, we model each agent's motion with the 2nd order dynamics:

$$\dot{x}_i = J_i^{-1}(q_i)v_i, \tag{1a}$$
$$M_i(x_i)\dot{v}_i + C_i(x_i, \dot{x}_i)v_i + g_i(x_i) = u_i, \tag{1b}$$

where $J_i : \mathbb{T} \to \mathbb{R}^{6 \times 6}$ is a *Jacobian matrix* that maps the Euler angle rates to v_i, given by

$$J_i(q_i) = \begin{bmatrix} I_3 & 0_{3 \times 3} \\ 0_{3 \times 3} & J_{q_i}(q_i) \end{bmatrix},$$

$$J_{q_i}(q_i) = \begin{bmatrix} 1 & 0 & \sin(\theta_i) \\ 0 & \cos(\phi_i) & -\cos(\theta_i)\sin(\phi_i) \\ 0 & \sin(\phi_i) & \cos(\phi_i)\cos(\theta_i) \end{bmatrix},$$

and $J_i^{-1}(q_i)$ is its matrix inverse. The matrix J_i is singular when $\det(J_i) = \cos(\theta_i) = 0 \Leftrightarrow \theta_i = \pm\frac{\pi}{2}$, which we refer to as *representation singularity*. The proposed controller will guarantee, however, that this is always avoided and thus (1a) is well defined.

Furthermore, $M_i : W \to \mathbb{R}^{6 \times 6}$ is the positive definite *inertia matrix*, $C_i : W \times \mathbb{R}^6 \to \mathbb{R}^{6 \times 6}$ is the *Coriolis matrix*, and $g_i : W \to \mathbb{R}^6$ is the *gravity vector*. We consider that the Coriolis and the inertia vector fields are *unknown*. Finally, $u_i \in \mathbb{R}^6$ is the control input vector representing the 6D generalized *actuation force* acting on agent $i \in \mathcal{V}$. Let us also define the stack vectors $x = [x_1^\top, \ldots, x_N^\top]^\top \in W^N$ and $v = [v_1^\top, \ldots, v_N^\top]^\top \in \mathbb{R}^{6N}$. In addition, the matrices $\dot{M}_i - 2C_i$ are skew-symmetric [32], i.e., $y^\top \left[\dot{M}_i - 2C_i \right] y = 0, \forall y \in \mathbb{R}^6, i \in \mathcal{V}$. For the state measurement of each agent, the following assumption is required.

Assumption 1. (Measurements Assumption) Each agent i can measure its own states $p_i, q_i, \dot{p}_i, v_i, i \in \mathcal{V}$, and has a limited sensing range of: $d_i > \max\{r_i + r_j : i, j \in \mathcal{V}, i \neq j\}$.

Therefore, by defining the neighboring set as $\mathcal{N}_i(x_i) = \{j \in \mathcal{V} : p_j \in \mathcal{B}(p_i, d_i), i \neq j\}$, in view of the aforementioned assumption, agent i knows at each configuration x_i all $p^i_{j/i}, q_{j/i}$ and, since it knows its own p_i, q_i, it can compute all $p_j, q_j, \forall j \in \mathcal{N}_i(x_i), x_i \in \mathcal{W}$. For the neighboring set $\mathcal{N}_i(x_i)$ define also $N_i(x_i) = |\mathcal{N}_i(x_i)|$.

Moreover, we consider that in the given workspace there exist $Z \in \mathbb{N}$ *static obstacles*, with $\mathcal{Z} \triangleq \{1, \ldots, Z\}$, modeled as the spheres $\mathcal{B}(p_{o_z}, r_{o_z})$, with centers and radii $p_{o_z} \in \mathbb{R}^3, r_{o_z} \in \mathbb{R}_{>0}, z \in \mathcal{Z}$, respectively. The geometry of two agents i, j and an obstacle z in the workspace W is depicted in Fig. 1.

Let us define the distances $d_{ij,a} : \mathbb{R}^6 \to \mathbb{R}_{\geq 0}$, $d_{iz,o} : \mathbb{R}^3 \to \mathbb{R}_{\geq 0}$, with: $d_{ij,a}(p_i, p_j) = \|p_i - p_j\|, d_{iz,o}(p_i) = \|p_i - p_{o_z}\|, \forall i, j \in \mathcal{V}, i \neq j, z \in \mathcal{Z}$, as well as the constants $\underline{d}_{ij,a} = r_i + r_j, \underline{d}_{iz,o} = r_i + r_{o_z}$, that represent the minimum distance such that agents i and j and agent i and object z, do not collide, respectively. The subscripts a and o stand for
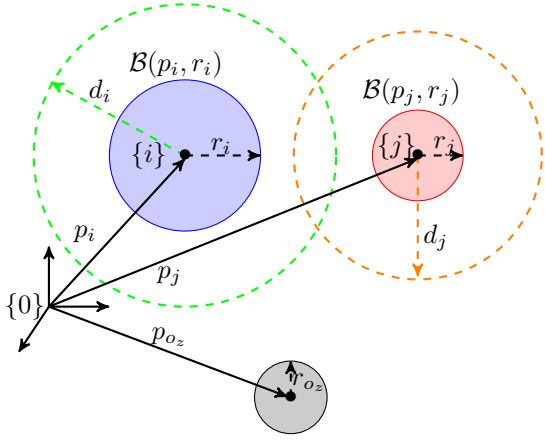
Fig. 1: Illustration of two moving agents $i, j \in \mathcal{V}$ and a static obstacle o_z in the workspace; $\{0\}$ is the inertial frame, $\{i\}, \{j\}$ are the frames attached to the agents' center of mass, $p_i, p_j, p_{o_z} \in \mathbb{R}^3$ are the positions of the center of mass of the agents i, j and the obstacle o_z, respectively, with respect to $\{0\}$; r_i, r_j, r_{o_z} are the radii of the agents i, j and the obstacle o_z, respectively; d_i, d_j with $d_i > d_j$ are the agents' sensing ranges. Note that the agents are not neighbors since $p_j \notin \mathcal{B}(p_i, d_i)$ and $p_i \notin \mathcal{B}(p_j, d_j)$.

agent and *obstacle*, respectively. The following assumption is required, for the feasibility of the problem:

Assumption 2. It holds that

1) $\|p_{o_z} - p_{o_{z'}}\| \geq 2 \max_{i \in \mathcal{V}}\{r_i\} + r_{o_z} + r_{o_{z'}} + \varepsilon_r, \forall z, z' \in \mathcal{Z}$, with $z \neq z'$,
2) $r_w - (\|p_{o_z}\| + r_{o_z}) \geq 2 \max_{i \in \mathcal{V}}\{r_i\} + \varepsilon_r, \forall z \in \mathcal{Z}$,

where ε_r is an arbitrarily small positive constant.

The aforementioned assumption states that there is enough space between the obstacles and the workspace boundary as well as the obstacles themselves for the agents to navigate among them.

B. Problem Statement

Due to the fact that the agents are not dimensionless and their sensing capabilities are limited, the control protocol, except from achieving desired position formation (define it by $p_{ij,\text{des}}$) and desired formation angles (define it by $q_{ij,\text{des}}$) for all neighboring agents $i \in \mathcal{V}, j \in \mathcal{N}_i(x_i(0))$, it should also guarantee for all $t \in \mathbb{R}_{\geq 0}$ that (i) all the agents avoid collision with every other agent; (ii) all the agents avoid collision with all the obstacles; (iii) all the agents avoid collision with the workspace boundary, (iv) all the initial edges are maintained, i.e., connectivity maintenance, and (v) the singularity of the Jacobian matrices J_i is avoided.

Definition 1. (Feasible Formation) Given the initial neighboring sets $\mathcal{N}_i(x_i(0)), i \in \mathcal{V}$, the desired displacements $x_{ij,\text{des}} = [p_{ij,\text{des}}^\top, q_{ij,\text{des}}^\top]^\top$ that characterize a formation configuration, are called *feasible* if $\bigcap_{i \in V}\{x_i \in W : \|x_i - x_j - x_{ij,\text{des}}\| = 0, d_{iz,o}(p_i) > 0, \|p_i\| + r_i < r_w, \forall z \in \mathcal{Z}, j \in \mathcal{N}_i(x_i(0))\}\} \neq \emptyset$.

Formally, the control problem under the aforementioned constraints is formulated as follows:

Problem 1. Consider N agents governed by the dynamics (1) and operating in a workspace W with Z spherical obstacles, with:

- $v_i(0) = 0_{6 \times 1}, \forall i \in \mathcal{V}$,
- $-\frac{\pi}{2} < -\bar{\theta} \leq \theta_i(0) \leq \bar{\theta} < \frac{\pi}{2}, \forall i \in \mathcal{V}$,
- $\|p_i(0) - p_j(0)\| > \underline{d}_{ij,a}, \forall i, j \in \mathcal{V}, i \neq j$,
- $\|p_i(0) - p_{o_z}(0)\| > \underline{d}_{iz,o}, \forall i \in \mathcal{V}, z \in \mathcal{Z}$,

i.e., singularity- and collision- free configurations at $t = 0$, where $\bar{\theta}$ is an arbitrary constant in the open set $(0, \frac{\pi}{2})$. Then, given a nonempty initial set $\mathcal{N}_i(x_i(0)) \neq \emptyset$, *feasible* inter-agent displacements $p_{ij,\text{des}}, q_{ij,\text{des}}, \forall i \in \mathcal{V}, j \in \mathcal{N}_i(x_i(0))$, such that $\underline{d}_{ij,a} < d_{ij,a}(p_i, p_j) < d_i, \forall(p_i, p_j) \in \{(p_i, p_j) \in \mathbb{R}^6 : \|x_i - x_j - x_{ij,\text{des}}\| = 0\}$, design decentralized control laws u_i, such that for every $i \in \mathcal{V}$ the following hold:

1) $\lim_{t \to \infty} \|x_i(t) - x_j(t) - x_{ij,\text{des}}\| = 0, \forall j \in \mathcal{N}_i(x_i(0))$,

2) $\|p_i(t) - p_j(t)\| > \underline{d}_{ij,a}, \forall j \in \mathcal{V}\backslash\{i\}, t \in \mathbb{R}_{\geq 0}$,

3) $\|p_i(t) - p_{o_z}(t)\| > \underline{d}_{iz,o}, \forall z \in \mathcal{Z}, t \in \mathbb{R}_{\geq 0}$,

4) $\|p_i(t)\| + r_i < r_w, \forall t \in \mathbb{R}_{\geq 0}$.

5) $\|p_i(t) - p_j(t)\| < d_i, \forall j \in \mathcal{N}_i(x_i(0)), t \in \mathbb{R}_{\geq 0}$,

6) $-\frac{\pi}{2} < -\bar{\theta} \leq \theta_i(t) \leq \bar{\theta} < \frac{\pi}{2}, \forall t \in \mathbb{R}_{\geq 0}$,

The aforementioned specifications imply the following: 1) stands for formation control (both position and orientation); 2) stands for inter-agent collision avoidance; 3) stands for collision avoidance between the agents and the obstacles; 4) stands for collision avoidance between the agents and the boundary; 5) stands for connectivity maintenance of the initially connected agents and finally, 6) stands for the representation of singularities avoidance.

IV. PROBLEM SOLUTION

In this section, a systematic solution to Problem 1 is introduced. In particular, the following analysis is performed: 1) The form of the proposed potential function along with its components is described in Section IV-A. 2) The proposed decentralized controllers that guarantee the satisfaction of all the control specifications are provided in Section IV-B. The required stability analysis is presented subsequently.

A. Decentralized Potential Functions

In order to solve the formation control problem with the collision- and singularity- avoidance as well as connectivity maintenance, we use a *decentralized potential function* for each agent $i \in \mathcal{V}$ as $\varphi_i(x)$, with the following properties:

(i) The function $\varphi_i(x)$, is not defined, i.e., $\varphi_i(x) = \infty$, $\forall i \in \mathcal{V}$, when a collision or a connectivity break occurs,

(ii) The critical points of φ_i where the vector field $\nabla_{x_i}\varphi_i(x)$ vanishes, i.e., the points where $\nabla_{x_i}\varphi_i(x) = 0$, consist of the goal configurations and a set of

configurations whose region of attraction (by following the vector field curves) is a set of measure zero.

More specifically, $\varphi_i(x)$ is a function of two main terms, a *goal function* $\gamma_i(x)$, that should vanish at the desired configuration, and an *obstacle function*, $\beta_i(x)$ is a function that encodes inter-agent collisions, collisions between the agents and the obstacle boundary/undesired regions of interest, connectivity losses between initially connected agents and singularities of the Jacobian matrix $J_i(q_i)$; $\beta_i(x)$ vanishes when one or more of the above situation occurs. Next, we provide a construction of the goal and obstacle terms. However, the construction of φ_i is out of the scope of this paper. Examples can be found in [33]–[35][1].

1) $\gamma_i(x)$ - Goal Function: The function $\gamma_i : W^N \to \mathbb{R}_{\geq 0}$ encodes the control objective of agent i, which is to achieve position and orientation formation with its neighboring agents. With that in mind, a reasonable choice of $\gamma_i(x)$ is:

$$
\begin{aligned}
\gamma_i(x) &= \sum_{j \in \mathcal{N}_i(0)} \left\{ \gamma_{ij,p}(p_i, p_j) + \gamma_{ij,q}(q_i, q_j) \right\}, \\
&= \sum_{j \in \mathcal{N}_i(0)} \gamma_{ij,x}(x_i, x_j), \quad (2)
\end{aligned}
$$

where $\gamma_{ij,p}(p_i, p_j) = \|p_i - p_j - p_{ij,\text{des}}\|^2$, $\gamma_{ij,q}(q_i, q_j) = \|q_i - q_j - q_{ij,\text{des}}\|^2$, $\gamma_{ij,x}(x_i, x_j) = \|x_i - x_j - x_{ij,\text{des}}\|^2$. The function $\gamma_i(x)$ reaches its unique global minimum when both $p_i - p_j = d_{ij,\text{des}}$ and $q_i - q_j = q_{ij,\text{des}}, \forall i \in \mathcal{V}$, i.e., when both formation and orientation alignment are achieved between all the neighboring agents.

2) $\beta_i(x)$ - Obstacle Function: The function $\beta_i(x) : W^N \to \mathbb{R}$, encodes all inter-agent collisions, collisions between the agents and obstacles, collisions with the boundary of the workspace, connectivity between initially connected agents and singularities of the Jacobian matrix $J_i(x_i)$. First, for each agent i, we define the functions $\eta_{ij,a} : \mathbb{R} \to \mathbb{R}_{\geq 0}, \eta_{iz,o} : \mathbb{R} \to \mathbb{R}_{\geq 0}, \eta_{ij,c} : \mathbb{R} \to \mathbb{R}_{\geq 0}$ where:

$$
\begin{aligned}
\eta_{ij,a}(d_{ij,a}) &= d_{ij,a}^2 - \underline{d}_{ij,a}^2, \\
\eta_{iz,o}(d_{iz,o}) &= d_{iz,o}^2 - \underline{d}_{iz,o}^2, \\
\eta_{ij,c}(d_{ij,a}) &= d_i^2 - d_{ij,a}^2.
\end{aligned}
$$

The subscripts j and z correspond to agent $j \in \mathcal{V}\setminus\{i\}$ and obstacle $z \in \mathcal{Z}$, respectively, whereas the subscript c stands for *connectivity*. Let us also define the functions $b_{ij,a} : \mathbb{R}_{\geq 0} \to [0,1]$, $b_{iz,o} : \mathbb{R}_{\geq 0} \to [0,1]$, $b_{ij,c} : \mathbb{R}_{\geq 0} \to [0,1]$, $b_{iw} : \mathbb{R}_{\geq 0} \to \mathbb{R}$, $b_{J_i} : [-\frac{\pi}{2}, \frac{\pi}{2}] \to [0,1]$, where:

$$
b_{ij,a}(x) = \begin{cases} \phi_{i,a}(x), & 0 \leq x < d_i^2 - \underline{d}_{ij,a}^2, \\ 1, & d_i^2 - \underline{d}_{ij,a}^2 \leq x, \end{cases}
$$

$$
b_{iz,o}(x) = \begin{cases} \phi_{i,o}(x), & 0 \leq x < d_i^2 - \underline{d}_{iz,o}^2, \\ 1, & d_i^2 - \underline{d}_{iz,o}^2 \leq x, \end{cases}
$$

[1]For instance, we can choose $\varphi_i = \frac{1}{1-\phi_i}$, where ϕ_i is the navigation function used in [34], [35].

$$
b_{ij,c}(x) = \begin{cases} 0 & x < 0, \\ \phi_{i,c}(x), & 0 \leq x < d_i^2 - \underline{d}_{ij,a}^2, \\ 1, & d_i^2 - \underline{d}_{ij,a}^2 \leq x, \end{cases}
$$

$$
b_{iw}(x) = \left[1 - \frac{x}{(r_w - r_i)^2} \right]^2,
$$

$$
b_{J_i}(x) = \cos^2(x).
$$

The functions $\phi_{i,a}, \phi_{i,o}, \phi_{i,c}$ are *increasing polynomials*, appropriately selected to guarantee that the functions $b_{ij,a}, b_{iz,o}, b_{ij,c}$, respectively, are twice continuously differentiable everywhere, with $\phi_{i,a}(0) = \phi_{i,o}(0) = \phi_{i,c}(0) = 0$, $\forall i \in \mathcal{V}$. An example of a function $b_{ij,a}$ along with its derivatives $\frac{\partial b_{ij,a}(x)}{\partial x}, \frac{\partial^2 b_{ij,a}(x)}{\partial x^2}$ is depicted in Fig. 2. The functions $b_{iz,o}, b_{ij,c}$ have an identical behavior. We can now choose the function $\beta_i : W^N \to [0,1]$ as the following product for every $i \in \mathcal{V}$:

$$
\beta_i(x) = b_{iw}(\|p_i\|^2) b_{J_i}(\theta_i) \left[\prod_{j \in \mathcal{V}\setminus\{i\}} b_{ij,a}(\eta_{ij,a}) \right]
$$
$$
\left[\prod_{z \in \mathcal{Z}} b_{iz,o}(\eta_{iz,o}) \right] \left[\prod_{j \in \mathcal{N}_i(0)} b_{ij,c}(\eta_{ij,c}) \right]. \quad (3)
$$

The functions $b_{ij,a}(\eta_{ij,a}), b_{iz,o}(\eta_{iz,o}), b_{ij,c}(\eta_{ij,c})$ correspond to inter-agent collision, collision with obstacles and connectivity maintenance, respectively, for agent $i \in \mathcal{V}$, while the functions $b_{iw}(\|p_i\|^2), b_{J_i}(\theta_i)$ correspond to collision with the workspace boundary and representation singularities. Each of these terms becomes zero when there is a collision, a connectivity break or a representation singularity. Note that all the aforementioned functions use only local information depending on the sensing range d_i of agent i.

Remark 1. Note that the choice of the functions γ_i and β_i from (2) and (3), respectively, renders φ_i from (2), (3) to be a function only of the neighboring states of x_i i.e., decentralized. The function γ_i depends on the initial neighboring set $\mathcal{N}_i(x_i(0))$ since the formation requirements need to be achieved between the agents that belong to the initial neighboring set $\mathcal{N}_i(x_i(0))$. Furthermore, the function β_i depends on the time-varying neighboring set $\mathcal{N}_i(x_i)$, since in order to capture the collision avoidance goals, the neighboring sets $\mathcal{N}_i(x_i)$ need to be updated with new potential neighbors.

With the introduced notation, the properties of the functions φ_i are:

(i) $\beta_i(x) \to 0 \Leftrightarrow \varphi_i(x) \to \infty, \forall i \in \mathcal{V}$,
(ii) $\nabla_{x_i}\varphi_i(x)|_{x_i=x_i^\star} = 0, \forall x_i^\star \in \mathcal{W}$ s.t. $\gamma_i(x_i^\star) = 0$ and the regions of attraction of the points $\{x \in \mathcal{W}^N : \nabla_{x_i}\varphi_i(x)|_{x_i=\widetilde{x}_i} = 0, \gamma_i(\widetilde{x}_i) \neq 0\}, i \in \mathcal{V}$, are sets of measure zero.

B. Control Design and Stability Analysis

Next, we design bounded controllers u_i such that all the specifications of Problem 1 are met, according to the
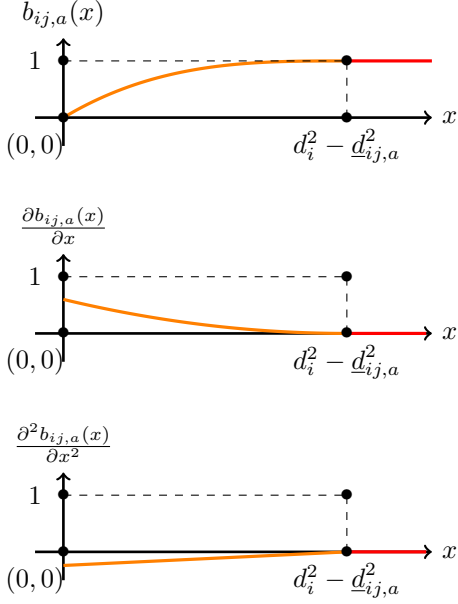
Fig. 2: The functions $b_{ij,a}(x)$ (top), $\frac{\partial b_{ij,a}(x)}{\partial x}$ (middle) and $\frac{\partial^2 b_{ij,a}(x)}{\partial x^2}$ (bottom), for $d_i^2 - \underline{d}_{ij,a}^2 = 5$ and $\phi_{i,a}(x) = 0.008x^3 - 0.12x^2 + 0.6x$.

following theorem, which summarizes the main results of this work.

Theorem 1. *The decentralized control law* $u_i : \mathbb{D}_i \times \mathbb{R}^6 \to \mathbb{R}^6$, *with* $\mathbb{D}_i = \{x \in \mathcal{W}^N : \beta_i(x) > 0\}$ *and*

$$u_i(x, v_i) = -\widetilde{k}_i v_i + g_i(x_i) - \left[J_i^{-1}(x_i) \right]^\top \nabla_{x_i} \varphi_i(x) \\ - \sum_{j \in \mathcal{N}_i(x_i)} \nabla_{x_i} \varphi_j(x) \qquad (4)$$

for each agent $i \in \mathcal{V}$, *with control gain* $\widetilde{k}_i > 0$, *brings agent* i *to its desired configuration from almost all initial conditions, while ensuring* $\beta_i > 0, \forall i \in \mathcal{V}$ *and the boundedness of all closed loop signals, providing, thus, a solution to Problem 1.*

Proof. Consider the nonnegative Lyapunov-like function for the system (1) $L : \mathbb{D}_1 \times \cdots \times \mathbb{D}_N \times \mathbb{R}^{6N}$, with:

$$L(x, v) = \sum_{i \in \mathcal{V}} \left\{ \varphi_i(x) + \frac{1}{2} v_i^\top M_i(x_i) v_i \right\}, \qquad (5)$$

Since the system configuration at $t = 0$ is singularity- and collision-free, the functions β_i are strictly positive at $t = 0, \forall i \in \mathcal{V}$. Furthermore, $v_i(0) = 0_{6 \times 1}, \forall i \in \mathcal{V}$. Thus, L is initially bounded, i.e., there exists a positive and finite constant M such that

$$L_0 \triangleq L(x(0), v(0)) \leq M. \qquad (6)$$

By differentiating (5) with respect to time, substituting the dynamics (1), using the skew-symmetry of $\dot{M}_i - 2C_i, \forall i \in \mathcal{V}$ and using the fact that $\sum_{i \in \mathcal{V}} ([\nabla_{x_i} \varphi_i(x)]^\top \dot{x}_i + \sum_{j \in \mathcal{N}_i(x_i)} [\nabla_{x_j} \varphi_i(x)]^\top \dot{x}_j) =$
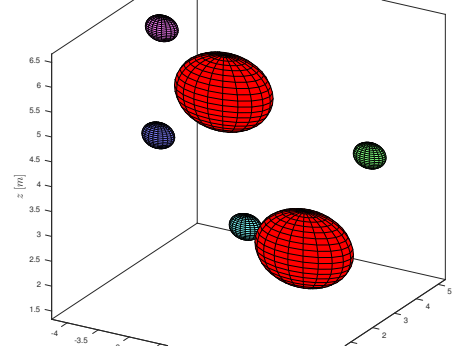

Fig. 3: The initial workspace of the simulated scenario ($t = 0$). Agent 1 (with blue), agent 2 (with green), agent 3 (with cyan) and agent 4 (with purple) and two obstacles (with red).

$\sum_{i \in \mathcal{V}} ([\nabla_{x_i} \varphi_i(x)]^\top + \sum_{j \in \mathcal{N}_i(x_i)} [\nabla_{x_i} \varphi_j(x)]^\top) \dot{x}_i$, we obtain:

$$\dot{L} = \sum_{i \in \mathcal{V}} v_i^\top \left[\left(J_i^{-1}(x_i) \right)^\top \nabla_{x_i} \varphi_i(x) + u_i - g_i(x_i) \right. \\ \left. + \sum_{j \in \mathcal{N}_i(x_i)} [\nabla_{x_i} \varphi_j(x)] \right],$$

which, by substituting the control law (4), becomes:

$$\dot{L} \leq -\sum_{i \in \mathcal{V}} \widetilde{k}_i \|v_i\|^2.$$

Therefore, L is non-increasing and hence, in view of (6), we conclude that

$$L(x(t), v(t)) \leq L_0 \leq M, \forall t \in \mathbb{R}_{\geq 0},$$

and the boundedness of all the terms $x(t)$, $v(t)$, $\varphi_i(x(t)), \forall t \in \mathbb{R}_{\geq 0}$. By invoking the properties of $\varphi_i(x)$, we also conclude that $\beta_i(x(t)) > 0, \forall t \in \mathbb{R}_{\geq 0}$ and hence, inter-agent collisions, collisions between the agents and the obstacles/workspace boundary as well as connectivity losses and singularities, are avoided.

Moreover, by invoking LaSalle's Invariance Principle, the state of the system converges to the largest invariant set contained in the set:

$$S = \left\{ x, v : \dot{L}(x, v) = 0 \right\} = \left\{ x, v : v_i = 0_{6 \times 1}, \forall i \in \mathcal{V} \right\}.$$

For the subset S to be invariant we require $\dot{v}_i = 0_{6 \times 1}$, from which, the closed-loop system implies $\left[J_i^{-1}(x_i) \right]^\top \nabla_{x_i} \varphi_i(x) = 0_{6 \times 1}$. Note that, since $\beta_i(x(t)) > 0, \forall t \in \mathbb{R}_{\geq 0}$, $J(x_i(t))$ is always nonsingular. Therefore, we conclude that the closed loop system will converge to the configuration where $\nabla_{x_i} \varphi_i(x) = 0_{6 \times 1}, \forall i \in \mathcal{V}$. According to the inherent properties of the functions $\varphi_i(x)$, this will happen from all initial conditions except for a set of measure zero, i.e., almost all initial conditions [35], [36]. Moreover, it can be shown that the control laws (4) stay bounded $\forall t \in \mathbb{R}_{\geq 0}, i \in \mathcal{V}$. $\qquad \square$

Remark 2. Note that the design of the obstacle functions (3) renders the control laws (4) decentralized, in the sense
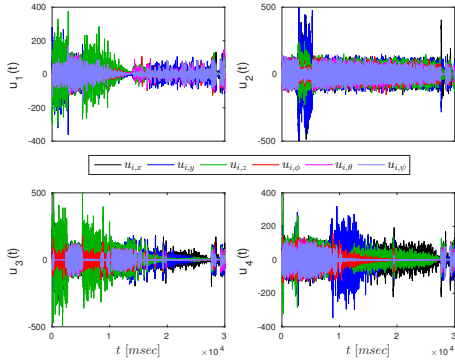
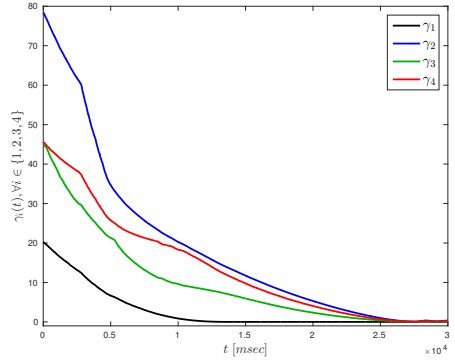Fig. 4: The resulting control inputs $u_i, \forall i \in \{1, \cdots, 4\}$.

Fig. 5: The evolution of the goal functions $\gamma_i, \forall i \in \{1, \cdots, 4\}$, which are shown to converge to zero.
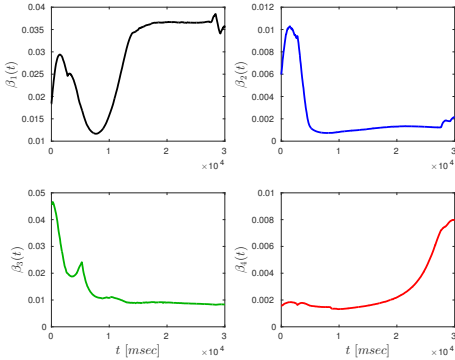

Fig. 6: The evolution of the obstacle functions $\beta_i, \forall i \in \{1, \cdots, 4\}$, which are shown to be always positive.

that each agent uses only local information with respect to its neighboring agents, according to its limited sensing radius. Each agent can obtain the necessary information to cancel the term $\sum_{j \in \mathcal{N}_i(x_i)} \nabla_{x_i} \varphi_j(x)$ from its neighboring agents.

V. SIMULATION RESULTS

To demonstrate the efficiency of the proposed control protocol, we consider a simulation example with $N = 4$, $\mathcal{V} = \{1, 2, 3, 4\}$ spherical agents of the form (1), with $r_i = 0.25$m and $d_i = 5$m,$\forall i \in \{1, \ldots, 4\}$. The initial conditions are set to $p_1(0) = [-3, 0, 5]^\top$ m, $p_2(0) = [-1, 4, 4]^\top$ m, $p_3(0) = [-3, 4, 2]^\top$ m, $p_4(0) = [-4, 3, 6]^\top$ m, $q_1(0) = q_2(0) = q_3(0) = q_4(0) =$
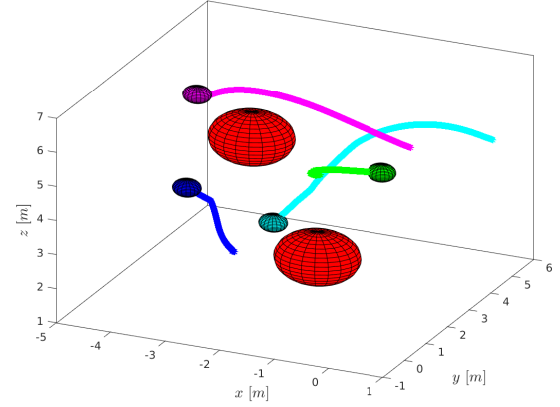

Fig. 7: The motion of the agents in the workspace for $t \in [0, 30]$s.

$[0, 0, 0]^\top$ r, which imply the initial neighboring sets $\mathcal{N}_1(0) = \{2\}, \mathcal{N}_2(0) = \{1, 3, 4\}$, $\mathcal{N}_3(0) = \{2, 4\}$, $\mathcal{N}_4(0) = \{2, 3\}$. The desired formation is defined by the feasible displacements $p_{12,\mathrm{des}} = -p_{21,\mathrm{des}} = [-1, -1, -2]^\top$ m, $p_{23,\mathrm{des}} = -p_{32,\mathrm{des}} = [-2, -3, 0]^\top$ m, $p_{24,\mathrm{des}} = -p_{42,\mathrm{des}} = [-1, -2, 0]^\top$ m, $p_{34,\mathrm{des}} = -p_{43,\mathrm{des}} = [1, 1, 0]^\top$ m, $q_{12,\mathrm{des}} = -q_{21,\mathrm{des}} = [-\frac{\pi}{4}, 0, -\frac{\pi}{4}]^\top$ r, $q_{23,\mathrm{des}} = -q_{32,\mathrm{des}} = [-\frac{\pi}{12}, 0, 0]^\top$ m, $q_{24,\mathrm{des}} = -q_{42,\mathrm{des}} = [-\frac{\pi}{8}, 0, 0]^\top$ r, $q_{34,\mathrm{des}} = -q_{43,\mathrm{des}} = [\frac{5\pi}{24}, 0, 0]^\top$ r. We consider a workspace of radius $r_w = 10$m containing two spherical static obstacles at $p_{o_1} = [-3, 3, 5]^\top$m, $p_{o_2} = [-1, 1, 3]^\top$m with radii $r_{o_1} = r_{o_2} = 0.75$m. An illustration of the workspace with the agents at $t = 0$ is given in Fig. 3. The simulation results are depicted in Fig. 5-7 for $t \in [0, 30]$s. In particular, Fig. 5 shows the evolution of the goal functions $\gamma_i, \forall i \in \{1, \cdots, 4\}$, which are decreasing to zero, whereas Fig. 6 depicts the obstacle functions $\beta_i, \forall i \in \{1, \cdots, 4\}$, which stay always positive. Furthermore, the control inputs are shown in Fig. 4. Finally, the navigation of the agents in the workspace is pictured in Fig. 7. As proven in the theoretical analysis, the formation is successfully achieved and all the specifications of Problem 1 are met.

VI. CONCLUSIONS AND FUTURE WORK

In this work we proposed a potential function- based decentralized control protocol for multi-agent systems which guarantees formation control with inter-agent collision avoidance, collision avoidance between the agents and the obstacles/workspace boundary, connectivity maintenance as well as singularity avoidance of multiple rigid bodies. Simulation results have verified the validity of the proposed approach. Future efforts will be devoted towards developing global results as well as real-time experiments.

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